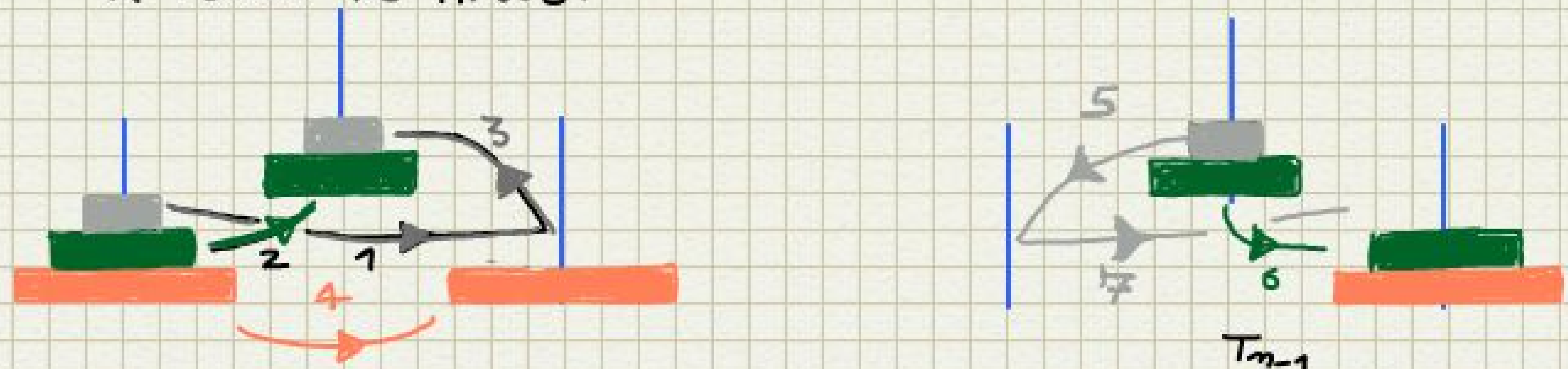
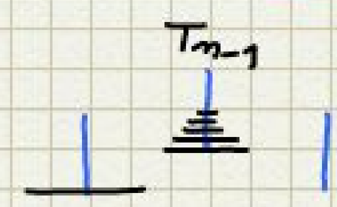
Fórmulas de recorrência

A TORRE DE HANOI



NO CASO DE n DISCOS SERIAM



$T_{n-1} + 1 + T_{n-1}$

$$\boxed{T_n = 2T_{n-1} + 1}$$

$$T_n + 1 = 2(T_{n-1} + 1)$$

$\tau_n = 2\tau_{n-1}$ solução α^n

$\alpha^n = 2\alpha^{n-1} \longrightarrow \alpha = 2$ solução 2^n

$T_n + 1 = 2^n \longrightarrow \boxed{T_n = 2^n - 1}$

SEQUÊNCIA DE FIBONACCI

1, 1, 2, 3, 5, 8, ...

$f_{n+2} = f_{n+1} + f_n$ $\quad \alpha^2 = \alpha + 1$

$\alpha = (1 \pm \sqrt{5})/2$

$$A\left(\frac{1-\sqrt{5}}{2}\right)^n + B\left(\frac{1+\sqrt{5}}{2}\right)^n$$

$n=1 \quad 1$
$n=2 \quad 1$

$$A\,\frac{1-\sqrt{5}}{2} + B\,\frac{1+\sqrt{5}}{2} = 1$$

$$A\left(\frac{1-\sqrt{5}}{2}\right)^2 + B\left(\frac{1+\sqrt{5}}{2}\right)^2 = 1$$

$$\left.\begin{array}{l} \frac{A+B}{2} + \frac{B-A}{2}\sqrt{5} = 1 \\ \frac{A+B}{4}\,6 + \frac{B-A}{4}\,2\sqrt{5} = 1 \end{array}\right\} \quad A+B=0\,!$$

