

Sequências

FIBONACCI $F(n) + F(n+1) = F(n+2)$

$$1, 1, 2, 3, 5, 8, 13, 21, \dots$$

$$F(n) = a \alpha_1^n + b \alpha_2^n$$

COM $\alpha_{1,2}$ DETERMINADOS RESOLVENDO A EQUAÇÃO QUE OBTENHAMOS DA SEQUÊNCIA, NESTE CASO $\alpha^n + \alpha^{n+1} = \alpha^{n+2} \rightarrow 1 + \alpha = \alpha^2$

$$\alpha_{1,2} = \frac{1 \pm \sqrt{5}}{2}$$

$$F(n) = a \left(\frac{1 + \sqrt{5}}{2} \right)^n + b \left(\frac{1 - \sqrt{5}}{2} \right)^n$$

PARA DETERMINAR a E b USAREMOS OS VALORES DE $F(1)$ E $F(2)$

$$F(1) = a \frac{1 + \sqrt{5}}{2} + b \frac{1 - \sqrt{5}}{2} = 1$$

$$F(2) = a \left(\frac{1 + \sqrt{5}}{2} \right)^2 + b \left(\frac{1 - \sqrt{5}}{2} \right)^2 = 1$$

ENTÃO TEREMOS $\frac{a+b}{2} + \sqrt{5} \frac{a-b}{2} = 1$ $\Rightarrow a+b=0$
 $3 \frac{a+b}{2} + \sqrt{5} \frac{a-b}{2} = 1$

CONSEQUENTEMENTE MANTER $a-b = 2/\sqrt{5}$

$$a = -b = 1/\sqrt{5}$$

$$F(n) = \frac{1}{\sqrt{5}} \left(\frac{1 + \sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1 - \sqrt{5}}{2} \right)^n$$

$$\lim_{n \rightarrow \infty} \frac{F(n+1)}{F(n)} = \frac{1 + \sqrt{5}}{2} \quad \text{RAZÃO ÁUREA}$$

O QUE ACONTECE SE MANTERMOS A FÓRMULA DA SEQUÊNCIA MAS MUDAMOS O SEGUNDO ELEMENTO

$$\tilde{F}(n) + \tilde{F}(n+1) = \tilde{F}(n+2) \quad 1, 3, 4, 7, 11, 18, 29, \dots$$

$$\tilde{F}(n) = \tilde{\alpha} \left(\frac{1+\sqrt{5}}{2} \right)^n + \tilde{\beta} \left(\frac{1-\sqrt{5}}{2} \right)^n$$

$$\tilde{F}(1) = \tilde{\alpha} \frac{1+\sqrt{5}}{2} + \tilde{\beta} \frac{1-\sqrt{5}}{2} = 1$$

$$\tilde{F}(2) = \tilde{\alpha} \left(\frac{1+\sqrt{5}}{2} \right)^2 + \tilde{\beta} \left(\frac{1-\sqrt{5}}{2} \right)^2 = 3$$

$$\begin{array}{l} \frac{\tilde{\alpha} + \tilde{\beta}}{2} + \sqrt{5} \frac{\tilde{\alpha} - \tilde{\beta}}{2} = 1 \quad \frac{\tilde{\alpha} + \tilde{\beta}}{2} + \sqrt{5} \frac{\tilde{\alpha} - \tilde{\beta}}{2} = 1 \\ 3 \frac{\tilde{\alpha} + \tilde{\beta}}{2} + \sqrt{5} \frac{\tilde{\alpha} - \tilde{\beta}}{2} = 3 \quad \frac{\tilde{\alpha} + \tilde{\beta}}{2} + \sqrt{5} \frac{\tilde{\alpha} - \tilde{\beta}}{6} = 1 \end{array}$$

$$\sqrt{5}(\tilde{\alpha} - \tilde{\beta}) \left(\frac{1}{2} - \frac{1}{6} \right) = 0$$

$$\tilde{\alpha} = \tilde{\beta}$$

$$\frac{\tilde{\alpha} + \tilde{\beta}}{2} = 1 \Rightarrow \tilde{\alpha} = \tilde{\beta} = 1$$

$$\tilde{F}(n) = \left(\frac{1+\sqrt{5}}{2} \right)^n + \left(\frac{1-\sqrt{5}}{2} \right)^n$$

CLARAMENTE $\lim_{n \rightarrow \infty} \frac{\tilde{F}(n+1)}{\tilde{F}(n)} = \lim_{n \rightarrow \infty} \frac{F(n+1)}{F(n)} = \frac{1+\sqrt{5}}{2}$

