Resolução Prova 1 - Jean Lucas - jean.stendoff@gmail.com

1) (i)

$\boxed{n=1}$ $\quad \frac{1}{2} = 1 - \frac{1}{2^1}$ ✓

$\boxed{n=k}$ $\quad \frac{1}{2} + \cdots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$ ✓

$\boxed{n=k+1}$ $\quad \left(\frac{1}{2} + \cdots + \frac{1}{2^k}\right) + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}} = 1 + \frac{(-2+1)}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$ ✓

(ii) $2n^3 > 3n^2 + 3n + 1 \quad (n \geq 3)$

$\boxed{n=3}$ $\quad 2.3^3 = 54 > 37 = 3.3^2 + 3.3 + 1$ ✓

$\boxed{n=k}$ $\quad 2k^3 > 3k^2 + 3k + 1$ ✓

$\boxed{n=k+1}$ $\quad 2(k+1)^3 = 2k^3 + 2(3k^2 + 3k + 1) > 3k^2 + 3k + 1 + 2(3k^2 + 3k + 1)$

$$
\begin{aligned}
&= 9k^2 + 9k + 3 \\
&= 3k^2 + 9k + (6k^2 + 3) \\
&> 3k^2 + 9k + 7 \\
&= 3(k+1)^2 + 3(k+1) + 4
\end{aligned}
$$ ✓

(iii)

$\boxed{n=1}$ $\quad \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right) - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right) = \frac{1+\sqrt{5}-1+\sqrt{5}}{2\sqrt{5}} = 1$ ✓

$\boxed{n=2}$ $\quad \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^2 - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^2 = \frac{1}{\sqrt{5}}\left(\frac{1+5+2\sqrt{5}}{4}\right) - \frac{1}{\sqrt{5}}\left(\frac{1+5-2\sqrt{5}}{4}\right)$

$$= \frac{1}{4\sqrt{5}}\left(6+2\sqrt{5} - 6 + 2\sqrt{5}\right) = 1$$ ✓

$\boxed{n=k}$ $\quad \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^k - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^k$ ✓

$\boxed{n=k+1}$ $\quad \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{k+1} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{k+1}$ ✓

$\boxed{n=k+2}$ $\quad \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{k+2} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{k+2} = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^k\left(\frac{1+\sqrt{5}}{2}\right)^2 - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^k\left(\frac{1-\sqrt{5}}{2}\right)^2$

$$= \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^k\left(\frac{1+\sqrt{5}}{2} + 1\right) - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^k\left(\frac{1-\sqrt{5}}{2} + 1\right)$$

$$= \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{k+1} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{k+1} + \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^k - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^k$$ ✓

$$\therefore F(k+2) = F(k+1) + F(k+1)$$

② Ⓐ $\frac{4!}{2!2!} = 6$ Ⓑ $\binom{4}{2}\binom{4}{2}+\binom{4}{1}\binom{4}{1}+\binom{4}{0}\binom{4}{0}=53$

Veja o Resumo 1.

③ Definimos: $A=\{1,4,\dots,100\} \Rightarrow \#A = 34$

$B=\{2,5,\dots,98\} \Rightarrow \#B = 33$

$C=\{3,6,\dots,99\} \Rightarrow \#C = 33$

Assim, se escolhemos 1, o conjunto K_1 de possibilidades está contido B, assim como K_2, associado ao primeiro número sendo 2, está contido em A:

$$\left.\begin{array}{l} 1 \to \#B \\ 2 \to \#A-1 \\ 3 \to \#C-1 \end{array}\right\} \quad \left.\begin{array}{l} 4 \to \#B-1 \\ 5 \to \#A-2 \\ 6 \to \#C-2 \end{array}\right\}$$

