MATEMÁTICA DISCRETA/P1

12 DE SETEMBRO DE 2012

PROF. STEFANO DE LEO

NOME:

RA:

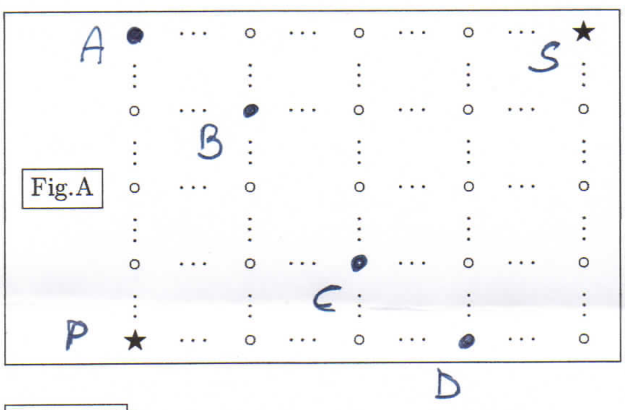
NUM:

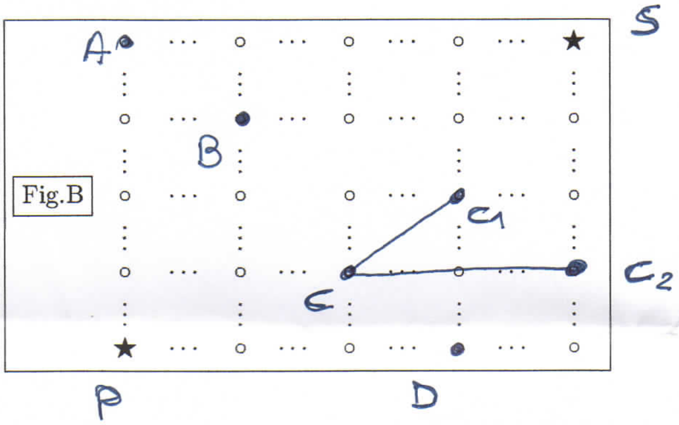
• 1 [0.5]

Usando o princípio de indução provar que os nťmeros de Fibonaccci satisfazem $\sum_{p=1}^{2n-1} F(p)F(p+1) = [F(2n)]^2$.

• 2 [1.0]

Quantos são os trajetos de comprimento mínimo que ligam os pontos ★?





• 3 - 0.5

Em uma urna há fichas numeradas de 1 até 10. Qual é a probabilidade de retirar 3 fichas cuja soma seja maior ou igual a 10?

• 4 - 1.5

Encontrar o número de soluções, em inteiros, de $y_1+y_2+y_3+y_4 = 25$, em que $-2 \le y_1 \le 4$, $4 \le y_2 \le 7$, $1 \le y_3 \le 8$ e com $y_4 \ge 6$ (caso A) e $y_4 = 6$ (caso B).

• 5 - 1.0

Quantas são as permutações das letras da palavra BOOKING e, que a I ocupa o primeiro lugar, ou a K o segundo lugar, ou que comecam e terminam por vogal.

Ex.	2 - Fig.A	2 -Fig.B	3	4 - A	4 - B	5
RES.	52	43	113/120	224	1	

1) $n=1$ $\quad F(1)\,F(2) = [F(2)]^2$ ✓ $\qquad F(1)=F(2)=1$

$n=k$ $\quad \sum_{p=1}^{2k-1} F(p)\,F(p+1) = [F(2k)]^2$ ✓

$n=k+1$ $\quad \sum_{p=1}^{2k+1} F(p)\,F(p+1) = [F(2k+2)]^2$?

$\rightarrow F(2k+1)\,F(2k+2) + F(2k)\,F(2k+1) + \underbrace{\sum_{p=1}^{2k-1} F(p)\,F(p+1)}_{[F(2k)]^2}$

$\rightarrow F(2k+1)\,F(2k+2) + F(2k)\,\underbrace{[F(2k+1)+F(2k)]}_{F(2k+2)}$

$\rightarrow \underbrace{[F(2k+1) + F(2k)]}_{F(2k+2)}\,F(2k+2)$ ✓

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2) Fig. A

PAS : 1

PBS : $\frac{4!}{1!3!}\cdot\frac{4!}{3!1!} = 16$

PCS : $\frac{3!}{2!1!}\cdot\frac{5!}{2!3!} = 30$

PDS : $1\cdot\frac{5!}{1!4!} = 5$

TOT $\boxed{52}$

Fig. B

PCC₁S : $\frac{3!}{2!1!}\,\frac{2!}{1!1!}\,\frac{3!}{1!2!} = 18$

PCC₂S : $\frac{3!}{2!1!}\;1\;1 = 3$

} 21

$30 \rightarrow 21$

TOT $\boxed{43}$

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3) TOT $\frac{10!}{3!7!} = \frac{10\cdot9\cdot8}{6} = 120$