CONSIDERAMOS AGORA LA SIGUIENTE SECUENCIA

$$2G(n) + G(n+1) = G(n+2)$$

$$1, 1, 3, 5, 11, 21, 42, \dots$$

CALCULAMOS $\alpha_{1,2}$ $2\alpha^n + \alpha^{n+1} = \alpha^{n+2}$

$$\alpha^2 - \alpha - 2 = 0 \quad \alpha_{1,2} = \frac{1 \pm \sqrt{9}}{2} \begin{array}{l} 2 \\ -1 \end{array}$$

$$G(n) = a 2^n + b (-1)^n$$

$$\left. \begin{array}{l} G(1) = 2a - b = 1 \\ G(2) = 4a + b = 1 \end{array} \right\} \quad 6a = 2 \Rightarrow a = \frac{1}{3}$$

$$b = \frac{2}{3} - 1 = -\frac{1}{3}$$

$$G(n) = [2^n - (-1)^n] / 3$$

$$\lim_{n \rightarrow \infty} \frac{G(n+1)}{G(n)} = 2$$

CONSIDEREMOS AGORA $-H(n) + 2H(n+1) = H(n+2)$

$$-1, 1, 3, 5, 7, 9, \dots$$

$$-\alpha^n + 2\alpha^{n+1} = \alpha^{n+2}$$

$$\alpha^2 - 2\alpha + 1 = 0 \quad \alpha_{1/2} = 1$$

$$H(n) = a \cdot 1^n + n b \cdot 1^n = a + nb$$

$$\left. \begin{array}{l} H(1) = a + b = -1 \\ H(2) = a + 2b = 1 \end{array} \right\} b = 2 \Rightarrow a = -3$$

$$H(n) = 2n - 3$$

$$\lim_{n \rightarrow \infty} \frac{H(n+1)}{H(n)} = 1$$

$$4S(n) + 4S(n+1) - S(n+2) = S(n+3)$$

$$1, 1, 1, 4+4-1=7, 4+4-7=1$$

$$4+28-1=31, \dots$$

$$4\alpha^n + 4\alpha^{n+1} - \alpha^{n+2} = \alpha^{n+3}$$

$$\alpha^3 + \alpha^2 - 4\alpha - 4 = 0$$

$$\alpha^2(\alpha+1) - 4(\alpha+1) = 0$$

$$(\alpha^2 - 4)(\alpha + 1) = 0 \quad \text{soluções } -1, \pm 2$$

$$S(n) = a2^n + b(-2)^n + c(-1)^n$$

$$S(1) = 2a - 2b - c = 1$$

$$S(2) = 4a + 4b + c = 1$$

$$S(3) = 8a - 8b - c = 1$$

$$6a + 2b = 2$$

$$12a - 4b = 2$$

$$a = \frac{1}{4}$$

$$2b = 2 - \frac{3}{2} = \frac{1}{2}$$

$$b = \frac{1}{4}, c = -1$$

$$S(n) = 2^n \frac{1+(-1)^n}{4} - (-1)^n$$

$$\lim_{n \rightarrow \infty} \frac{S(2n+2)}{S(2n)} = 1$$

COMO CRIAR EXERCÍCIOS

$$(x-1)^2(x^2-4)=0 \Rightarrow a1^n + b n 1^n + c(-2)^n + d 2^n$$
$$(x^3-2x+1)(x^2-4) = x^4 - 2x^3 + x^2 - 4x^2 + 8x - 4$$
$$= x^4 - 2x^3 - 3x^2 + 8x - 4$$

$$F(n+4) = 2F(n+3) + 3F(n+2) - 8F(n+1) + 4F(n)$$

$$a=b=c=d=1 \quad F(n) = 1 + n + [(-1)^n + 1]2^n$$

$$F(1)=2, F(2)=11, F(3)=4, F(4)=37$$

$$\lim_{n \rightarrow +\infty} \frac{F(2n+2)}{F(2n)} = \lim_{n \rightarrow +\infty} \frac{1+2n+5+3 \cdot 2^{2n+2}}{1+2n+2 \cdot 2^{2n}} = 4$$

$$\lim_{n \rightarrow +\infty} \frac{F(2n+3)}{F(2n+1)} = \lim_{n \rightarrow +\infty} \frac{1+2n+3}{1+2n+1} = 1$$

$$x^4 - 2x^3 - 3x^2 + 8x - 4 = 0$$

$$x^4 - 2x^3 + x^2 - 4x^2 + 8x - 4 = 0$$

$$x^2(x^2-2x+1) - 4(x^2-2x+1) = 0$$

$$(x^2-4)(x-1)^2 = 0$$

$$a1^n + b n 1^n + c(-2)^n + d 2^n$$

$$a+b-2c+2d=2 \quad (1) \quad L(1)-(3):$$

$$a+2b+4c+4d=11 \quad (2) \quad 3a+b=4$$

$$a+3b-8c+8d=4 \quad (3) \quad 4(2)-(4):$$

$$a+4b+16c+16d=37 \quad (4) \quad 3a+4b=7$$

$$a=b=1$$

$$(1) \quad d=c \quad (2) \quad 3+8c=11 \Rightarrow c=1$$

$$F(n) = 1 + n + [1 + (-1)^n]2^n \quad \checkmark$$