$A = -B$

$B = 1/\sqrt{5}$

solução $\boxed{\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n}$

$$f_n = 2f_{n-1} - 3f_{n-2} + 6f_{n-3}$$

$$\alpha^3 - 2\alpha^2 + 3\alpha - 6 = 0$$

$$\alpha^2(\alpha-2) + 3(\alpha-2) = 0$$

$$(\alpha^2+3)(\alpha-2) = 0 \Longrightarrow \boxed{\alpha = \pm i\sqrt{3}, \ \alpha = 2}$$

$f_0 = 4 \quad f_1 = f_2 = 2$

$$A + B + C = 4$$
$$i\sqrt{3}(A - B) + 2C = 2$$
$$-3(A+B) + 4C = 2$$

$$A+B+C=4$$
$$-(A+B)+\frac{4}{3}C=\frac{2}{3}$$
$$\frac{7}{3}C=\frac{14}{3} \qquad \boxed{C=2}$$

$$A+B=2$$
$$i\sqrt{3}(A-B)=-2$$
$$\cancel{2}A=\cancel{2}-\frac{2}{i\sqrt{3}} \qquad \boxed{A=1+\frac{i}{\sqrt{3}}}$$

$$B=2-A \qquad \boxed{B=1-\frac{i}{\sqrt{3}}}$$

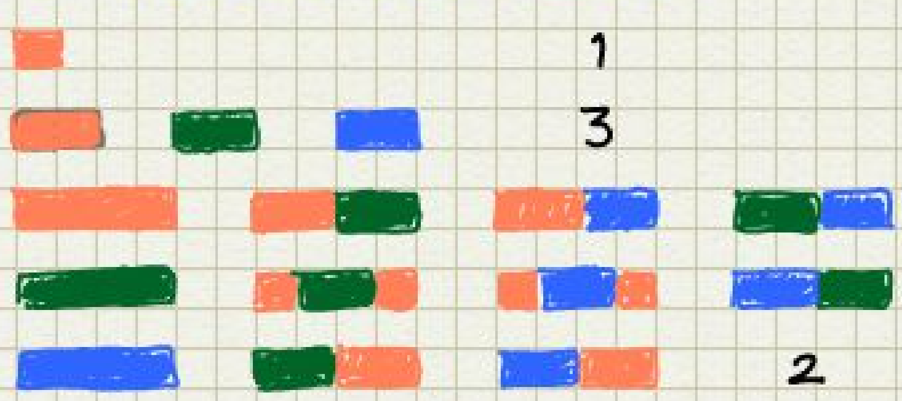
solução $\boxed{\left(1+\frac{i}{\sqrt{3}}\right)(i\sqrt{3})^n+\left(1-\frac{i}{\sqrt{3}}\right)(-i\sqrt{3})^n+2\,2^n}$

Blocos de comprimento 1cm laranjas
Blocos de comprimentos 2 cm verdes azuis
Quantos blocos de n cm podemos fazer?

$n=1$ simples

$n=2$

$n=4$



1

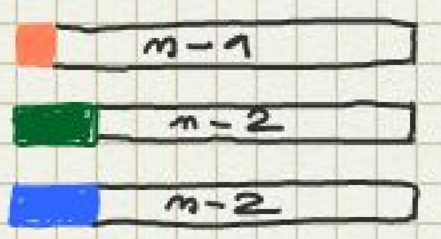
3

11

2

3 3 3

n cm



$$C_n=C_{n-1}+2C_{n-2}$$

$$\alpha^2-\alpha-2\alpha=0 \qquad \alpha=(1\pm3)/2$$
$$-1,\ 2$$

Solução $\quad A(-1)^n+B(2)^n$

$n=1 \quad -A+2B=1 \;\rightarrow\; 6B=4 \;\rightarrow\; B=\frac{2}{3} \qquad A=2B-1=\frac{1}{3}$

$n=2 \quad A+4B=3$

$$\boxed{\frac{1}{3}(-1)^n+\frac{2}{3}(2)^n}$$

controle $\quad n=4 \quad \frac{1}{3}+\frac{32}{3}=11$