# Equações lineares com coeficientes unitários

OBJETIVO ENCONTRAR O NÚMERO DE SOLUÇÕES DA EQUAÇÃO

$$x_1 + x_2 + \dots + x_k + x_{k+1} = n+1$$

ONDE $x_i$ INTEIROS $\geq 1$

EXEMPLO: $x_1 + x_2 = 5$ SOLUÇÕES $(1,4)\ (2,3)\ (3,2)\ (4,1)$ $\boxed{4}$

$x_1 + x_2 + x_3 + x_4 = 11$ COMPLICADO!

$1+1+1+1+1+1+1+1+1+1+1 = 11$ $(x_1, x_2, x_3, x_4)$

$+ \qquad + \qquad +$ $(1, 4, 4, 2)$

$+ \qquad + \qquad +$ $(2, 2, 3, 4)$

$10 +$ $3+$

$$\binom{10}{3} = \frac{10!}{3!\,7!} = \frac{10\cdot\cancel{9}^{3}\cdot\cancel{8}^{4}}{\cancel{3}\cdot\cancel{2}} = \boxed{120}$$

$$\boxed{x_1 + x_2 + \dots + x_k + x_{k+1} = n+1 \Rightarrow \binom{n}{k} \text{ SOLUÇÕES} \quad x_i \geq 1}$$

O QUE ACONTECE SE QUEREMOS O NÚMERO DE SOLUÇÕES PARA INTEIROS NÃO NEGATIVOS?

$y_1 + y_2 + \dots + y_k + y_{k+1} = n+1$ $\qquad y_i \geq 0$

MUDAMOS VARIÁVEL $x_i = y_i + 1$

$x_1 + x_2 + \dots + x_k + x_{k+1} = n+1+k+1 = n+k+2$ $\qquad \boxed{\binom{n+k+1}{k}}$

$y_1 + y_2 + y_3 + y_4 + y_5 = 12$ $\qquad k=4,\ n=11$ $\qquad \binom{11+4+1}{4} = \binom{16}{4}$

$y_1 + y_2 + y_3 + y_4 = 11$ $\qquad k=3,\ n=10$ $\qquad \binom{10+3+1}{3} = \binom{14}{3} = \boxed{364}$

$$\boxed{y_1 + y_2 + \dots + y_k + y_{k+1} = n+1 \Rightarrow \binom{n+k+1}{k} \quad y_i \geq 0}$$

$y_1 + y_2 + y_3 = 17$ COM $y_i > 3$

$x_i = y_i - 3 \rightarrow x_1 + 3 + x_2 + 3 + x_3 + 3 = 17$

$x_1 + x_2 + x_3 = 8$ $\qquad \binom{7}{2} = \boxed{21}$

$0 < x_1 + x_2 + x_3 + x_4 \leq 6$

$=4: 1$ $\qquad =5: \binom{4}{3} = 4$ $\qquad =6: \binom{5}{3} = 10$ $\qquad \boxed{15}$

Encontrar o número de soluções de $y1 + y2 + y3 + y4 = 22$ com $5 \leq y1 \leq 7$ $4 \leq y2 \leq 6$ $1 \leq y3 \leq 4$ $y4 \geq 5$

$x_1 = y_1 - 4$ $\quad 1 \leq x_1 \leq 3$
$x_2 = y_2 - 3$ $\quad 1 \leq x_2 \leq 3$
$x_3 = y_3$ $\quad 1 \leq x_3 \leq 4$
$x_4 = y_4 - 4$ $\quad x_4 \geq 1$ ✓

$\rightarrow x_1 + 4 + x_2 + 3 + x_3 + x_4 + 4 = 22 \Rightarrow x_1 + x_2 + x_3 + x_4 = 11$

SOLUÇÕES SEM RESTRIÇÕES $\binom{10}{3} = 120$

| | | | | | |
|---|---|---|---|---|---|
| A | $>3$ | $\tilde{x}_1 = x_1 + 3$ | $\tilde{x}_1 + x_2 + x_3 + x_4 = 11$ | $7!/3!4!$ | 35 |
| B | $>3$ | $\tilde{x}_2 = x_2 + 3$ | $x_1 + \tilde{x}_2 + x_3 + x_4 = 11$ | $7!/3!4!$ | 35 |
| C | $>4$ | $\tilde{x}_3 = x_3 + 4$ | $x_1 + x_2 + \tilde{x}_3 + x_4 = 11$ | $6!/3!3!$ | 20 |
| AB | | | $\tilde{x}_1 + \tilde{x}_2 + x_3 + x_4 = 11$ | $4!/3!1!$ | 4 |
| AC | | | $\tilde{x}_1 + x_2 + \tilde{x}_3 + x_4 = 11$ | $3!/3!0!$ | 1 |
| BC | | | $x_1 + \tilde{x}_2 + \tilde{x}_3 + x_4 = 11$ | $3!/3!0!$ | 1 |

$N(A \cup B \cup C) = 35 + 35 + 20 - 4 - 1 - 1 = 84$ $\quad \cancel{ABC}$

RESPOSTA : $120 - 84 = \boxed{36}$

Encontrar o número de soluções de $y1+y2+y3+y4=1$ com $-3 \leq yi \leq 3$

$y_i + 4 = x_i \Rightarrow 1 \leq x_i \leq 7$

$\boxed{x_1 + x_2 + x_3 + x_4 = 17}$

A > 7
B > 7
C > 7
D > 7

$\binom{16-7}{3} = \binom{9}{3} = \frac{\overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{8}} \cdot 7}{\cancel{3} \cdot \cancel{2}} = 84$ $\quad \cancel{AB}$ $\binom{16-14}{3}$

TOT : $\binom{16}{3} = \frac{16 \cdot \overset{5}{\cancel{15}} \cdot \overset{7}{\cancel{14}}}{\cancel{3} \cdot \cancel{2}} = 560$

$560 - 4 \cdot 84 = \boxed{224}$
$336$

$y_1 + y_2 = 1 \rightarrow x_1 + x_2 = 9$ $\quad$ TOT : $\binom{8}{1} = 8$

A > 7 $\binom{8-7}{1} = 1$ $\quad$ B > 7 $\binom{8-7}{1} = 1$ $\quad \cancel{AB}$ $\quad 8 - 1 - 1 = \boxed{6}$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| -3 | ~~-3~~ | ~~-2~~ | ~~-1~~ | ~~0~~ | ~~1~~ | ~~2~~ | ~~3~~ |
| -2 | ~~-3~~ | ~~-2~~ | ~~-1~~ | ~~0~~ | ~~1~~ | ~~2~~ | 3 |
| -1 | ~~-3~~ | ~~-2~~ | ~~-1~~ | ~~0~~ | ~~1~~ | 2 | ~~3~~ |
| 0 | ~~-3~~ | ~~-2~~ | ~~-1~~ | ~~0~~ | 1 | ~~2~~ | ~~3~~ |
| 1 | ~~-3~~ | ~~-2~~ | ~~-1~~ | 0 | ~~1~~ | ~~2~~ | ~~3~~ |
| 2 | ~~-3~~ | ~~-2~~ | -1 | ~~0~~ | ~~1~~ | ~~2~~ | ~~3~~ |
| 3 | ~~-3~~ | -2 | ~~-1~~ | ~~0~~ | ~~1~~ | ~~2~~ | ~~3~~ |

$\boxed{6}$