Sequências

FIBONACCI $\qquad F(n)+F(n+1)=F(n+2)$

1, 1, 2, 3, 5, 8, 13, 21, ...

$$F(n)=a\,\alpha_1^n+b\,\alpha_2^n$$

COM $\alpha_{1,2}$ DETERMINADOS RESOLVENDO A EQUAÇÃO QUE OBTEMOS DA SEQUÊNCIA, NESTE CASO $\alpha^n+\alpha^{n+1}=\alpha^{n+2} \rightarrow 1+\alpha=\alpha^2$

$$\alpha_{1,2}=\frac{1\pm\sqrt{5}}{2}$$

$$F(n)=a\left(\frac{1+\sqrt{5}}{2}\right)^n+b\left(\frac{1-\sqrt{5}}{2}\right)^n$$

PARA DETERMINAR a E b USAREMOS OS VALORES DE $F(1)$ E $F(2)$

$$F(1)=a\,\frac{1+\sqrt{5}}{2}+b\,\frac{1-\sqrt{5}}{2}=1$$

$$F(2)=a\left(\frac{1+\sqrt{5}}{2}\right)^2+b\left(\frac{1-\sqrt{5}}{2}\right)^2=1$$

ENTÃO TEREMOS
$$\frac{a+b}{2}+\sqrt{5}\,\frac{a-b}{2}=1$$
$$3\,\frac{a+b}{2}+\sqrt{5}\,\frac{a-b}{2}=1 \qquad \Rightarrow a+b=0$$

CONSEQUENTEMENTE $a-b=2/\sqrt{5}$

$$a=-b=1/\sqrt{5}$$

$$F(n)=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n-\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$\lim_{n\to\infty}\frac{F(n+1)}{F(n)}=\frac{1+\sqrt{5}}{2} \qquad \text{RAZÃO ÁUREA}$$

O QUE ACONTECE SE MANTEMOS A FÓRMULA DA SEQUÊNCIA MAS MUDAMOS O SEGUNDO ELEMENTO

$\tilde{F}(n)+\tilde{F}(n+1)=\tilde{F}(n+2) \qquad$ 1, 3, 4, 7, 11, 18, 29 ...

$$\tilde{F}(n) = \tilde{a}\left(\frac{1+\sqrt{5}}{2}\right)^n + \tilde{b}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$\tilde{F}(1) = \tilde{a}\,\frac{1+\sqrt{5}}{2} + \tilde{b}\,\frac{1-\sqrt{5}}{2} = 1$$

$$\tilde{F}(2) = \tilde{a}\left(\frac{1+\sqrt{5}}{2}\right)^2 + \tilde{b}\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$$

$$\frac{\tilde{a}+\tilde{b}}{2} + \sqrt{5}\,\frac{\tilde{a}-\tilde{b}}{2} = 1 \qquad \frac{\tilde{a}+\tilde{b}}{2} + \sqrt{5}\,\frac{\tilde{a}-\tilde{b}}{2} = 1$$

$$3\,\frac{\tilde{a}+\tilde{b}}{2} + \sqrt{5}\,\frac{\tilde{a}-\tilde{b}}{2} = 3 \qquad \frac{\tilde{a}+\tilde{b}}{2} + \sqrt{5}\,\frac{\tilde{a}-\tilde{b}}{6} = 1$$

$$\sqrt{5}\,(\tilde{a}-\tilde{b})\left(\frac{1}{2}-\frac{1}{6}\right) = 0$$

$$\tilde{a} = \tilde{b}$$

$$\frac{\tilde{a}+\tilde{b}}{2} = 1 \Rightarrow \tilde{a} = \tilde{b} = 1$$

$$\tilde{F}(n) = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$$

CLARAMENTE $\displaystyle \lim_{n\to\infty} \frac{\tilde{F}(n+1)}{\tilde{F}(n)} = \lim_{n\to\infty} \frac{F(n+1)}{F(n)} = \frac{1+\sqrt{5}}{2}$

CONSIDERAMOS AGORA A SEGUINTE SEQUÊNCIA

$$2\,G(n) + G(n+1) = G(n+2)$$

$$1, 1, 3, 5, 11, 21, 42, \dots$$

CALCULAMOS $\alpha_{1,2}$ $\qquad 2\alpha^n + \alpha^{n+1} = \alpha^{n+2}$

$$\alpha^2 - \alpha - 2 = 0 \qquad \alpha_{1,2} = \frac{1 \pm \sqrt{9}}{2} \begin{cases} 2 \\ -1 \end{cases}$$

$$G(n) = a\,2^n + b\,(-1)^n$$

$$\left.\begin{array}{l} G(1) = 2a - b = 1 \\ G(2) = 4a + b = 1 \end{array}\right\} \qquad 6a = 2 \Rightarrow a = 1/3$$

$$b = 2/3 - 1 = -1/3$$

$$G(n) = [2^n - (-1)^n]/3$$

$$\lim_{n\to\infty} \frac{G(n+1)}{G(n)} = 2$$

CONSIDERAMOS AGORA $\quad -H(n) + 2H(n+1) = H(n+2)$

$$-1, 1, 3, 5, 7, 9, \ldots$$

$$-\alpha^n + 2\alpha^{n+1} = \alpha^{n+2}$$

$$\alpha^2 - 2\alpha + 1 = 0 \qquad \alpha_{1/2} = 1$$

$$H(n) = a\,1^n + n\,b\,1^n = a + nb$$

$$\left.\begin{array}{l} H(1) = a + b = -1 \\ H(2) = a + 2b = 1 \end{array}\right\} b = 2 \Rightarrow a = -3$$

$$H(n) = 2n - 3$$

$$\lim_{n\to\infty} \frac{H(n+1)}{H(n)} = 1$$

$$4S(n) + 4S(n+1) - S(n+2) = S(n+3)$$

$1, 1, 1, 4+4-1=7, 4+4-7=1$

$4+28-1=31, \ldots$

$$4\alpha^n + 4\alpha^{n+1} - 4\alpha^{n+2} = \alpha^{n+3}$$

$$\alpha^3 + \alpha^2 - 4\alpha - 4 = 0$$

$$\alpha^2(\alpha+1) - 4(\alpha+1) = 0$$

$(\alpha^2 - 4)(\alpha+1) = 0 \quad$ soluções $\quad 1, \pm 2$

$$S(n) = a\,2^n + b(-2)^n + c(-1)^n$$

$$\left.\begin{array}{l} S(1) = 2a - 2b - c = 1 \\ S(2) = 4a + 4b + c = 1 \end{array}\right\} 6a + 2b = 2$$

$$\left.\begin{array}{l} S(3) = 8a - 8b - c = 1 \end{array}\right\} 12a - 4b = 2$$

$$12a + 4b = 4$$
$$12a - 4b = 2$$
$$\overline{\quad 24a = 6 \quad}$$

$a = 1/4$

$2b = 2 - 3/2 = 1/2$

$b = 1/4, \; c = -1$

$$S(n) = 2^n\,\frac{1 + (-1)^n}{4} - (-1)^n$$

$$\lim_{n\to\infty} \frac{S(2n+2)}{S(2n)} = 4$$