$\boxed{\text{Gabarito prova 2}}$

1) $$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}3 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

$(3-\lambda)(1-\lambda)+1=0$ $\quad \lambda^2-4\lambda+4=0$

$\boxed{\lambda=2}$ JORDAN FORM

$$\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}\begin{pmatrix}2 & 1\\ 0 & 2\end{pmatrix} = \begin{pmatrix}3 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}$$

$\boxed{\begin{matrix}2\alpha\\ 3\alpha-\gamma\end{matrix}}$ $\boxed{\begin{matrix}\alpha+2\beta\\ 3\beta-\delta\end{matrix}}$ $\quad \alpha=\gamma=1 \quad \beta=0 \quad \delta=-1$

$$\boxed{S=\begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix}}$$

sol $\exp[Mt]\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}$

$$e^{2t}\, S\begin{pmatrix}1 & t\\ 0 & 1\end{pmatrix}S^{-1}\begin{bmatrix}1\\ 0\end{bmatrix} = e^{2t}\begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix}\begin{pmatrix}1 & t\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$$
$$= e^{2t}\begin{pmatrix}1 & t\\ 1 & t-1\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix} = e^{2t}\begin{pmatrix}1+t\\ t\end{pmatrix}$$

$$\boxed{x_1(t) = (t+1)\,e^{2t} \qquad x_2(t) = t\,e^{2t}}$$

2) $$\begin{bmatrix}\ddot{x}(t)\\ \dot{x}(t)\end{bmatrix} = \begin{bmatrix}2 & -2\\ 1 & 0\end{bmatrix}\begin{bmatrix}\dot{x}(t)\\ x(t)\end{bmatrix} \qquad \begin{bmatrix}\dot{x}(t)\\ x(t)\end{bmatrix} = e^{Mt}\begin{bmatrix}\dot{x}(0)\\ x(0)\end{bmatrix}$$

$(2-\lambda)(-\lambda)+2=0$ $\quad \lambda^2-2\lambda+2=0$ $\quad \boxed{\lambda = 1\pm i}$

$$\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}\begin{pmatrix}1+i & 0\\ 0 & 1-i\end{pmatrix} = \begin{pmatrix}2 & -2\\ 1 & 0\end{pmatrix}\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}$$

$(1+i)\alpha = 2\alpha-2\gamma$ $\quad \boxed{\begin{matrix}\alpha=2\\ \beta=2\end{matrix}}$ $\quad (1+i)2 = 4-2\gamma$ $\quad 1+i = 2-\gamma$ $\quad \boxed{\begin{matrix}\gamma=1-i\\ \delta=1+i\end{matrix}}$

$(1-i)\beta = 2\beta-2\delta$ $\quad (1-i)2 = 4-2\delta$ $\quad 1-i = 2-\delta$

$$S=\begin{pmatrix}2 & 2\\ 1-i & 1+i\end{pmatrix} \qquad S^{-1} = \frac{1}{4i}\begin{pmatrix}1+i & -2\\ i-1 & 2\end{pmatrix}$$

$2(1+i)-2(1-i) = 4i$

$$\begin{pmatrix}2 & 2\\ 1-i & 1+i\end{pmatrix} e^{t}\begin{pmatrix}e^{it} & 0\\ 0 & e^{-it}\end{pmatrix}\frac{1}{4i}\begin{pmatrix}1+i & -2\\ i-1 & 2\end{pmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$$
$$\frac{e^{t}}{4i}\begin{bmatrix}2e^{it} & 2e^{-it}\\ (1-i)e^{it} & (1+i)e^{-it}\end{bmatrix}\begin{bmatrix}1+i\\ i-1\end{bmatrix} = \frac{e^{t}}{4i}\begin{bmatrix}2(1+i)e^{it}+2e^{-it}(i-1)\\ 2e^{it}-2e^{it}\end{bmatrix}$$

$\boxed{x(t) = e^{t}\,\mathrm{sen}\,t}$ $\qquad \dot{x}(t) = e^{t}(\mathrm{sen}\,t+\cos t)$ $\qquad \begin{matrix}x(0)=0\\ \dot{x}(0)=1\end{matrix}$

3) $$H=\begin{pmatrix}2 & 1-i\\ 1+i & 4\end{pmatrix} \qquad (2-\lambda)(4-\lambda)-2=0$$

$\lambda^2-6\lambda+6=0$ $\qquad \boxed{\lambda_{1/2} = 3\pm\sqrt{3}}$

$$\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}\begin{pmatrix}3+\sqrt{3} & 0\\ 0 & 3-\sqrt{3}\end{pmatrix} = \begin{pmatrix}2 & 1-i\\ 1+i & 4\end{pmatrix}\begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix}$$

$\alpha(3+\sqrt{3}) = 2\alpha + \delta(1-i)$ $\qquad$ $\alpha(1+\sqrt{3}) = \delta(1-i)$

$\beta(3-\sqrt{3}) = 2\beta + (1-i)\delta$ $\qquad$ $\beta(1-\sqrt{3}) = \delta(1-i)$

