Parte A

$$\left(\begin{array}{ccc|c|ccc} 1 & 2 & B & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 3 & 1 & A & 8 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c|ccc} 1 & 2 & B & 1 & 1 & 0 & 0 \\ 0 & 1 & B & 1 & 1 & -1 & 0 \\ 0 & 5 & 3B-A & -5 & 3 & 0 & -1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c|ccc} 1 & 2 & B & 1 & 1 & 0 & 0 \\ 0 & 1 & B & 1 & 1 & -1 & 0 \\ 0 & 0 & 2B+A & 10 & 2 & -5 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & B & 0 \\ 0 & 1 & B & 0 \\ 0 & 0 & A+2B & 0 \end{array}\right)$$

CONDIÇÃO LINEAR ~~DEPEN~~ DEPENDÊNCIA

$$\boxed{A = -2B} \quad (1)$$

$$x_1 = -2x_2 - B = B$$
$$x_2 = -B$$

$$\boxed{B, -B, 1} \quad (2)$$

Controle

$$B\begin{pmatrix}1\\1\\3\end{pmatrix} - B\begin{pmatrix}2\\1\\1\end{pmatrix} + \begin{pmatrix}B\\0\\-2B\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

ok!

"A = B"

$$x_3 = 10/3A$$
$$x_2 = -Ax_3 + 1 = -7/3$$
$$x_1 = -2x_2 - Ax_3 + 1$$
$$= 14/3 - 10/3 + 1 = 7/3$$

$$\boxed{\left(\frac{7}{3}, -\frac{7}{3}, \frac{10}{3A}\right)} \quad (3)$$

$$S_1 = \begin{pmatrix}1\\1\\3\end{pmatrix} \qquad S_2 = \begin{pmatrix}2\\1\\1\end{pmatrix} - \frac{(1\ 1\ 3)\begin{pmatrix}2\\1\\1\end{pmatrix}}{(1\ 1\ 3)\begin{pmatrix}1\\1\\3\end{pmatrix}}\begin{pmatrix}1\\1\\3\end{pmatrix} = \begin{pmatrix}2\\1\\1\end{pmatrix} - \frac{6}{11}\begin{pmatrix}1\\1\\3\end{pmatrix}$$

$$= \frac{1}{11}\begin{pmatrix}22-6\\11-6\\11-18\end{pmatrix} = \frac{1}{11}\underbrace{\begin{pmatrix}16\\5\\-7\end{pmatrix}}_{\text{segundo vetor}}$$

$$S_3 = \begin{pmatrix}A\\0\\A\end{pmatrix} - \frac{(1\ 1\ 3)\begin{pmatrix}A\\0\\A\end{pmatrix}}{(1\ 1\ 3)\begin{pmatrix}1\\1\\3\end{pmatrix}}\begin{pmatrix}1\\1\\3\end{pmatrix} - \frac{(16\ 5\ -7)\begin{pmatrix}A\\0\\A\end{pmatrix}}{(16\ 5\ -7)\begin{pmatrix}16\\5\\-7\end{pmatrix}}\begin{pmatrix}16\\5\\-7\end{pmatrix}$$

$$= \begin{pmatrix}A\\0\\A\end{pmatrix} - \frac{4A}{11}\begin{pmatrix}1\\1\\3\end{pmatrix} - \frac{\cancel{9}^{3}A}{\cancel{330}_{110}}\begin{pmatrix}16\\5\\-7\end{pmatrix} = \frac{A}{110}\begin{pmatrix}110-40-48\\-40-15\\110-120+21\end{pmatrix}$$

$$= \frac{A}{110}\begin{pmatrix}22\\-55\\11\end{pmatrix} \Rightarrow \begin{pmatrix}2\\-5\\1\end{pmatrix}$$

$$\boxed{\begin{pmatrix}1\\1\\3\end{pmatrix}, \begin{pmatrix}16\\5\\-7\end{pmatrix}, \begin{pmatrix}2\\-5\\1\end{pmatrix}} \quad (4)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & A & 1 & 0 & 0 \\ 0 & 1 & A & 1 & -1 & 0 \\ 0 & 0 & 3A & 2 & -5 & 1 \end{array}\right)$$

$$(5) \quad \boxed{M^{-1} = \frac{1}{3}\begin{pmatrix}-1 & 1 & 1 \\ 1 & 2 & -1 \\ 2/A & -5/A & 1/A\end{pmatrix}}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -A & -1 & 2 & 0 \\ 0 & 1 & A & 1 & -1 & 0 \\ 0 & 0 & A & 2/3 & -5/3 & 1/3 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & 1/3 & 1/3 \\ 0 & 1 & 0 & 1/3 & 2/3 & -1/3 \\ 0 & 0 & A & 2/3 & -5/3 & 1/3 \end{array}\right)$$

