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1) $n=1$ $\quad \frac{1}{4-1} = \frac{1}{2+1}$ ✓ $\qquad n=k$ $\quad \sum_1^k i \ \frac{1}{4i^2-1} = \frac{k}{2k+1}$ ✓

$n=k+1$ $\quad \sum_1^{k+1} i \ \frac{1}{4i^2-1} = \sum_1^k i \ \frac{1}{4i^2-1} + \frac{1}{4(k+1)^2-1} = \frac{k}{2k+1} + \frac{1}{4k^2+8k+3}$

$(2k+1)(k+1)$ $\qquad (2k+1)(2k+3)$

$= \frac{k(2k+3)+1}{(2k+1)(2k+3)} = \frac{2k^2+3k+1}{(2k+1)(2k+3)}$

$= \frac{k+1}{2k+3}$ ✓

2)

A	B	¬ (A ∨ B)	⟺	¬A ∧ ¬B
V	V	F V	V	F F F
V	F	F V	V	F F V
F	V	F V	V	V F F
F	F	V F	V	V V V

3) ANASTASIA

G1) Vog Con S x x x x x A $\{$ 1 3 5!/3! ; A 3 5!/2!

G2) Vog S N Vog x x x x A $\{$ 1A 4!/2! ; A1 4!/2! ; A4 4!

G3) A S S A x x x x Vog 1 4!/2! ; A 4!

G12) Vog S ~~N~~ ~~X~~ $\qquad$ G23) A S ~~S~~ ~~X~~

G13) A SSA x x x x A $\quad 4!$

NÚMEROS DE ANAGRAMAS $\quad 3\frac{5!}{3!} + 3\frac{5!}{2!} + \frac{4!}{2!} + \frac{4!}{2!} + 4! + \frac{4!}{2!} + \cancel{4!} - \cancel{4!}$

$\frac{5}{2}4! + \frac{15}{2}4! + \frac{5}{2}4! = \frac{25}{2} \cdot 24 = \boxed{300}$

4)

$\binom{4}{0}\binom{8}{8}$ $\quad \binom{4}{1}\binom{3}{2}\binom{5}{5}$ $\quad \binom{6}{4}\binom{3}{1}\binom{3}{3}\binom{3}{2}\binom{3}{2}$ $\quad \binom{8}{8}\binom{4}{0}$ $\quad \binom{8}{7}\binom{2}{1}\binom{2}{0}$

$1 + 12 + 45 + 135 + 16 + 1 = \boxed{210}$

5) $y_1 + y_2 + y_3 + y_4 = 4$

$-1 \le y_1 \le 4$, $-2 \le y_{2,3} \le 3$, $-3 \le y_4 \le N$

$y_1 = x_1 - 2$
$y_{2,3} = x_2 - 3$
$y_4 = x_3 - 4$

$4+2+3+3+4$

$\boxed{x_1 + x_2 + x_3 + x_4 = 16}$

A (>6) B (>6) C (>6) $\binom{9}{3}$

AB, AC, BC $\binom{3}{3}$

AD, BD, CD $\binom{5-N}{3}$

D (> N+4) $\binom{11-N}{3}$

$N \ge -3$ $\qquad$ ~~ABC~~,... $\qquad$ ~~ABCD~~

TOT $\binom{15}{3}$

$-3 \le N \le 2$ $\quad \binom{15}{3} - 3\binom{9}{3} - \binom{11-N}{3} + 3\binom{3}{3} + 3\binom{5-N}{3}$

$3 \le N \le 8$ $\quad \binom{15}{3} - 3\binom{9}{3} - \binom{11-N}{3} + 3\binom{3}{3}$

$N \ge 9$ $\quad \binom{15}{3} - 3\binom{9}{3} + 3\binom{3}{3}$

EXEMPLOS $N=2$ $\quad \binom{15}{3} - 4\binom{9}{3} + 6\binom{3}{3} = 35\cdot 13 - 48\cdot 7 + 6$

$= 455 - 336 + 6 = 125$ //

$N \ge 9$ $\quad 455 - 252 + 3 = 206$ //

6) $x_1 + x_2 + x_3 + x_4 = 12$ $\quad 1 \le x_i \le 6$

TOT $\binom{11}{3}$

A (>6)
B (>6) $\binom{5}{3}$
C (>6)
D (>6)

~~AB)...~~ ~~ABC~~D
~~ABC)...~~

$P_A = \dfrac{\binom{11}{3} - 4\binom{5}{3}}{6^4} = \dfrac{11\cdot 10\cdot 9 - 40}{6^4} = \dfrac{125}{6^4}$

$\dfrac{P_A}{P_B} = \dfrac{250}{245} = \boxed{\dfrac{50}{49}}$

$x_1 + x_2 + x_3 + x_4 + x_5 = 19$

TOT $\binom{18}{4}$

3064 - 2175 + 150

$\dfrac{\binom{18}{4} - 5\binom{12}{4} + 10\binom{6}{4}}{6^5} = \dfrac{245}{6^4 \cdot 2}$

7)

A	B	C
xxoxx	xx oo	xxx

EXEMPLO

NÚMERO DE POSSIBILIDADES $\dfrac{12!}{3!\,9!}$

" TIRO LONGARIA "

xxooo	xxxx	xxx	10
	$\binom{5}{3}$	$\binom{4}{0}$	$\binom{3}{0}$
xxxxx	xooo	xxx	4
	$\binom{5}{0}$	$\binom{4}{3}$	$\binom{3}{0}$
xxxxx	xxxx	ooo	1
	$\binom{5}{0}$	$\binom{4}{0}$	$\binom{3}{3}$

TOT 15

xxxxo	xxxo	xxo	60
	$\binom{5}{1}$	$\binom{4}{1}$	$\binom{3}{1}$

RESPOSTA $\dfrac{60}{15} = \boxed{4}$