Exercícios

1) QUAL É A PROBABILIDADE DE EXTRAIR DE UMA CAIXA CONTENDO 100 FICHAS NUMERADAS DE 1 ATÉ 100 DUAS FICHAS CUJA SOMA SEJA UM MÚLTIPLO DE 3

NÚMERO TOTAL DE POSSIBILIDADE 100×99

$A: 1, 4, 7, \dots, 97, 100$ $\quad$ AB, BA, CC

$B: 2, 5, 8, \dots, 98$ $\quad$ $34 \cdot 33, 33 \cdot 34, 33 \cdot 32$

$C: 3, 6, 9, \dots, 99$ $\quad$ $33(68+32)$

PROBABILIDADE $33 \cdot 100 / 99 \cdot 100 = \boxed{1/3}$

2) TENDO 12 FICHAS NUMERADAS DE 1 ATÉ 12, QUAL É A PROBABILIDADE DE RETIRAR 3 FICHAS CUJA SOMA SEJA INFERIOR O IGUAL A 10?

TOTAL $12 \cdot 11 \cdot 10$

$\dfrac{\cancel{6} \cdot \cancel{11}}{2\ \cancel{12} \cdot \cancel{11} \cdot 10}$ $\quad$ $\boxed{1/20}$

123 $\quad$ 134 $\quad$ 145 $\quad$ 234 $\quad$ $11 \cdot 3!$

4 $\quad$ 5 $\quad$ 5

5 $\quad$ 6

6

7

3) ENCONTRAR O NÚMERO DE SOLUÇÕES EM INTEIROS DE $y_1 + y_2 + y_3 + y_4 = 25$

COM $-2 \le y_1 \le 4$ $\quad$ $4 \le y_2 \le 7$ $\quad$ $1 \le y_3 \le 8$ E $y_4 \ge 6$ CASO A

$y_4 = 6$ CASO B

CASO A

$y_1 = x_1 - 3$

$y_2 = x_2 + 3$

$y_3 = x_3$

$y_4 = x_4 + 5$

$x_1 + x_2 + x_3 + x_4 = 25 + 3 - 3 - 5 = 20$

TOT $\binom{19}{3} = \dfrac{19 \cdot \cancel{18}^{3} \cdot 17}{\cancel{6}}$

$G1(>7)\ \binom{12}{3}$ $\quad$ $G2(>4)\ \binom{15}{3}$ $\quad$ $G_3(>8)\ \binom{11}{3}$

$G12(>11)\ \binom{8}{3}$ $\quad$ $G13(>15)\ \binom{4}{3}$ $\quad$ $G23(>12)\ \binom{7}{3}$

~~$G123(>19)$~~

$(12 \cdot 11 \cdot 10 + \underbrace{15 \cdot 14 \cdot 13}_{5 \cdot 7 \cdot 13} + \underbrace{11 \cdot 10 \cdot 9}_{55 \cdot 3} - 8 \cdot 7 \cdot 6 - 4 \cdot 3 \cdot 2 - 7 \cdot 6 \cdot 5)/6$

$220 + 35 \cdot 13 + 165 - 56 - 4 - 35$

$220 + 35 \cdot 12 + 105$

$\begin{array}{r} 70 \\ 35 \\ \hline 420 \end{array}$ $\quad$ $\boxed{745}$

$19 \cdot 51 = (20-1)51 = 1020 - 51 = \boxed{969}$

$\begin{array}{r} 969 \\ 745 \\ \hline 224 \end{array}$ $\quad$ $\boxed{\text{RES. } 224}$

18 $\quad$ CASO B $\quad$ $y_4 = 6$ $\quad$ $y_1 + y_2 + y_3 = 19$

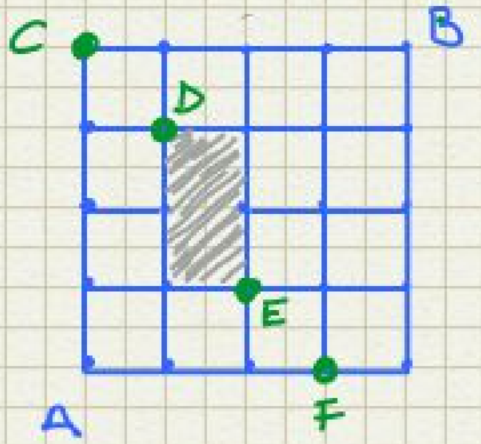
$x_1 + x_2 + x_3 = 19$ $\quad$ TOT $\binom{18}{2} = 9 \cdot 17 = 153$