Controles

(3) $\frac{7}{3}\begin{pmatrix}1\\1\\3\end{pmatrix} - \frac{7}{3}\begin{pmatrix}2\\1\\1\end{pmatrix} + \frac{10}{3A}\begin{pmatrix}A\\0\\A\end{pmatrix} = \frac{1}{3}\begin{pmatrix}7-14+10\\7-7+0\\21-7+10\end{pmatrix} = \frac{1}{3}\begin{pmatrix}3\\0\\24\end{pmatrix} = \begin{pmatrix}1\\0\\8\end{pmatrix}$ ✓

(4) $(1,1,3)\cdot(16,5,-7) = 16+5-21 = 0$ ✓
$(1,1,3)\cdot(2,-5,1) = 2-5+3=0$ ✓
$(2,-5,1)\cdot(16,5,-7) = 32-25-7=0$ ✓

(5) $\begin{pmatrix}1&2&A\\1&1&0\\3&1&A\end{pmatrix}\frac{1}{3}\begin{pmatrix}-1&1&1\\1&2&-1\\\frac{2}{A}&-\frac{5}{A}&\frac{1}{A}\end{pmatrix} = \frac{1}{3}\begin{pmatrix}-1+2+2&1+4-5&1-2+1\\1-1&1+2&1-1\\-3+1+2&3+2-5&3-1+1\end{pmatrix}$ ✓

$\theta = \frac{2\pi}{3}$ $\quad \cos\frac{\theta}{2} = \cos\frac{\pi}{3} = \frac{1}{2}$ $\quad \sin\frac{\theta}{2} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad \frac{1}{\sqrt{3}}(1,1,1)$

$$\frac{1}{2}\begin{pmatrix}1+i&1+i\\-1+i&1-i\end{pmatrix}\begin{pmatrix}i&1+iB\\-1+iB&-i\end{pmatrix}\frac{1}{2}\begin{pmatrix}1-i&-(1+i)\\1-i&1+i\end{pmatrix}$$

$$\frac{1}{4}\begin{pmatrix}(1+i)(i-1+iB)&(1+i)(1+iB-i)\\(1-i)(-i-1+iB)&(1-i)(-1-iB-i)\end{pmatrix}\begin{pmatrix}1-i&-(1+i)\\1-i&1+i\end{pmatrix}$$

$$\frac{1}{4}\begin{pmatrix}2(\cancel{i}\cancel{-1}+iB+\cancel{1}+iB\cancel{-i})&(1+i)^2(-i+1\cancel{-iB}+1\cancel{+iB}-i)\\(1-i)^2(-i-1\cancel{+iB}-1\cancel{-iB}-i)&2(\cancel{+i}\cancel{+1}-iB\cancel{-1}-iB\cancel{-i})\end{pmatrix}$$

$$\frac{1}{4}\begin{pmatrix}4iB&2(1-i)(1+i)^2\\-2(1+i)(1-i)^2&-4iB\end{pmatrix}$$

$$\begin{pmatrix}iB&1+i\\-1+i&-iB\end{pmatrix} \Rightarrow \boxed{(1,1,B)} \quad (6)$$

$$\begin{array}{cccc|c}x_1&x_2&x_3&x_4&\\1&-1&0&0&(1-a)N\\1&1&0&-2&0\\0&1&-1&0&aN\\0&1&1+a&-1&0\\\hline 1&-1&0&0&(1-a)N\\0&2&0&-2&(a-1)N\\0&1&-1&0&aN\\0&0&2+a&-1&-aN\\\hline 1&-1&0&0&(1-a)N\\0&1&0&-1&(a-1)N/2\\0&0&-1&1&(a+1)N/2\\0&0&2+a&-1&-aN\end{array}$$

