Funções Geradoras

De quantas maneiras diferentes podemos escolher 12 cervejas se existem 5 marcas de cervejas?

$$f(x) = (1+x+x^2+\dots+x^{12})^5 = (1-x^{13})^5 \sum_{0}^{\infty}{}_s \binom{4+s}{s} x^s$$

$$= (1-5\cancel{x^{13}}+\dots) \sum_{0}^{\infty}{}_s \binom{4+s}{s} x^s$$

$s=12$

$$\binom{16}{12} = \frac{\cancel{16}\cdot\cancel{15}\cdot 14\cdot 13}{\cancel{24}}$$

(2, 5)

$14 \times 13 = 182$, $\times 10$

$\boxed{1820}$

$a_k = 1$, $b_k = k$, $c_k = k^2$, $d_k = 1/k$

ENCONTRAR AS FUNÇÕES GERADORAS

$a_k = 1$ $\quad \sum_{0}^{\infty}{}_k x^k = 1 + x + x^2 + \dots = \dfrac{1}{1-x}$

$$\partial_x \sum_{0}^{\infty}{}_k x^k = \sum_{0}^{\infty}{}_k k x^{k-1} \Rightarrow x\partial_x \sum_{0}^{\infty}{}_k x^k = \sum_{0}^{\infty}{}_k k x^k$$

$b_k = k$ $\quad x\left(\dfrac{1}{1-x}\right)' = \boxed{\dfrac{x}{(1-x)^2}}$

$d_k = k^2$ $\quad x\left[\dfrac{x}{(1-x)^2}\right]' = x\left[\dfrac{1}{(1-x)^2} + \dfrac{2x}{(1-x)^3}\right] = \boxed{x\,\dfrac{1+x}{(1-x)^3}}$

$c_k = \dfrac{1}{k}$ $\quad \int dx \sum_{0}^{\infty}{}_k x^k = \sum_{0}^{\infty}{}_k \dfrac{x^{k+1}}{k+1} = \sum_{1}^{\infty}{}_s \dfrac{x^s}{s}$ $\qquad \int dx \dfrac{1}{1-x} = \boxed{-\ln(1-x)}$

$1: \dfrac{1}{1-x}$ $\qquad k: \dfrac{x}{(1-x)^2}$ $\qquad k^2: \dfrac{x(1+x)}{(1-x)^3}$ $\qquad \dfrac{1}{k}: -\ln(1-x)$

DE QUANTAS MANEIRAS PODEMOS ESCOLHER 3n LETRAS DE UM CONJUNTOS DE 3n A, 3n B, 3n C E 2n A, 2n B, 2n C

$$(1+x+x^2+\dots+x^{3n})^3 = \left(\frac{1-x^{3n+1}}{1-x}\right)^3$$

$$= \sum_{0}^{3}{}_k \binom{3}{k}(-1)^k x^{(3n+1)k} \sum_{0}^{\infty}{}_s \binom{2+s}{s} x^s$$

$k=0 \quad s=3n \qquad \binom{3n+2}{3n} \Rightarrow \boxed{(3n+2)(3n+1)/2}$

Controle $n=1$ $\quad$ 3 LETRAS DE UM CONJUNTO DE 3A, 3B, 3C

$(1+x+x^2+x^3)^3$

$(1+x+x^2+x^3)^2 \simeq (1+x^2+2x+2x^2+2x^3+2x^3) = (1+2x+3x^2+4x^3)$

$(1+x+x^2+x^3)(1+2x+3x^2+4x^3) \to 4+3+2+1 = 10$

AAA BBB CCC   AAB AAC   BBA BBC   CCA CCB   ABC $\boxed{10}$

3m LETRAS   2m A, 2m B, 2m C

$$(1+x+x^2+\dots+x^{2m})^3 = \left(\frac{1-x^{2m+1}}{1-x}\right)^3 = \sum_{0}^{3}{}_k \binom{3}{k}(-1)^k x^{(2m+1)k}$$

$$\sum_{0}^{\infty}{}_s \binom{2+s}{s} x^s$$

$k=0 \quad s=3m \quad \binom{3m+2}{3m}$

$k=1 \quad s=3m-2m-1 = m-1 \quad -3\binom{m+1}{m-1}$

$$\frac{(3m+2)(3m+1)}{2} - 3\frac{(m+1)m}{2} = \boxed{3m^2+3m+1}$$

$m=1$ $\boxed{7}$ VER O CASO ANTERIOR TIRANDO ~~AAA~~ ~~BBB~~ ~~CCC~~

ANAGRAMAS DE 3m LETRAS   2m A, 2m B, 2m C

$$\left(1+x+\frac{x^2}{2!}+\dots+\frac{x^{2m}}{2m!}\right)^3$$ COMPLICADO !!!

3m A, 3m B, 3m C

$$\left(1+x+\frac{x^2}{2!}+\dots+\frac{x^{3m}}{3m!}\right)^3 \to e^{3x}$$

$$e^x = \sum_{0}^{\infty}{}_s \frac{x^s}{s!} \qquad \frac{(3x)^{3m}}{3m!}$$

coeficiente $\boxed{3^{3m}}$

exemplo $m=1$ 27

AAA	1	AAB	3	ABC 6
BBB	1	AAC	3	
CCC	1	BBA	3	
		BBC	3	
		CCA	3	
		CCB	3	

$\boxed{27}$

CALCULAR $1^2+2^2+3^2+4^2+\dots+m^2$

função geradora k^2 $x(1+x)/(1-x)^3$

$$(1+x+x^2+\dots)(x+4x^2+9x^3+\dots)$$

$$x+4x^2+9x^3+x^2+4x^3+x^3+\dots$$

$$x+(1+4)x^2+(1+4+9)x^3+\dots$$

$$\frac{x(1+x)}{(1-x)^4}$$

$$(x+x^2)\sum_{0}^{\infty}{}_s \binom{3+s}{s}x^s \to \binom{3+m-1}{m-1}+\binom{3+m-2}{m-2}$$

$$\binom{m+2}{m-1}+\binom{m+1}{m-2}$$

$$\frac{(m+2)(m+1)m}{3!} + \frac{(m+1)m(m-1)}{3!} = \boxed{\frac{m(m+1)(2m+1)}{6}}$$