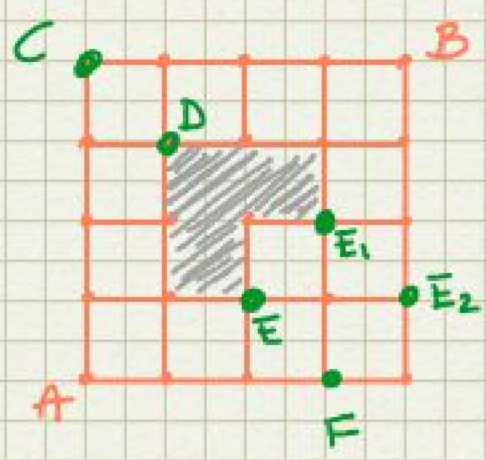
$G_1(>7)\ \binom{11}{2}$ $\quad$ $G_2(>4)\ \binom{14}{2}$ $\quad$ $G_3(>8)\ \binom{10}{2}$ $\quad$ $55 + 91 + 45$ $\quad$ $153 - 152 = \boxed{1}$

$G_{12}(>11)\ \binom{7}{2}$ $\quad$ $G_{13}(>15)\ \binom{3}{2}$ $\quad$ $G_{23}(>12)\ \binom{6}{2}$ $\quad$ $-21 - 3 - 15$ $\quad$ $55 + 70 + 27 = 152$

4) ENCONTRE TODOS OS POSSÍVEIS CAMINHOS QUE LIGAM A a B

CASO 1





CASO 2

CASO 1

ACB $\binom{4}{0}\cdot\binom{4}{4}$ 1

ADB $\binom{4}{1}\cdot\binom{4}{3}$ 16

AEB $\binom{3}{2}\cdot\binom{5}{2}$ 30

AFB $\binom{3}{0}\cdot\binom{5}{1}$ 5

TOT $\boxed{52}$

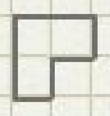
ACB : 1   ADB : 16

AEE_1B : $\binom{3}{2}\binom{2}{1}\binom{3}{1}$ 18

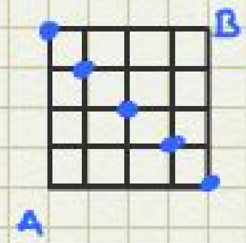
AEE_2B : $\binom{3}{2}\binom{2}{2}\binom{3}{0}$ 3

AFB : 5

TOT : $\boxed{43}$

REMOVENDO  E  OS CAMINHOS SERIAM $\binom{8}{4} = \frac{\cancel{8}\cdot 7\cdot \overset{2}{\cancel{6}}\cdot 5}{\cancel{24}}$

$\boxed{70}$



PODEMOS CHEGAR A $\binom{8}{4}$ TAMBÉM DECOMPONDO OS CAMINHOS

$$\binom{4}{0}\binom{4}{4}+\binom{4}{1}\binom{4}{3}+\binom{4}{2}\binom{4}{2}+\binom{4}{3}\binom{4}{1}+\binom{4}{4}\binom{4}{0}$$