$\vdots$

, Assim, multiplicando as cardinalidades relativas a cada $i\in\{1,\dots,100\}$:

$$32!(33!)^2$$

④ $1+2+3$ $2+3+4$

$1+2+4$ $2+3+5$

$1+2+5$ $\therefore 7$

$1+2+6$

$1+2+7$

⑤ $y_1+y_2+y_3+y_4=21$ $(*)$

$-2\leq y_1 \leq 3$ $y_1 = x_1-3$ $1\leq x_1 \leq 6$ em $(*)$:

$4\leq y_2 \leq 6$ $y_2 = x_2+3$ $\Rightarrow 1\leq x_2 \leq 3$ $(x_1-\cancel{3})+(x_2+\cancel{3})+x_3+(x_4+4)=21$

$1\leq y_3 \leq 6$ $y_3 = x_3$ $1\leq x_3\leq 6$ $x_1+x_2+x_3+x_4=17$

$5\leq y_4$ $y_4 = x_4+4$ $1\leq x_4$ $\therefore \binom{16}{3}$

$A(>6)$ e $C(>6) \Rightarrow \binom{10}{3}$ $(A\cap B)$ e $(B\cap C) \Rightarrow \binom{7}{3}$

$B(>3) \Rightarrow \binom{13}{3}$ $A\cap C \Rightarrow \binom{4}{3}$ $\nexists\ A\cap B\cap C$

$$\therefore \binom{16}{3} - 2\binom{10}{3} - \binom{13}{3} + 2\binom{7}{3} + \binom{4}{3}$$

$$= 40.14 - 2.12.10 - 13.22 + 2.35 + 4$$

$$= 560 - 240 - 286 + 74$$

$$= 108$$

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⑥ $y_1 + y_2 + y_3 = 14 \quad (*)$

$-2 \leq y_1 \leq N$ $\quad y_1 = x_1 - 3$ $\quad 1 \leq x_1 \leq N+3$ $\quad$ em $(*)$:

$5 \leq y_2 \leq 7$ $\quad y_2 = x_2 + 4$ $\Rightarrow$ $1 \leq x_2 \leq 3$ $\quad x_1 + x_2 + x_3 = 8$

$6 \leq y_3$ $\quad y_3 = x_3 + 5$ $\quad 1 \leq x_3$

$\therefore \binom{7}{2}$

$A(> N+3) \Rightarrow \binom{7-N-3}{2}$ $\qquad B(>3) \Rightarrow \binom{4}{2}$

Sabemos então que:

$$14 = \binom{7}{2} - \binom{4-N}{2} - \binom{4}{2}$$

$$14 = 21 - \frac{(4-N)!}{2!(2-N)!} - 6$$

$$1 = \frac{(4-N)(3-N)}{2} \Rightarrow N = 3 \text{ ou } N = 4$$

obs: $\binom{n}{p} = \frac{n!}{p!(n-p)!}$

obs2: $\frac{\alpha!}{(\alpha-\beta)!} = \alpha(\alpha-1)\dots(\alpha-\beta+1)$; $\alpha, \beta \in \mathbb{N}$

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⑦ Total: $5!$

Dividimos em 3 casos que não podem acontecer, e no final tiramos desse total:

Caso 1: Quantas maneiras botamos os 6 de modo que apenas 1 casal fique junto:

(i) escolher o casal que terá privilégio: 3

(ii) permuta entre tal homem e seu par: 2

(iii) escolher o vizinho de tal homem: 4

(iv) escolher o vizinho deste vizinho: 2

(v) escolher o seguinte: 1

(vi) último: 1

$\therefore 48$

Caso 2: Exatamente 2 casais juntos:
(i) escolher os dois casais: $\binom{3}{2} = 3$
(ii) permuta no 1º casal: 2
(iii) permuta no 2º casal: 2
(iv) colocações do último casal: 2
$\therefore$ 24

Caso 3: Três casais juntos:
(i) permuta no 1º casal: 2
(ii) permuta no 2º casal: 2
(iii) permuta no 3º casal: 2
(iv) permuta entre os casais: 2
$\therefore$ 16

Finalmente: $120 - 48 - 24 - 16 = 32$