Resolução Prova 1 - 2012-

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① $F(1) = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right) - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right) = \frac{1+\sqrt{5}-1+\sqrt{5}}{2\sqrt{5}} = 1$

$F(2) = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^2 - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^2 = \frac{1}{4\sqrt{5}}\left(6+2\sqrt{5}-6+2\sqrt{5}\right) = 1$

$\boxed{n=1}$ $F(1)F(2) = [F(2)]^2$ ✓

$\boxed{n=p}$ $\sum_{p=1}^{2n-1} F(p)F(p+1) = [F(2n)]^2$ ✓

$\boxed{n=p+1}$ $\sum_{p=1}^{2n+1} F(p)F(p+1) = \sum_{p=1}^{2n-1} F(p)F(p+1) + [F(2n)F(2n+1)] + [F(2n+1)F(2n+2)]$

$= [F(2n)]^2 + F(2n)F(2n+1) + F(2n+1)F(2n+2)$

$= F(2n)\underbrace{[F(2n)+F(2n+1)]}_{(*)\,F(2n+2)} + F(2n+1)F(2n+2)$

$= F(2n)F(2n+2) + F(2n+1)F(2n+2)$

$= F(2n+2)[F(2n)+F(2n+1)]$

$= [F(2n+2)]^2$ ✓

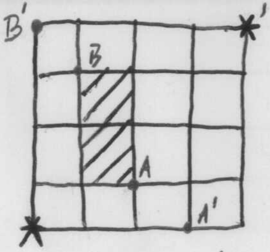
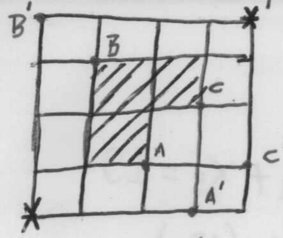
(*): lembrando que para os números de Fibonacci: $F(\varepsilon)+F(\varepsilon+1) = F(\varepsilon+2)$

③ A: "conjunto de números com soma menor que 10, de 1 a 10 e tomados 3 a 3"

$A = \{\{1;2;3\}, \{1;2;4\}, \{1;2;5\}, \{1;2;6\}, \{1;3;4\}, \{1;3;5\}, \{2;3;4\}\}$

$\therefore$ Probabilidade $(\geq 10) = \frac{\text{Total} - \#A}{\text{Total}} = \frac{113}{120} \sim 0,942$

obs: denotamos "#(b)" como sendo a cardinalidade de um conjunto b arbitrário.

② (A)  (B) 

obs: $\binom{n}{p} = \frac{n!}{p!(n-p)!}$

Defino $\{\alpha;\beta\}$ como sendo o número de maneiras para irmos de α a β. Assim, em (A):

$$\{*;*'\} = \{*;A\}\{A;*'\} + \{*;A'\}\{A';*'\}$$

$$+\{*;B\}\{B;*'\} + \{*;B'\}\{B';*'\}$$

$$= \binom{3}{1}\binom{5}{2} + \binom{3}{0}\binom{5}{1} + \binom{4}{1}\binom{4}{1} + \binom{4}{0}\binom{4}{4}$$

$$= 3.10 + 1.5 + 16 + 1 = 52;$$

em (B):

$$\{*;*'\} = \{*;A\}\{A;C\}\{C;*'\} + \{*;A\}\{A;C'\}\{C';*'\}$$

$$+\{*;A'\}\{A';*'\} + \{*;B\}\{B;*'\} + \{*;B'\}\{B';*'\}$$

$$= \binom{3}{1}\binom{2}{1}\binom{3}{1} + \binom{3}{1}\binom{2}{0}\binom{3}{0} + \binom{3}{0}\binom{5}{1} + \binom{4}{1}\binom{4}{1} + \binom{4}{0}\binom{4}{0}$$

$$= 18 + 3 + 5 + 16 + 1 = 43$$

④ $y_1 + y_2 + y_3 + y_4 = 25$ (CASO A)

$-2 \le y_1 \le 4$	$y_1 = x_1 - 3$	$1 \le x_1 \le 7$	$x_1 + x_2 + x_3 + x_4 = 20$
$4 \le y_2 \le 7$	$y_2 = x_2 + 3$	$\Rightarrow$ $1 \le x_2 \le 4$	Total: $\binom{19}{3}$
$1 \le y_3 \le 8$	$y_3 = x_3$	$1 \le x_3 \le 8$	
$6 \le y_4$	$y_4 = x_4 + 5$	$1 \le x_4$	

$A(>7) \Rightarrow \binom{12}{3}$ $\quad AB \Rightarrow \binom{8}{3}$ $\quad \nexists A \cap B \cap C$ pois implicaria em $\binom{0}{3}$, e não

$B(>4) \Rightarrow \binom{15}{3}$ $\quad AC \Rightarrow \binom{4}{3}$ $\quad$ existe $\binom{\alpha}{\beta}$ com $\alpha < \beta$.