$$1 + 16 + 36 + 16 + 1$$

$$\sum_{k=0}^{n}\binom{n}{k}\binom{n}{n-k}=\binom{2n}{n}$$

INTERESSANTE PARA CONSTRUIR IDENTIDADES COM OS COEFICIENTES BINOMIAIS

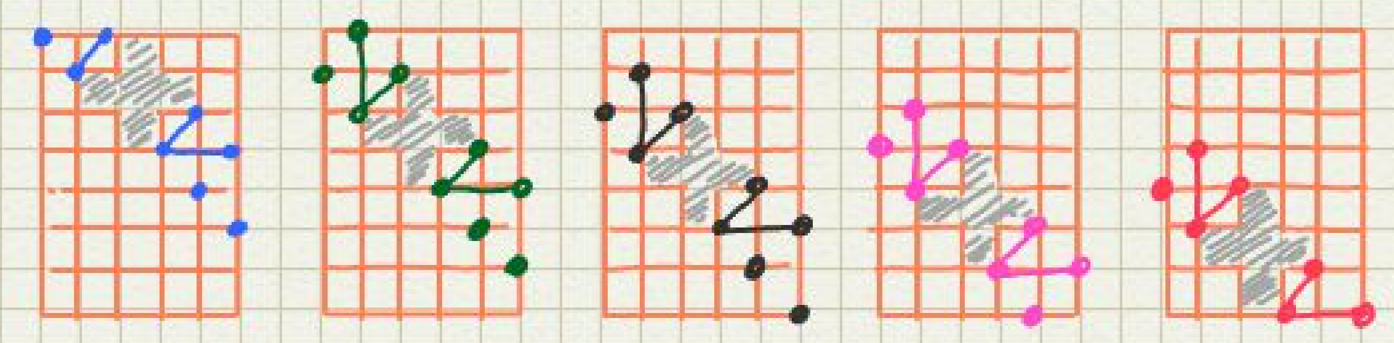
$$\binom{3}{0}\binom{3}{3}+\binom{3}{1}\binom{3}{2}+\binom{3}{2}\binom{3}{1}+\binom{3}{3}\binom{3}{0}$$

$$1 + 9 + 9 + 1$$

$$20 = \binom{6}{3}$$

$$\boxed{\sum_{0}^{n}{}_{k}\binom{n}{k}^2=\binom{2n}{n}}$$

PROVAR POR INDUÇÃO



$\binom{7}{0}\binom{5}{5}$ $\quad$ $\binom{6}{0}\binom{6}{5}$ $\quad$ $\binom{5}{0}\binom{7}{5}$ $\quad$ $\binom{4}{0}\binom{8}{5}$ $\quad$ $\binom{3}{0}\binom{9}{5}$

$\binom{6}{1}\binom{2}{0}\binom{4}{4}$ $\quad$ $\binom{5}{1}\binom{2}{0}\binom{5}{4}$ $\quad$ $\binom{4}{1}\binom{2}{0}\binom{6}{4}$ $\quad$ $\binom{3}{1}\binom{2}{0}\binom{7}{4}$

$\binom{7}{1}\binom{2}{1}\binom{3}{3}$ $\quad$ $\binom{6}{1}\binom{2}{1}\binom{4}{3}$ $\quad$ $\binom{5}{1}\binom{2}{1}\binom{5}{3}$ $\quad$ $\binom{4}{1}\binom{2}{1}\binom{6}{3}$ $\quad$ $\binom{3}{1}\binom{2}{0}\binom{7}{3}$

$\binom{7}{3}\binom{2}{1}\binom{3}{1}$ $\quad$ $\binom{6}{3}\binom{2}{1}\binom{4}{1}$ $\quad$ $\binom{5}{3}\binom{2}{1}\binom{5}{1}$ $\quad$ $\binom{4}{3}\binom{2}{1}\binom{6}{1}$ $\quad$ $\binom{3}{3}\binom{2}{1}\binom{7}{1}$

$\binom{7}{3}\binom{2}{2}\binom{3}{0}$ $\quad$ $\binom{6}{3}\binom{2}{2}\binom{4}{0}$ $\quad$ $\binom{5}{3}\binom{2}{2}\binom{5}{0}$ $\quad$ $\binom{4}{3}\binom{2}{2}\binom{6}{0}$ $\quad$ $\binom{3}{3}\binom{2}{0}\binom{7}{0}$

$\binom{7}{4}\binom{5}{1}$ $\quad$ $\binom{6}{4}\binom{6}{1}$ $\quad$ $\binom{5}{4}\binom{7}{1}$ $\quad$ $\binom{4}{4}\binom{8}{1}$

