Teste do dia 30/10

1) DO CONJUNTO DE 9 LETRAS: AABBBCCCC DE QUANTAS MANEIRAS PODEMOS ESCOLHER 7 LETRAS E QUANTOS ANAGRAMAS PODEMOS FORMAR

PARA CALCULAR O NÚMERO DE MANEIRAS DE ESCOLHER 7 LETRAS DO CONJUNTO DE 9 LETRAS DADO USAREMOS A SEGUINTE FUNÇÃO GERADORA

$$(1+x+x^2)(1+x+x^2+x^3)(1+x+x^2+x^3+x^4)$$

RESOLUÇÃO STANDARD

$$\frac{(1-x^3)(1-x^4)(1-x^5)}{(1-x)^3} = \frac{(1-x^3)(1-x^4-x^5+\cancel{x^9})}{(1-x)^3}$$

$$= \frac{1-x^4-x^5-x^3+x^7+\cancel{x^8}}{(1-x)^3}$$

$$(1-x^3-x^4-x^5+x^7)\sum_{r=0}^{\infty}\binom{r+2}{2}x^r$$

$$\Rightarrow \binom{9}{2}-\binom{6}{2}-\binom{5}{2}-\binom{4}{2}+\binom{2}{2} = 36-15-10-6+1 = \boxed{6}$$

RESOLUÇÃO SIMPLIFICADA FUNÇÃO GERADORA: $(1+x+x^2)(x+x^2+x^3)(x^2+x^3+x^4)$

PORQUE DEVE APARECER PELO MENOS UMA B E DUAS C

$$x^3(1+x+x^2)^3 = x^3\left(\frac{1-x^3}{1-x}\right)^3 = \frac{x^3(1-3x^3+\cancel{3x^6}-\cancel{x^9})}{(1-x)^3}$$

$$(x^3-3x^6)\sum_{r=0}^{\infty}\binom{r+2}{2}x^r \Rightarrow \binom{6}{2}-3\binom{3}{2} = \boxed{6}$$

DETERMINAMOS O NÚMERO DE ANAGRAMAS

FUNÇÃO GERADORA $\left(1+x+\frac{x^2}{2}\right)\left(x+\frac{x^2}{2}+\frac{x^3}{3!}\right)\left(\frac{x^2}{2}+\frac{x^3}{3!}+\frac{x^4}{4!}\right)$

$$\frac{x^3}{2}\left(1+x+\frac{x^2}{2}\right)\underbrace{\left(1+\frac{x}{2}+\frac{x^2}{6}\right)\left(1+\frac{x}{3}+\frac{x^2}{12}\right)}$$

$$\begin{array}{ccccc}
1 & \frac{1}{3} & \frac{1}{12} & & \\
 & \frac{1}{2} & \frac{1}{6} & \frac{1}{24} & \\
 & & \frac{1}{6} & \frac{1}{18} & \frac{1}{72} \\
\hline
\end{array}$$

$$1+\frac{5}{6}x+\frac{5}{12}x^2+\frac{7}{72}x^3+\frac{1}{72}x^4$$

$$\begin{array}{cccccccc}
 & 1 & \frac{5}{6} & \frac{5}{12} & \frac{7}{72} & \frac{1}{72} & & \\
 & & 1 & \frac{5}{6} & \frac{5}{12} & \frac{7}{72} & \cancel{\frac{1}{72}} & \\
 & & & \frac{1}{2} & \frac{5}{12} & \frac{5}{24} & \cancel{\frac{7}{144}} & \cancel{\frac{1}{144}} \\
\frac{x^3}{2} & 1 & x & x^2 & x^3 & x^4 & \cancel{x^5} & \cancel{x^6}
\end{array}$$

$$\begin{array}{r}
23 \\
35 \\
\hline
115 \\
69 \\
\hline
805
\end{array}$$

$$\cancel{\frac{1}{2}}\ \frac{23}{\cancel{72}}\ 7\cdot\cancel{6}\cdot5\cdot\cancel{4}\cdot\cancel{3}\ \cancel{2} = 23\cdot35 = \boxed{805}$$