$C(>8) \Rightarrow \binom{11}{3}$ $\quad BC \Rightarrow \binom{7}{3}$

$$\therefore \binom{19}{3} - \binom{12}{3} - \binom{15}{3} - \binom{11}{3} + \binom{8}{3} + \binom{4}{3} + \binom{7}{3}$$

$$\frac{1}{6}(19.18.17 - 12.11.10 - 15.14.13 - 11.10.9 + 8.7.6 + 7.6.5) + 4 = 224$$

(CASO B)

$y_1 + y_2 + y_3 = 19$

$$\begin{cases} y_1 = x_1 - 3 \\ y_2 = x_2 + 3 \\ y_3 = x_3 \end{cases} \Rightarrow \begin{cases} 1 \le x_1 \le 7 \\ 1 \le x_2 \le 4 \\ 1 \le x_3 \le 8 \end{cases} \quad \Bigg| \quad x_1 + x_2 + x_3 = 19 \quad \text{Total: } \binom{18}{2}$$

$A(>7) \Rightarrow \binom{11}{2}$ $\quad AB \Rightarrow \binom{7}{2}$ $\quad \nexists ABC.$

$B(>4) \Rightarrow \binom{14}{2}$ $\quad AC \Rightarrow \binom{3}{2}$

$C(>8) \Rightarrow \binom{10}{2}$ $\quad BC \Rightarrow \binom{6}{2}$

$$\therefore \binom{18}{2} - \binom{11}{2} - \binom{14}{2} - \binom{10}{2} + \binom{7}{2} + \binom{3}{2} + \binom{6}{2}$$

$$= 9.17 - 11.5 - 7.13 - 5.9 + 7.3 + 3 + 15$$

$$= 1$$

5) Defino A_B como sendo letra A na posição B. Assim:

$$\underbrace{I_1 \Rightarrow \frac{6!}{2!}}_{A} \; ; \; \underbrace{K_2 \Rightarrow \frac{6!}{2!}}_{B} \; ; \; \underbrace{O_1 O_7 \Rightarrow 5! \; ; \; O_1 I_7 \Rightarrow 5! \; ; \; I_1 O_7 \Rightarrow 5!}_{C}$$

obs 1: V denota ou;

obs 2: $A_\alpha B_\alpha = \begin{cases} 1, \text{ se } A = B \\ 0, \text{ se } A \neq B \end{cases}$, pois letras diferentes não ocupam o mesmo lugar ao mesmo tempo.

$AB \Rightarrow I_1 K_2 \Rightarrow \frac{5!}{2}$

$AC \Rightarrow \cancel{I_1 O_1} O_7 \; V \; \cancel{I_1 O_1} O_7 \; V \; I_1 I_1 O_7 \Rightarrow I_1 O_7 \Rightarrow 5!$

$BC \Rightarrow O_1 K_2 O_7 \; V \; O_1 K_2 I_7 \; V \; I_1 K_2 O_7 \Rightarrow 4! + 4! + 4!$

$ABC \Rightarrow I_1 \cancel{O_1 K_2} O_7 \; V \; I_1 \cancel{O_1 K_2} I_7 \; V \; I_1 I_1 K_2 O_7 = I_1 K_2 O_7 \Rightarrow 4!$

Assim: $\frac{6!}{2!} + \frac{6!}{2!} + \cancel{5!} + 5! + 5! - \frac{5!}{2} - \cancel{5!} - 3.4! + 4!$

$$= 6! + \frac{3}{2}.5! - 2.4! = 6.5.4! + \frac{15}{2}.4! - 2.4!$$

$$= 4!\,(30 + 15/2 - 2) = 4!\,\frac{71}{2} = 12.71 = 852$$