$\binom{7}{5}\binom{5}{0}$ $\quad$ $\binom{6}{5}\binom{6}{0}$ $\quad$ $\binom{5}{5}\binom{7}{0}$

1	6	21	56	126
	6	25	60	105
14	48	100	160	210
210	160	100	48	14
35	20	10	4	1
175	90	35	8	
21	6	1		
$\boxed{456}$	$\boxed{336}$	$\boxed{292}$	$\boxed{336}$	$\boxed{456}$

- $y_1+y_2+y_3 = 25$ $\qquad$ $3\le y_1\le N$, $2\le y_2\le 4$, $1\le y_3\le 8$

$y_1 = x_1+2$
$y_2 = x_2+1$
$y_3 = x_3$

$\boxed{x_1+x_2+x_3 = 22}$ $\qquad$ TOT $\binom{21}{2}$

A $(>N-2)$ $\binom{23-N}{2}$ $\qquad$ AB $(>N+1)$ $\binom{20-N}{2}$ $\qquad$ ABC $(>N+9)$ $\cancel{\binom{12-N}{2}}$

B (>3) $\binom{18}{2}$ $\qquad$ AC $(>N+6)$ $\binom{15-N}{2}$

C (>8) $\binom{13}{2}$ $\qquad$ BC (>11) $\binom{10}{2}$

MINIMAL N $\quad$ N=13 GARANTE UMA SOLUSÃO

$N=13$ (CONTROLE) $\binom{21}{2}-\cancel{\binom{10}{2}}-\binom{18}{2}-\binom{13}{2}+\binom{7}{2}+\binom{2}{2}+\cancel{\binom{10}{2}}$

$210 \quad -153 \quad -78 \quad +21 \quad +1 \qquad = 1$!!!

$N\geq 14$ $\binom{21}{2}-\binom{23-N}{2}-\binom{18}{2}-\binom{13}{2}+\binom{20-N}{2}+\binom{10}{2}$

$21 \cdot 20 - (23-N)(22-N) - 18 \cdot 17 - 13 \cdot 12 + (20-N)(19-N) + 10 \cdot 9 = 2 \cdot sol$

$-78 + 6N = 2\, sol$ $\qquad \boxed{\text{soluções} = 3N - 39 \qquad 14 \le N \le 18}$

$\binom{20-N}{2}!$

$N=14$	3
$N=15$	6
$N=16$	9
$N=17$	12
$N=18$	15

$19 \le N \le 21$ $\qquad 21 \cdot 20 - (23-N)(22-N) - 18 \cdot 17 - 13 \cdot 12 + 10 \cdot 9 = 2 \cdot sol$

$48 - (23-N)(22-N) = 2\, sol$

$\boxed{sol = 24 - \frac{(23-N)(22-N)}{2}}$

$N=19$	$24-6$	18
20	$24-3$	21
21	$24-1$	23

$N \ge 22$ $\qquad 48 = 2\, sol$ $\qquad \cancel{\boxed{sol = 24}}$ $\qquad (22,2,1) + 23$

Teste do dia 13 de setembro

BB, BC, CB, BR, RB $\qquad$ RC, CR

Ex1) BABACAR

TOT $\frac{7!}{3!\,2!}$

1) Con Con A x x x x $\qquad 5 \cdot \frac{4!}{2!} \qquad 2 \cdot \frac{4!}{2!\,2!}$

2) B x A x x x A $\qquad 4!$

3) x B x x x x A $\qquad 5!/2!$

12) B Con A x x x A $\qquad 3 \cdot 3!$

13) Con B A x x x A $\qquad 3 \cdot 3!$

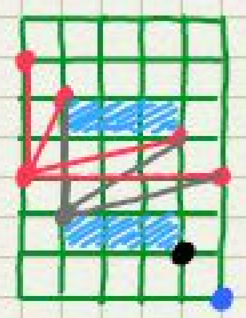
23) B B A x x x A $\qquad 3!$

123) B B A x x x A $\qquad 3!$

$3 \cdot 4! + 4! + \frac{5!}{2!} - 6 \cdot 3! \cancel{-3!} \cancel{+3!}$ $\qquad 16 \cdot 3! + 10 \cdot 3! - 6 \cdot 3! = 20 \cdot 3!$

