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EX. 1

$$2\begin{pmatrix}1\\2\\5\end{pmatrix}+2\begin{pmatrix}1\\5/2\\4\end{pmatrix}+3\begin{pmatrix}2\\11/3\\4\end{pmatrix}=\begin{pmatrix}10\\20\\30\end{pmatrix}=\alpha\begin{pmatrix}1\\3\\1\end{pmatrix}+\beta\begin{pmatrix}2\\1\\2\end{pmatrix}+\gamma\begin{pmatrix}1\\1\\3\end{pmatrix}$$

$$\left(\begin{array}{ccc|c}1&2&1&10\\3&1&1&20\\1&2&3&30\end{array}\right)$$

$$\begin{pmatrix}1&0&0\\a&1&0\\b&c&1\end{pmatrix}\begin{pmatrix}1&2&1\\3&1&1\\1&2&3\end{pmatrix}=\begin{pmatrix}1&2&1\\0&x&x\\0&0&x\end{pmatrix}$$

$$a+3=0 \qquad \boxed{a=-3}$$

$$b+3c+1=0 \to 2b+6c+2=0$$

$$2b+c+2=0 \qquad 2b+c+2=0$$

$$\boxed{c=0} \qquad \boxed{b=-1}$$

$$\left(\begin{array}{ccc|ccc|c}1&0&0&1&2&1&10\\-3&1&0&3&1&1&20\\-1&0&1&1&2&3&30\end{array}\right)$$

$$\left(\begin{array}{ccc|c}1&2&1&10\\0&-5&-2&-10\\0&0&2&20\end{array}\right)$$

$\gamma=10$

$-5\beta=2\gamma-10 \to \beta=-2$

$\alpha=-2\beta-\gamma+10=4$

RES: $(4,-2,10)$

EX. 2

$$\vec{u}_1=\begin{pmatrix}1\\3\\1\end{pmatrix}$$

$$\vec{u}_2=\begin{pmatrix}2\\1\\2\end{pmatrix}-\frac{\begin{pmatrix}1\\3\\1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\2\end{pmatrix}}{1+9+1}\begin{pmatrix}1\\3\\1\end{pmatrix}=\begin{pmatrix}2\\1\\2\end{pmatrix}-\frac{7}{11}\begin{pmatrix}1\\3\\1\end{pmatrix}$$

$$=\frac{1}{11}\begin{pmatrix}22-7\\11-21\\22-7\end{pmatrix}=\cancel{\frac{5}{11}}\begin{pmatrix}3\\-2\\3\end{pmatrix}$$

$$\vec{u}_2=\begin{pmatrix}3\\-2\\3\end{pmatrix}$$

$$\vec{u}_3=\begin{pmatrix}1\\1\\3\end{pmatrix}-\frac{\begin{pmatrix}1\\3\\1\end{pmatrix}\cdot\begin{pmatrix}1\\1\\3\end{pmatrix}}{1+9+1}\begin{pmatrix}1\\3\\1\end{pmatrix}-\frac{\begin{pmatrix}3\\-2\\3\end{pmatrix}\cdot\begin{pmatrix}1\\1\\3\end{pmatrix}}{9+4+9}\begin{pmatrix}3\\-2\\3\end{pmatrix}$$

$$=\begin{pmatrix}1\\1\\3\end{pmatrix}-\frac{7}{11}\begin{pmatrix}1\\3\\1\end{pmatrix}-\frac{\overset{5}{\cancel{10}}}{\underset{11}{\cancel{22}}}\begin{pmatrix}3\\-2\\3\end{pmatrix}=\frac{1}{11}\begin{pmatrix}11-7-15\\11-21+10\\33-7-15\end{pmatrix}$$

$$=\begin{pmatrix}-1\\0\\1\end{pmatrix}$$

RES: $\left\{\begin{pmatrix}1\\3\\1\end{pmatrix},\begin{pmatrix}3\\-2\\3\end{pmatrix},\begin{pmatrix}-1\\0\\1\end{pmatrix}\right\}$

EX. 3

$$(1,0,1)\to V_0=\begin{pmatrix}i&1\\-1&-i\end{pmatrix}$$

$$x+2y+z=0 \to (\alpha,\beta,\gamma)=\frac{1}{\sqrt{6}}(1,2,1)$$

$$\theta=\pi \to R(\pi)=\frac{1}{\sqrt6}\begin{pmatrix}i&1+2i\\-1+2i&-i\end{pmatrix}$$

$$R(-\pi)=-R(\pi)$$

$$V_0'=R(\pi)V_0R(-\pi)=-\frac{1}{6}\underbrace{\begin{pmatrix}i&1+2i\\-1+2i&-i\end{pmatrix}\begin{pmatrix}i&1\\-1&-i\end{pmatrix}}_{\begin{pmatrix}i^2-1-2i & i-i+2\\-i-2+i&-1+2i-1\end{pmatrix}\to -2\begin{pmatrix}1+i&-1\\1&1-i\end{pmatrix}}\begin{pmatrix}i&1+2i\\-1+2i&-i\end{pmatrix}$$

$$V_0' = \frac{1}{3}\begin{pmatrix} 1+i & -1 \\ 1 & 1-i \end{pmatrix}\begin{pmatrix} i & 1+2i \\ -1+2i & -i \end{pmatrix} = \frac{1}{3}\begin{pmatrix} i-1+1-2i & 1+i+2i-2+i \\ i-1+i+2i+2 & 1+2i-i-1 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} -i & -1+4i \\ 1+4i & i \end{pmatrix} \rightarrow \boxed{\left(-\frac{1}{3}, \frac{4}{3}, -\frac{1}{3}\right)}$$

EX. 4

$$\begin{matrix} 1 & 2 & 1 & 1 \\ 3 & 1 & 1 & 1 \\ 1 & 2 & 3 & 1 \end{matrix} \Rightarrow \begin{matrix} 1 & 2 & 1 & 1 \\ 0 & 5 & 2 & 2 \\ 1 & 2 & 3 & 1 \end{matrix} \Rightarrow \begin{matrix} 1 & 2 & 1 & 1 \\ 0 & 5 & 2 & 2 \\ 0 & 0 & 2 & 0 \end{matrix} \rightarrow \gamma = 0$$

$$\begin{matrix} \alpha + 2\beta + \delta = 0 \\ 5\beta + 2\delta = 0 \end{matrix} \rightarrow \beta = -\frac{2}{5}\delta \rightarrow \alpha = -\frac{1}{5}\delta$$

$$\boxed{\text{RES: } (-1, -2, 0, 5)\,r}$$

EX. 5

$$\begin{pmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{pmatrix} \Rightarrow \lambda^2 - 5\lambda + 6 = 0$$

$$\Downarrow$$

$$\alpha = 2 \quad \beta = 3$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{pmatrix} 2a & 3b \\ 2c & 3d \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ 4c-2a & 4d-2b \end{pmatrix} \Rightarrow \begin{matrix} a = c \\ 2b = d \end{matrix}$$

$$S = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \Downarrow S^{-1} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\exp\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} e^2 & 0 \\ 0 & e^3 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} e^2 & e^3 \\ e^2 & 2e^3 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 2e^2 - e^3 & e^3 - e^2 \\ 2(e^2 - e^3) & 2e^3 - e^2 \end{pmatrix}}$$