CONTROLAMOS AS RESPOSTAS DADAS

BBBCCCC	$7!/3!4!$
ABBCCCC	$7!/2!4!$
ABBBCCC	$7!/3!3!$
AABCCCC	$7!/2!4!$
AABBCCC	$7!/2!2!3!$
AABBBCC	$7!/2!2!3!$

TOT $\boxed{6}$

$$7!\left(\frac{1}{3!4!}+\frac{1}{4!}+\frac{1}{3!3!}+\frac{1}{2!3!}\right)$$

$$7!\left[\frac{1}{3!}\left(\frac{1}{24}+\frac{1}{6}+\frac{1}{2}\right)+\frac{1}{4!}\right]$$

$$7!\left[\frac{1}{6}\,\frac{17}{24}+\frac{1}{24}\right] = 7\cdot\cancel{6}\cdot5\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\;\frac{23}{\cancel{6}\cdot\cancel{24}}$$

$$= 35\cdot23 = \boxed{805}$$

2) LANÇANDO 6 DADOS CALCULAR A PROBABILIDADE DE FAZER 20, 21 ou 22 E COMPARE ELA COM A PROBABILIDADE DE FAZER 6, 7 ou 8 LANÇANDO 2 DADOS

FUNÇÃO GERADORA $(x+x^2+x^3+x^4+x^5+x^6)^N = x^N\left(\frac{1-x^6}{1-x}\right)^N$

$$= x^N \sum_0^N {}_s \binom{N}{s}(-1)^s x^{6s} \sum_0^\infty {}_r \binom{N-1+r}{N-1} x^r$$

$N=2$ $\quad x^2 \sum_0^2 {}_s \binom{2}{s}(-1)\, x^{6s} \sum_0^\infty {}_r \binom{1+r}{1} x^r$

"6" $\quad s=0 \quad r=4 \quad \binom{2}{0}\binom{5}{1}/6^2 \qquad 5/36$

"7" $\quad s=0 \quad r=5 \quad \binom{2}{0}\binom{6}{1}/6^2 \qquad 6/36 \qquad$ TOT $\frac{16}{36} = \boxed{\frac{4}{9}}$

"8" $\quad s=0 \quad r=6 \quad \binom{2}{0}\binom{7}{1}/6^2 \qquad 5/36$

$\quad s=1 \quad r=0 \quad -\binom{2}{1}\binom{1}{1}/6^2$

$N=6$ $\quad x^6 \sum_0^6 {}_s \binom{6}{s}(-1)^s x^{6s} \sum_0^\infty {}_r \binom{5+r}{5} x^r$

"20" $\quad s=0 \quad r=14 \quad \binom{6}{0}\binom{19}{5}$

$\quad s=1 \quad r=8 \quad -\binom{6}{1}\binom{13}{5} \qquad \frac{11.628-7722+315}{6^6} = \frac{4221}{6^6}$

$\quad s=2 \quad r=2 \quad \binom{6}{2}\binom{7}{5}$

"21" $\quad s=0 \quad r=15 \quad \binom{6}{0}\binom{20}{5}$

$\quad s=1 \quad r=9 \quad -\binom{6}{1}\binom{14}{5} \qquad \frac{15.504-12012+840}{6^6} = \frac{4332}{6^6}$

$\quad s=2 \quad r=3 \quad \binom{6}{2}\binom{8}{5}$

"22" $\quad s=0 \quad r=16 \quad \binom{6}{0}\binom{21}{5}$

$\quad s=1 \quad r=10 \quad -\binom{6}{1}\binom{15}{5} \qquad \frac{20.349-18.018+1890}{6^6} = \frac{4221}{6^6}$

