Formulas úteis trabalhando com funções geradoras

$$\frac{1}{(1-x)^N} = (1-x)^{-N} \qquad f(x) = f(0) + f'(0)\,x + f''(0)\,\frac{x^2}{2!} + \dots$$

TAYLOR EXPANSION

$$f(0) = 1 \qquad f'(x) = -N(1-x)^{-N-1}(-1) \qquad f'(0) = N$$

$$f''(x) = N(N+1)(1-x)^{-N-2} \qquad f''(0) = N(N+1)$$

TERMO DE ORDEM "r" $\quad f^{(r)}(0) = N(N+1)\dots(N+r-1)$

$$= (N+r-1)!\,/\,(N-1)!$$

$$f^{(r)}(0)\,\frac{x^r}{r!} \rightarrow \frac{(N+r-1)!}{(N-1)!\,r!}\,x^r$$

$$\frac{1}{(1-x^{\alpha})^N} = \sum_{r=0}^{\infty} \binom{N+r-1}{r} x^{\alpha r}$$

exemplo $\quad \frac{1}{1-x} = 1 + x + x^2 + \dots$

$$(1+x)^N = \overset{1}{(1+x)}\,\overset{2}{(1+x)} \cdots \overset{N}{(1+x)}$$

$$1 + Nx + N(N-1)x^2 + \dots + x^N$$

$$(1+x)^N = \sum_{r=0}^{N} \binom{N}{r} x^r$$

$$(1+x^{\alpha})^N = \sum_{r=0}^{N} \binom{N}{r} x^{\alpha r}$$

$$1 + x + x^2 + \dots + x^N = \;?$$

$$(1-x)(1+x+x^2+\dots+x^N) = 1 - x^{N+1}$$

$$1 + \cancel{x} + \cancel{x^2} + \dots + \cancel{x^N}$$

$$- \cancel{x} - \cancel{x^2} - \dots - \cancel{x^N} - x^{N+1}$$

$$\boxed{1 + x + x^2 + \dots + x^N = \frac{1-x^{N+1}}{1-x}}$$

EXEMPLO: ENCONTRAR A PROBABILIDADE DE FAZER 12, 13, 14, 15, 16 LANÇANDO 4 DADOS

$(x + x^2 + x^3 + x^4 + x^5 + x^6)^4$ É A FUNÇÃO GERADORA

$$x^4(1 + x + x^2 + x^3 + x^4 + x^5)^4 = x^4\left(\frac{1-x^6}{1-x}\right)^4 = x^4(1-x^6)^4\,\frac{1}{(1-x)^4}$$

$$x^4 \sum_{r=0}^{4} \binom{4}{r} x^{6r}(-1)^r \sum_{s=0}^{\infty} \binom{4+s-1}{s} x^s$$

$\boxed{12:}$ $\quad r=0 \quad \binom{4}{0} \quad s=8 \quad \binom{11}{8} \qquad \binom{11}{8} \qquad \frac{11\cdot 10\cdot 9}{3\cdot 2} \qquad 165$ $\qquad \boxed{125}$

$\quad 1 \quad \binom{4}{1}(-1) \quad s=2 \quad \binom{5}{2} \qquad -4\binom{5}{2} \qquad -4\,\frac{5\cdot 4}{2} \qquad -40$

$\boxed{13:}$ $\quad r=0 \quad \binom{4}{0} \quad s=9 \quad \binom{12}{9} \qquad \binom{12}{9} \qquad \frac{12\cdot 11\cdot 10}{6} \qquad 220$ $\qquad \boxed{140}$

$\quad 1 \quad \binom{4}{1}(-1) \quad s=3 \quad \binom{6}{3} \qquad -4\binom{6}{3} \qquad -4\,\frac{6\cdot 5\cdot 4}{6} \qquad -80$

$\boxed{14}$ $\quad 0, 10;\; 1, 4 \qquad \binom{13}{10} - 4\binom{7}{4} = \frac{13\cdot 12\cdot 11}{6} - 4\,\frac{7\cdot 6\cdot 5}{6} = 286 - 140 \qquad \boxed{146}$

$\boxed{15}$ $\quad 0, 11;\; 1, 5 \qquad \binom{14}{11} - 4\binom{8}{5} = \frac{14\cdot 13\cdot 12}{6} - 4\,\frac{8\cdot 7\cdot 6}{6} = 364 - 224 \qquad \boxed{140}$

$\boxed{16}$ $\quad 0, 12;\; 1, 6;\; 2, 0 \qquad \binom{15}{12} - 4\binom{9}{6} + 6\binom{3}{0} = \frac{15\cdot 14\cdot 13}{3\cdot 2} - 4\,\frac{9\cdot 8\cdot 7}{3\cdot 2} + 6$

NEW $\quad \binom{4}{2} = 6$

$$= 13\cdot 35 - 16\cdot 21 + 6$$

$$= 455 - 336 + 6$$

$$= \boxed{125}$$