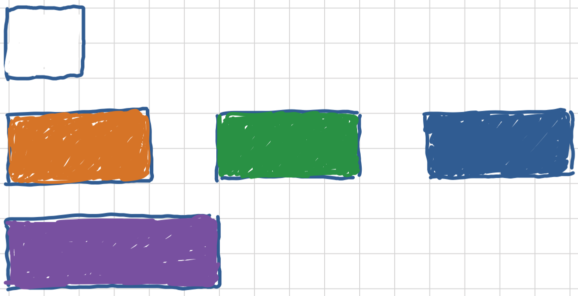
EX. 4

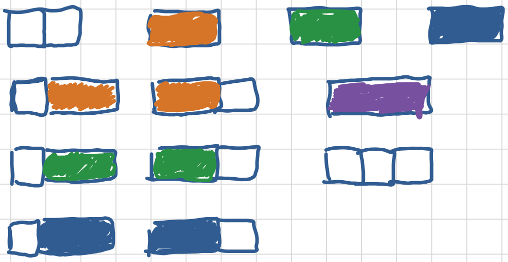
BLOCOS



$C(1) = 1$

$C(2) = 4$

$C(3) = 8$



$$C(n+3) = C(n+2) + 3C(n+1) + C(n)$$

$$x^3 - x^2 - 3x - 1 = 0 \Rightarrow (x+1)(x^2-2x-1) = 0$$

$$\text{"}-1\text{"} \qquad \text{"}1 \pm \sqrt{2}\text{"}$$

$$C(n) = a(-1)^n + b\left(1-\sqrt{2}\right)^n + c\left(1+\sqrt{2}\right)^n$$

$$-a + b(1-\sqrt{2}) + c(1+\sqrt{2}) = 1 \qquad (1)$$

$$a + b(1-\sqrt{2})^2 + c(1+\sqrt{2})^2 = 4 \qquad (2)$$

$$-a + b(1-\sqrt{2})^3 + c(1+\sqrt{2})^3 = 8 \qquad (3)$$

$(1)+(2)$ $\quad b(1-\sqrt{2})(1+1-\sqrt{2}) + c(1+\sqrt{2})(1+1+\sqrt{2}) = 5$

$(2)+(3)$ $\quad b(1-\sqrt{2})^2(1+1-\sqrt{2}) + c(1+\sqrt{2})^2(1+1+\sqrt{2}) = 12$

$$b(1-\sqrt{2})(2-\sqrt{2}) + c(1+\sqrt{2})(2+\sqrt{2}) = 5$$

$$b(1-\sqrt{2})^2(2-\sqrt{2}) + c(1+\sqrt{2})^2(2+\sqrt{2}) = 12$$

$$\hookrightarrow \sqrt{2}(\sqrt{2}-1) \qquad \hookrightarrow \sqrt{2}(1+\sqrt{2})$$

$$-b\sqrt{2}(1-\sqrt{2})^2 + c\sqrt{2}(1+\sqrt{2})^2 = 5 \qquad \times (1-\sqrt{2})$$

$$-b\sqrt{2}(1-\sqrt{2})^3 + c\sqrt{2}(1+\sqrt{2})^3 = 12$$

$$-b\sqrt{2}(1-\sqrt{2})^3 + c\sqrt{2}(1-\sqrt{2})(1+\sqrt{2})^2 = 5(1-\sqrt{2})$$

SUBTRAINDO. $\quad c\sqrt{2}(1+\sqrt{2})^2 \underbrace{(\cancel{1}+\sqrt{2}-\cancel{1}+\sqrt{2})}_{2\sqrt{2}} = 12 - 5 + 5\sqrt{2} = 7 + 5\sqrt{2}$

$$4c = (7+5\sqrt{2})/(1+\sqrt{2})^2$$

$$= (7+5\sqrt{2})(\sqrt{2}-1)/(1+\sqrt{2}) = (7\sqrt{2}-7+10-5\sqrt{2})/(1+\sqrt{2})$$

$$C=\frac{3+2\sqrt2}{4(1+\sqrt2)}$$
$$=\frac{(1+\sqrt2)^2}{4(1+\sqrt2)}=\frac{1+\sqrt2}{4}$$