$\quad s=2 \quad r=4 \quad \binom{6}{2}\binom{9}{5}$

$$\frac{12.774}{6^6} = \frac{2129}{6^5} \text{ vs } \frac{16}{6^2} \qquad \frac{2129}{6^{\cancel{5}\,3}}\,\frac{\cancel{6^2}}{16} = \frac{2129}{3456} \approx 0.62$$

"19" $\binom{6}{0}\binom{18}{5}-\binom{6}{1}\binom{12}{5}+\binom{6}{2}\binom{6}{5} \qquad 8568-4752+90 \qquad 3906/6^6$

"23" $\binom{6}{0}\binom{22}{5}-\binom{6}{1}\binom{16}{5}+\binom{6}{2}\binom{10}{5} \qquad 26334-26208+3780 \qquad 3906/6^6$

19, 20, 21, 22, 23 $\quad \frac{30586}{6^6} = \frac{3431}{6^5} \approx 0.441 \qquad \frac{4}{9} \approx 0.444$

3) CALCULAR O NÚMERO DE SOLUÇÕES EM INTEIROS POSITIVOS, NEGATIVOS E NULOS DO SISTEMA

$y_1 + y_2 + y_3 = M$ PARA $M = 0, N, 2N, 3N, 4N$ E $5N$

$0 \le y_1 \le N-1$
$-N \le y_2 \le N-1$
$N \le y_3 \le 3N-1$

FUNÇÃO GERADORA $\frac{1-x^N}{1-x} \cancel{\frac{1}{x^N}} \frac{1-x^{2N}}{1-x} \cancel{x^N} \frac{1-x^{2N}}{1-x}$

$$\frac{(1-x^N)(1-x^{2N})^2}{(1-x)^3}$$

$$(1 - x^N - 2x^{2N} + 2x^{3N} + x^{4N} - x^{5N}) \sum_{r=0}^{\infty} \binom{r+2}{2} x^r$$

$$\binom{M+2}{2} - \binom{M-N+2}{2} - 2\binom{M-2N+2}{2} + 2\binom{M-3N+2}{2} + \binom{M-4N+2}{2} - \binom{M-5N+2}{2}$$

$\boxed{M=0}$ $\binom{2}{2}$ $\boxed{M=0 \text{ SOL: } 1}$

$\boxed{M=N}$ $\binom{N+2}{2} - \binom{2}{2} = \frac{(N+2)(N+1)}{2} - 1 = \boxed{\frac{N}{2}(N+3)}$

$\boxed{M=2N}$ $\binom{2N+2}{2} - \binom{N+2}{2} - 2\binom{2}{2} = \boxed{\frac{3}{2}N(N+1) - 2}$

$\boxed{M=3N}$ $\binom{3N+2}{2} - \binom{2N+2}{2} - 2\binom{N+2}{2} + 2\binom{2}{2} = \boxed{\frac{3}{2}N(N-1)}$

$\boxed{M=4N}$ $\binom{4N+2}{2} - \binom{3N+2}{2} - 2\binom{2N+2}{2} + 2\binom{N+2}{2} + \binom{2}{2} = \boxed{\frac{N}{2}(N-3) + 1}$

$\boxed{M=5N}$ $\binom{5N+2}{2} - \binom{4N+2}{2} - 2\binom{3N+2}{2} + 2\binom{2N+2}{2} + \binom{N+2}{2} - \binom{2}{2}$ $\boxed{M=5N \text{ SOL: } 0}$

EXEMPLOS $N=1$ $0, 1^2, 2$

$M=0$	1
$M=1$	2
$M=2$	1
$M=3$	0
$M=4$	0
$M=5$	0

$N=2$ $0, 1^3, 2^5, 3^7, 4^7, 5^5, 6^3, 7$

$M=0$	1
$M=2$	5
$M=4$	7
$M=6$	3
$M=8$	0
$M=10$	0