$\frac{20 \cdot 3!}{7!/3!\,2!} = \frac{\cancel{10}\,2 \cdot \cancel{3!} \cdot \cancel{3}}{7 \cdot \cancel{6}\,5 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{2!}} = \boxed{\frac{2}{7}}$

Ex. 2



$\binom{3}{0}$ $\binom{3}{0}\binom{6}{5}$ 6

$\binom{3}{1}\binom{6}{4}$ 45

$\binom{5}{4}\binom{4}{1}$ 20

$\binom{5}{5}\binom{4}{0}$ 1

$\binom{5}{4}\binom{7}{1}$ 35

$\binom{3}{1}$ $\binom{3}{0}\binom{6}{4}$ 45

$\binom{5}{3}\binom{4}{1}$ 120

$\binom{5}{4}\binom{4}{0}$ 15

$\binom{5}{5}\binom{7}{0}$ 1

CAMINHOS $\boxed{288}$

TOT $\frac{12!}{5!\,7!} = \frac{\cancel{12} \cdot 11 \cdot \cancel{10} \cdot 9 \cdot 8}{\cancel{120}} = \boxed{792}$

$\boxed{\frac{288}{792} = \frac{4}{11}}$

EX 3)

$$y_1 + y_2 + y_3 = 25$$
$$-N \leq y_1 \leq N$$
$$2 \leq y_2 \leq 4$$
$$1 \leq y_3 \leq 8$$

$y_1 = x_1 - N - 1$

$y_2 = x_2 + 1$

$y_3 = x_3$

$$\boxed{x_1 + x_2 + x_3 = 25 + N}$$

TOT $\binom{24+N}{2}$

A $(>2N+1)$ $\binom{23-N}{2}$

B (>3) $\binom{21+N}{2}$

C (>8) $\binom{16+N}{2}$

AB $(>2N+4)$ $\binom{20-N}{2}$

AC $(>2N+9)$ $\binom{15-N}{2}$

BC (>11) $\binom{13+N}{2}$

ABC $(>2N+12)$ $\cancel{\binom{12-N}{2}}$

MÍNIMO N É 13

N=13 TEREMOS UMA SOLUÇÃO

$N=13$ $\binom{24+N}{2} - \binom{23-N}{2} - \binom{21+N}{2} - \binom{16+N}{2} + \binom{20-N}{2} + \binom{15-N}{2} + \binom{13+N}{2} = \text{SOL}$

$\binom{37}{2} - \binom{10}{2} - \binom{34}{2} - \binom{29}{2} + \binom{7}{2} + \binom{2}{2} + \binom{26}{2} = 1$ V

$14 \leq N \leq 18$ $\binom{24+N}{2} - \binom{23-N}{2} - \binom{21+N}{2} - \binom{16+N}{2} + \binom{20-N}{2} + \binom{13+N}{2} = \text{SOL}$

$19 \leq N \leq 21$ $\binom{24+N}{2} - \binom{23-N}{2} - \binom{21+N}{2} - \binom{16+N}{2} + \binom{13+N}{2} = \text{SOL}$

$N \geq 22$ $\binom{24+N}{2} - \binom{21+N}{2} - \binom{16+N}{2} + \binom{13+N}{2} = \text{SOL}$

$$(24+N)(23+N) - (21+N)(20+N) - (16+N)(15+N) + (13+N)(12+N) = 2\,\text{SOL}$$

$$552 + \cancel{47N} + \cancel{N^2} - 420 - \cancel{41N} - \cancel{N^2} - 240 - \cancel{31N} - \cancel{N^2} + 136 + \cancel{25N} + \cancel{N^2} = 2\,\text{SOL}$$

$48 = 2$ SOL $\boxed{\text{SOL} = 24}$