$\alpha = 1 \qquad (1+i)(1+\sqrt{3}) = 2\delta$

$\beta = 1 \qquad (1+i)(1-\sqrt{3}) = 2\delta$

$$\boxed{\begin{bmatrix} 1 \\ \frac{(1+i)(1+\sqrt{3})}{2} \end{bmatrix} (3+\sqrt{3}) \qquad \begin{bmatrix} 1 \\ \frac{(1-i)(1+\sqrt{3})}{2} \end{bmatrix} (3-\sqrt{3})}$$

$$\begin{pmatrix} 2 & 1-i \\ 1+i & 4 \end{pmatrix} \begin{bmatrix} 1 \\ \frac{(1+i)(1+\sqrt{3})}{2} \end{bmatrix} = \begin{bmatrix} 2+1+\sqrt{3} \\ (1+i)(1+2+2\sqrt{3}) \end{bmatrix}$$

$$= \begin{bmatrix} 3+\sqrt{3} \\ (1+i)(3+2\sqrt{3}) \end{bmatrix} = (3+\sqrt{3}) \begin{bmatrix} 1 \\ \frac{(1+i)(1+\sqrt{3})}{2} \end{bmatrix} \quad \checkmark$$

$$\begin{pmatrix} 2 & 1-i \\ 1+i & 4 \end{pmatrix} \begin{bmatrix} 1 \\ \frac{(1+i)(1-\sqrt{3})}{2} \end{bmatrix} = \begin{bmatrix} 2+1-\sqrt{3} \\ (1+i)(3-2\sqrt{3}) \end{bmatrix} = (3-\sqrt{3}) \begin{bmatrix} 1 \\ \frac{(1+i)(1-\sqrt{3})}{2} \end{bmatrix} \quad \checkmark$$

4) $A = \begin{pmatrix} a & 1 \\ 2 & 0 \end{pmatrix} \qquad B = D = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} b & 1 \\ 0 & 2 \end{pmatrix}$

$A^{-1} = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 2 & -a \end{pmatrix} \qquad B^{-1} = D^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} \qquad C^{-1} = \frac{1}{2b} \begin{pmatrix} 2 & -1 \\ 0 & b \end{pmatrix}$

$\tilde{A} = (A-C)^{-1}$

$\boxed{\tilde{B} = -\tilde{A}}$

$$\tilde{A} = \begin{pmatrix} a-b & 0 \\ 2 & -2 \end{pmatrix}^{-1} = \frac{1}{-2(a-b)} \begin{bmatrix} -2 & 0 \\ -2 & a-b \end{bmatrix}$$

$$\boxed{\tilde{A} = \frac{1}{2(a-b)} \begin{bmatrix} 2 & 0 \\ 2 & b-a \end{bmatrix}}$$

$\tilde{C} = (B - AC^{-1}D)^{-1}$

$\tilde{D} = -\tilde{C}AC^{-1}$

$$AC^{-1} = \begin{pmatrix} a & 1 \\ 2 & 0 \end{pmatrix} \frac{1}{2b} \begin{pmatrix} 2 & -1 \\ 0 & b \end{pmatrix} = \boxed{\frac{1}{2b} \begin{pmatrix} 2a & b-a \\ 4 & -2 \end{pmatrix}}$$

$$\tilde{C} = \left[\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} - \frac{1}{2b} \begin{pmatrix} 2a & b-a \\ 4 & -2 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \right]^{-1} = 2b \left[\begin{pmatrix} 4b & 0 \\ 2b & 2b \end{pmatrix} - \begin{pmatrix} 4a+b-a & b-a \\ 6 & -2 \end{pmatrix} \right]^{-1}$$

$$= 2b \begin{pmatrix} 3(b-a) & a-b \\ 2b-6 & 2b+2 \end{pmatrix}^{-1} = \frac{\cancel{2b}}{4\,\cancel{8b}(b-a)} \begin{bmatrix} 2(b+1) & b-a \\ 6-2b & 3(b-a) \end{bmatrix} = \tilde{C}$$

det: $6(b-a)(b+1) - (a-b)\,2(b-3)$

$(b-a)[6b + \cancel{6} + 2b - \cancel{6}] = 8b(b-a)$

$$\tilde{D} = \frac{1}{4(a-b)} \begin{pmatrix} 2(1+a) & b-a \\ 2(3-a) & a-b \end{pmatrix}$$

$$\frac{1}{4(a-b)} \begin{bmatrix} 4 & 0 & -4 & 0 \\ 4 & 2(b-a) & -4 & 2(a-b) \\ -2(1+b) & a-b & 2(1+a) & b-a \\ 2(b-3) & 3(a-b) & 2(3-a) & a-b \end{bmatrix}$$

$a = b = 1$ $\cancel{M^{-1}}$

$a = 1$
$b = 2$

$$M^{-1} = \frac{1}{4}\begin{pmatrix} -4 & 0 & 4 & 0 \\ -4 & -2 & 4 & 2 \\ 6 & 1 & -4 & -1 \\ 2 & 3 & -4 & 1 \end{pmatrix}$$

$a = 0$
$b = 1$

$$M^{-1} = \frac{1}{4}\begin{pmatrix} -4 & 0 & 4 & 0 \\ -4 & -2 & 4 & 2 \\ 4 & 1 & -2 & -1 \\ 4 & 3 & -6 & 1 \end{pmatrix}$$