RESOLUÇÃO 2ª PROVA

① PARADO TOTAL: $\frac{6!}{2!} = 360$

A. Con __ Con __ __ __

B. __ Vog __ __ __ __

C. O __ __ __ __ __

D __ __ A __ __ __

$N(A) = C_3^1 \cdot C_2^1 \cdot 4!/2! = 72$

$N(B) = 5! + \frac{5!}{2} = 180$

$N(C) = \frac{5!}{2!} = 60$

$N(D) = 5! = 120$

$N(A\cap B) = C_3^1 \cdot C_2^1 \cdot 3! + C_3^1 \cdot C_2^1 \cdot \frac{3!}{2!} = 36 + 18 = 54$

$N(A\cap C) = \emptyset$

$N(A\cap D) = \emptyset$

$N(B\cap C) = 4! = 24$

$N(B\cap D) = 2\cdot 4! = 48$

$N(C\cap D) = 4! = 24$

$N(A\cap B\cap C) = \emptyset$

$N(A\cap B\cap D) = \emptyset$

$N(A\cap C\cap D) = \emptyset$

$N(B\cap C\cap D) = 3! = 6$ $N(A\cap B\cap C\cap D) = 0$

Resp $= 72 + 180 + 60 + 120 - 54 - 24 - 48 - 24 + 6 = \boxed{288}$ //

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② 5A 6B 5C 4D

"1" "1" "0" "0"

$$(x+x^2+x^3+x^4+x^5)(x+x^2+x^3+x^4+x^5+x^6)(1+x+x^2+x^3+x^4+x^5)(1+x+x^2+x^3+x^4) =$$

$$= x(1+x+x^2+x^3+x^4)\,x(1+x+x^2+x^3+x^4+x^5)(1+x+x^2+x^3+x^4+x^5)(1+x+x^2+x^3+x^4) =$$

$$= x^2(1+x+x^2+x^3+x^4)^2(1+x+x^2+x^3+x^4+x^5)^2 =$$

$$= x^2 \cdot \frac{(1-x^5)^2}{(1-x)^2} \cdot \frac{(1-x^6)^2}{(1-x)^2} = \frac{x^2(1-2x^5+x^{10})(1-2x^6+x^{12})}{(1-x)^4} = \frac{(x^2-2x^7+\cancel{x^{12}})(1-2x^6+\cancel{x^{12}})}{(1-x)^4} =$$

$$= \frac{(x^2-2x^7)(1-2x^6)}{(1-x)^4} = \frac{(x^2-2x^8-2x^7+\cancel{4x^{13}})}{(1-x)^4} = \frac{(x^2-2x^7-2x^8)}{(1-x)^4}$$

$$\frac{1}{(1-x)^4} = \sum_{n=0}^{\infty}\binom{4+n-1}{n}x^n = \binom{3}{0}+\binom{4}{1}x+\binom{5}{2}x^2+\binom{6}{3}x^3+\binom{7}{4}x^4+\binom{8}{5}x^5+\binom{9}{6}x^6+\binom{10}{7}x^7+\binom{11}{8}x^8$$

$$= 1 + 4x + 10x^2 + 20x^3 + 35x^4 + 56x^5 + 84x^6 + 120x^7 + 165x^8$$

Selecionando-se: 8 letras $\Rightarrow (x^2-2x^7-2x^8)(1+4x+\dots+84x^6)$

$$84x^8-8x^8-2x^8=\boxed{74}$$

9 letras $\Rightarrow (x^2-2x^7-2x^8)(1+4x+10x^2+\dots+120x^7)$

$$120x^9-20x^9-8x^9=\boxed{92}$$

10 letras $\Rightarrow (x^2-2x^7-2x^8)(1+4x+10x^2+20x^3+\dots+165x^8)$

$$165x^{10}-40x^{10}-20x^{10}=\boxed{105}$$

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③ 3A 4B 3C 3D

"1" "1" "0" "0"