$x_1 + aN = N + x_2$
$x_1 + x_2 = 2x_4$
$aN + x_3 = x_2$
$x_4 = x_2 + a x_3 + x_3$

$\}\ (a+1)x_3 = (1-a)N/2$

$x_3 = \frac{1-a}{2(1+a)}N$

$x_1 = x_2 + (1-a)N$
$x_2 = x_4 + (a-1)N/2$
$x_4 = x_3 + (a+1)N/2 = [(1+a)^2 + 1 - a]N/2 = \frac{a^2+a+2}{2(1+a)}N$
$x_2 = (a^2+a+2+a^2-1)N/2(1+a) = \frac{2a^2+a+1}{2(1+a)}N$
$x_1 = [2a^2+a+1+2(1-a^2)]N/2(1+a) = \frac{3+a}{2(1+a)}N$

$$\boxed{(x_1,x_2,x_3,x_4) = \frac{N}{2(1+a)}(3+a,\ 1+a+2a^2,\ 2+a+a^2,\ 1-a)} \quad (7)$$

$ax_3 = \frac{a(1-a)}{2(1+a)} N$ min para $\boxed{a = 0 \text{ o } a = 1 \qquad O_{min} = 0 \quad L_{max} = 1}$ (7)

para encontrar o max $\frac{\partial}{\partial a} \frac{a(1-a)}{1+a} = 0$

$$(a-a^2)'(1+a) - (a-a^2)(1+a)' = 0$$
$$(1-2a)(1+a) - a + a^2 = 0$$
$$1 + \cancel{a} - 2a - 2a^2 - \cancel{a} + a^2 = 0$$
$$a^2 + 2a - 1 = 0$$
$$a = -1 \pm \sqrt{2} \rightarrow \sqrt{2} - 1$$

$$\boxed{\begin{array}{l} O_{max},\ L_{min} \quad a = \sqrt{2} - 1 \\ O_{max}: \left(\frac{3}{2} - \sqrt{2}\right) N \qquad L_{min}: \left(\sqrt{2} - \frac{1}{2}\right) N \end{array}}$$ (7)

$$\begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} \begin{pmatrix} t \\ at+b \end{pmatrix} = \begin{bmatrix} (1+ca)t + bc \\ (d+a)t + b \end{bmatrix} = \begin{pmatrix} s \\ \tilde{a}s + \tilde{b} \end{pmatrix}$$

$$(1+ca)t + bc = s \Rightarrow t = (s - bc)/(1+ca)$$

$$\frac{(d+a)(s-bc)}{1+ca} + b = \tilde{a}s + \tilde{b} \Rightarrow \boxed{\begin{array}{l} \tilde{a} = \frac{d+a}{1+ca} \\ \tilde{b} = b - bc\,\frac{d+a}{1+ca} \end{array}}$$ (8)

Ponto $\begin{bmatrix} \overbrace{(1+ca)}^{=0} t + bc \\ \underbrace{(d+a)}_{=0} t + b \end{bmatrix}$ $\quad c = -1/a$, $d = -a$

$$\boxed{\text{Ponto} \begin{pmatrix} 1 & -1/a \\ -a & 1 \end{pmatrix}}$$ (8)

reta paralela eixo x
$d + a = 0 \quad (1 + ca \neq 0)$

paralela eixo y
$1 + ca = 0 \quad (d + a \neq 0)$

$$\boxed{\begin{array}{lll} /\!/x & \begin{pmatrix} 1 & c \\ -a & 1 \end{pmatrix} & c \neq -1/a \\ /\!/y & \begin{pmatrix} 1 & -1/a \\ d & 0 \end{pmatrix} & d \neq -a \end{array}}$$ (8)

mapeamento em $y = x$ $\quad \tilde{a} = 1 \Rightarrow d + a = 1 + ca \qquad d + \cancel{a} = 1 + \cancel{a}$

$\tilde{b} = 0 \Rightarrow (1 - c)b = 0 \Rightarrow c = 1$

$$\boxed{y = x \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}}$$ (8)