$1+2+3,\ +4,\ +5,\ +6$ } 4 $\qquad 1+3+4,\ 5$ } 2 $\qquad 2+3+4$ } 1

TOT 7

PROBABILIDADE ≥ 10 3 FICHAS $\quad \frac{113}{120} \sim 94.17\%$

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4) CASO A $\quad y_1+y_2+y_3+y_4 = 25$

$-2 \leq y_1 \leq 4$ $\quad y_1 = x_1 - 3$ $\quad x_1 \leq 7$

$4 \leq y_2 \leq 7$ $\quad y_2 = x_2 + 3$ $\quad x_2 \leq 4$

$1 \leq y_3 \leq 8$ $\quad y_3 = x_3$ $\quad x_3 \leq 8$

$6 \leq y_4$ $\quad y_4 = x_4 + 5$ $\quad x_4$

$\rightarrow x_1 \cancel{-3} + x_2 \cancel{+3} + x_3 + x_4 + 5 = 25$

$\boxed{x_1+x_2+x_3+x_4 = 20}$

TOT $\quad 19!/3!16! = 19\cdot18\cdot17/6 = 19\cdot3\cdot17$

A(>7) $\quad 12!/3!9! = 2\cdot11\cdot10$

B(>4) $\quad 15!/3!12! = 5\cdot7\cdot13$

C(>8) $\quad 11!/3!8! = 11\cdot5\cdot3$

AB(>11) $\quad 8!/3!5! = 8\cdot7$

AC(>15) $\quad 4!/3!1! = 4$

BC(>12) $\quad 7!/3!4! = 7\cdot5$

~~ABC(>19)~~

$19\cdot51 - 220 - 35\cdot13 - 11\cdot15 + 56 + 4 + 35$

19×51: $19 + 95 = 969$ $\qquad 35 \times 13$: $105 + 35 = 455$ $\qquad 15 \times 15 = 165$

$969 + 56 + 4 + 35 = 1064$ $\quad - \quad 220 + 455 + 165 = 840$

$\boxed{224}$

CONTROLE

$-2 \leq y_1 \leq 4$ $\quad 7$

$4 \leq y_2 \leq 7$ $\quad 4$

$1 \leq y_3 \leq 8$ $\quad 8$

$7 \times 4 \times 8$: $28 \times 8 = 224$ ✓

4) CASO B $y_1 + y_2 + y_3 = 19 \Rightarrow x_1 + x_2 + x_3 = 19 \Rightarrow$ tot $\frac{18!}{2!\,16!} = 153$

$y_1 \to x_1 - 3$, $y_2 \to x_2 + 3$, $y_3 \to x_3$

$x_1 \le 7,\ x_2 \le 4,\ x_3 \le 8$

A (>7) $11!/2!\,9! = 55$

B (>4) $14!/2!\,12! = 91$

C (>8) $10!/2!\,8! = 45$

AB (>11) $7!/2!\,5! = 21$

AC (>15) $3!/2!\,1! = 3$

BC (>12) $6!/2!\,4! = 15$

$\cancel{ABC\ (>19)}$

$153 - 55 - 91 - 45 + 21 + 3 + 15$

$$\begin{array}{r} 153\ + \\ 21\ + \\ 3\ + \\ 15 \\ \hline 192 \end{array} \quad - \quad \begin{array}{r} 55 \\ 91 \\ 45 \\ \hline 191 \end{array} \quad = \boxed{1}$$

1 corretta

ossia $\left.\begin{array}{l} x_1 = 7 \\ x_2 = 4 \\ x_3 = 8 \end{array}\right\} \underline{\underline{19}}$

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5)

	B	O	O	K	I	N	G	
G_1	I							$6!/2!$
G_2		K						$6!/2!$
G_3	Voc						Voc	$3 \times 5!$
G_{12}	I	K						$5!/2$
G_{13}	I						Voc	$5!$
G_{23}	Vo	K					Voc	$3 \times 4!$
G_{123}	I	K					Voc	$4!$

G_1, G_2, G_3: $9 \times 5!$

G_{12}, G_{13}, G_{23}: $\frac{3}{2} 5! + 3 \times 4!$

$$N(G_1 \cup G_2 \cup G_3) = N(G_1) + N(G_2) + N(G_3) - \overbrace{N(G_1 \cap G_2)}^{G_{12}} - \overbrace{N(G_1 \cap G_3)}^{G_{13}}$$
$$- \underbrace{N(G_2 \cap G_3)}_{G_{23}} + \underbrace{N(G_1 \cap G_2 \cap G_3)}_{G_{123}}$$

$$\left(9 - \frac{3}{2}\right) 5! \underbrace{- 3 \times 4! + 4!}_{-2 \times 4!}$$

$$\frac{15}{2}\, 5!$$

$$\left(\frac{75}{2} - 2\right) 4!^{\,\to 24} = 71 \times 12$$

$\boxed{852}$