$$b\cancel{\sqrt2}(1-\sqrt2)^2=c\cancel{\sqrt2}(1+\sqrt2)^2-\frac{5}{\sqrt2}$$
$$b(1-\sqrt2)^2=\frac{(1+\sqrt2)^3}{4}-\frac{10\sqrt2}{4}$$
$$b(1+\sqrt2)(1-\sqrt2)^2=\frac{(1+\sqrt2)^4-10\sqrt2(1+\sqrt2)}{4}$$
$$-b(1-\sqrt2)=[(3+2\sqrt2)^2-10\sqrt2-20]/4$$
$$=(17+12\sqrt2-10\sqrt2-20)/4$$
$$=(2\sqrt2-3)/4$$
$$=-(1-\sqrt2)^2/4$$

$$b=\frac{1-\sqrt2}{4}$$

$$a=b(1-\sqrt2)+c(1+\sqrt2)-1$$
$$=[(1-\sqrt2)^2+(1+\sqrt2)^2-4]/4=(3-2\sqrt2+3+2\sqrt2-4)/4=\frac{1}{2}$$

Solução

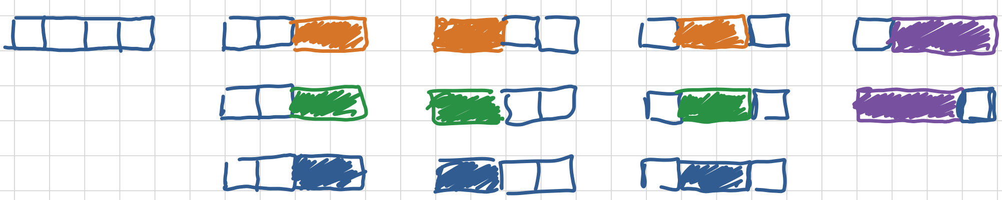
$$C(n)=\frac{(-1)^n}{2}+\frac{(1-\sqrt2)^{n+1}}{4}+\frac{(1+\sqrt2)^{n+1}}{4}$$

$$C(n)=[2(-1)^n+(1-\sqrt2)^{n+1}+(1+\sqrt2)^{n+1}]/4$$

$$C(1)=(-2+3-\cancel{2\sqrt2}+3+\cancel{2\sqrt2})/4=1$$
$$C(2)=[2+(3-2\sqrt2)(1-\sqrt2)+(3+2\sqrt2)(1+\sqrt2)]/4=1$$
$$=(2+7-\cancel{5\sqrt2}+7+\cancel{5\sqrt2})/4=4$$
$$C(3)=[-2+(3-2\sqrt2)(3-2\sqrt2)+(3+2\sqrt2)(3+2\sqrt2)]/4$$
$$=[-2+17-\cancel{12\sqrt2}+17+\cancel{12\sqrt2})/4=8$$

$$C(4)=[+2+(17-12\sqrt2)(1-\sqrt2)+(17+12\sqrt2)(1+\sqrt2)]/4$$
$$=(2+41-\cancel{29\sqrt2}+41+\cancel{29\sqrt2})/4=21$$

Possibilidades



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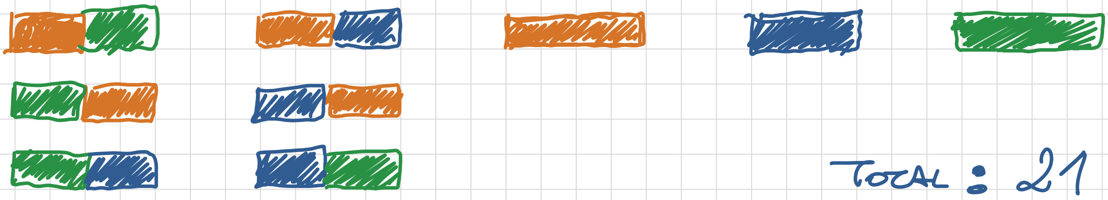

TOTAL : 21

$$\lim_{n \to \infty} \frac{C(n+1)}{C(n)} = 1+\sqrt{2} \sim 2.414$$

$$\{1, 4, 8, 21, 49, 120, 288, 697, 1681, 4060\}$$

PRIMEIROS 10 TERMOS

BLOCOS DE 10cm

A IMPORTÂNCIA DO LIMITE !!!

IMAGINAMOS DE TER CALCULADO O LIMITE
E COSTRUIMOS A SÉRIE APROXIMADA PARA TER UMA IDÉIA

$$\{1, 4, 8, 21, 21(1+\sqrt{2}), 21(1+\sqrt{2})^2, 21(1+\sqrt{2})^3, 21(1+\sqrt{2})^4, 21(1+\sqrt{2})^5, 21(1+\sqrt{2})^6\}$$

$$\{1, 4, 8, 21, 51, 122, 295, 713, 1722, 4158\}$$