$$\left(x+\frac{x^2}{2!}+\frac{x^3}{3!}\right)\left(x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}\right)\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}\right)\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}\right)=$$

$$=\left(x+\frac{x^2}{2}+\frac{x^3}{6}\right)\left(x+\frac{x^2}{2}+\frac{x^3}{6}+\cancel{\frac{x^4}{24}}\right)\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}\right)^2=$$

$$=\left(x^2+\frac{x^3}{2}+\frac{x^4}{6}+\frac{x^3}{2}+\frac{x^4}{4}+\cancel{\frac{x^5}{12}}+\frac{x^4}{6}+\cancel{\frac{x^5}{12}}+\cancel{\frac{x^6}{36}}\right)\left(1+2x+2x^2+\dots\right)=\left(x^2+x^3+\frac{7x^4}{12}\right)\left(1+2x+2x^2\right)$$

$$=2x^4+2x^3+\frac{7x^4}{12}=\left(\frac{24+24+7}{12}\right)\cdot 24\cdot\frac{x^4}{4!}=\frac{55\cdot \cancel{24}^{2}}{\cancel{12}}\cdot\frac{x^4}{4}\Rightarrow\boxed{110}\ \frac{x^4}{4!}$$

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④ $x_1+2x_2+3x_3+x_4+x_5$

$2\leqslant x_1\leqslant 4$ $\qquad x_1\to(x^2+x^3+x^4)$

$1\leqslant x_{2,3,4}$ $\qquad 2x_2\to(x^2+x^4+x^6+x^8+\dots)$

$x_5=0,1$ $\qquad 3x_3\to(x^3+x^6+x^9+x^{12}+\dots)$

$x_4\to(x+x^2+x^3+x^4+\dots)$

$x_5\to(1+x)$

$$(x^2+x^3+x^4)(x^2+x^4+x^6+x^8+\dots)(x^3+x^6+x^9+x^{12}+\dots)(x+x^2+x^3+x^4+\dots)(1+x)$$

$$= x^2(1+x+x^2)\,x^2(1+x^2+x^4+x^6+\dots)\,x^3(1+x^3+x^6+x^9+\dots)\,x(1+x+x^2+x^3+\dots)\cdot(1+x) =$$

$$= x^8\cdot\frac{(1-x^3)}{(1-x)}\cdot\frac{1}{(1-x^2)}\cdot\frac{1}{(1-x^3)}\cdot\frac{1}{(1-x)}\cdot(1+x) = \frac{x^8\,\cancel{(1-x^3)}\cdot\cancel{(1+x)}}{(1-x)^2(1-x)\cancel{(1+x)}\cancel{(1-x^3)}} = \frac{x^8}{(1-x)^3} =$$

$$\frac{1}{(1-x)^3} = \sum_{n=0}^{\infty}\binom{3+n-1}{n}x^n = \binom{2}{0}+\binom{3}{1}x+\binom{4}{2}x^2+\binom{5}{3}x^3+\binom{6}{4}x^4+\binom{7}{5}x^5+\binom{8}{6}x^6+\binom{9}{7}x^7 =$$

$$= 1+3x+6x^2+10x^3+15x^4+21x^5+28x^6+36x^7 =$$

$$= x^8(1+3x+6x^2+10x^3+15x^4+21x^5+28x^6+36x^7)$$

Soluções que garantem soma:

$9 \Rightarrow x^8\cdot 3x = 3x^9 \Rightarrow$ $\boxed{\text{3 soluções}}$

$11 \Rightarrow x^8\cdot 10x^3 = 10x^{11} \Rightarrow$ $\boxed{\text{10 soluções}}$

$13 \Rightarrow x^8\cdot 21x^5 = 21x^{11} \Rightarrow$ $\boxed{\text{21 soluções}}$

$15 \Rightarrow x^8\cdot 36x^7 = 36x^{11} \Rightarrow$ $\boxed{\text{36 soluções}}$