Fibonacci

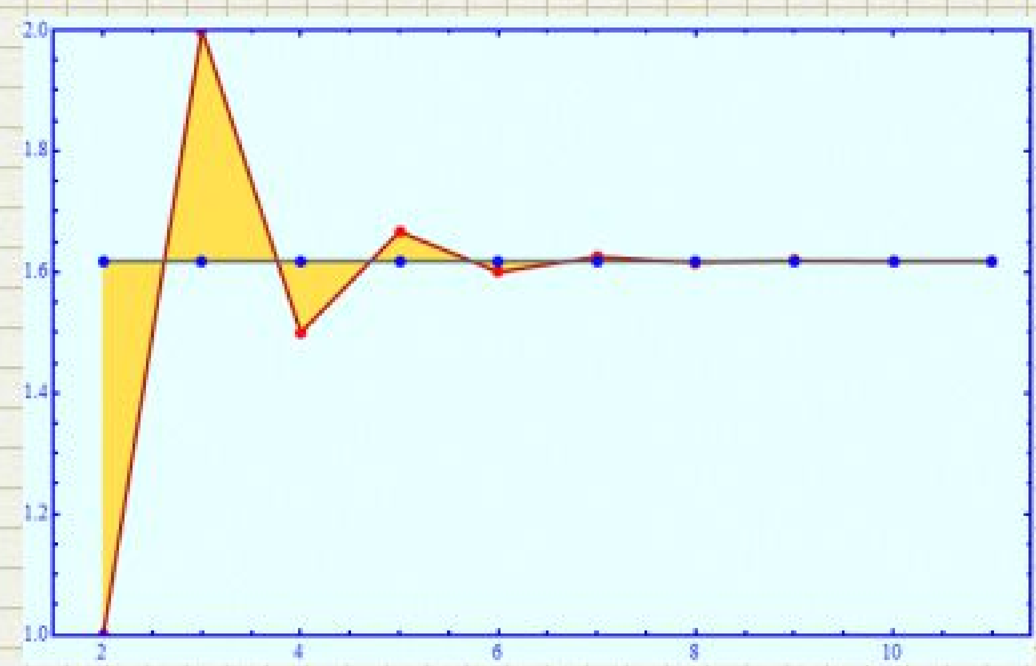
$$f_{n+2} = f_{n+1} + f_n$$

$f_1 = 1 \qquad f_2 = 1 \qquad \sqrt{5}\, f_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$

$$\lim_{n\to\infty} \frac{f_{n+1}}{f_n} = \frac{1+\sqrt{5}}{2} \approx 1.618$$

$\{1/1, 2/1, 3/2, 5/3, 8/5, 13/8, \ldots\}$

$\{1, 2, 1.5, 1.67, 1.6, 1.625, \ldots\}$



COMO MUDA 1.618 SE $f_2 = 3$?

$\alpha^2 = \alpha + 1$ sempre dará $\frac{1 \pm \sqrt{5}}{2}$

MAS AGORA TEREMOS

$$A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right) = 1$$

$$A\left(\frac{1+\sqrt{5}}{2}\right)^2 + B\left(\frac{1-\sqrt{5}}{2}\right)^2 = 3$$

$$\frac{A+B}{2} + \sqrt{5}\,\frac{A-B}{2} = 1 \quad \Rightarrow \quad A+B = 2 \quad \Rightarrow \quad A-B = 0 \quad \Rightarrow \quad A=B=1$$

$$3\,\frac{A+B}{2} + \sqrt{5}\,\frac{A-B}{2} = 3$$

$$g_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$$

$g_1 = 1 \qquad g_2 = \frac{3}{2} + \cancel{\frac{\sqrt{5}}{2}} + \frac{3}{2} - \cancel{\frac{\sqrt{5}}{2}} = 3$ ✓ $\qquad \lim_{n\to\infty} \frac{g_{n+1}}{g_n} = \frac{1+\sqrt{5}}{2}$

$\{1, 3, 4, 7, 11, 18, 29, 47, 76, 123\}$ SOMANDO TEREMOS 319 (29×11)

AS CONDIÇÕES INICIAIS NÃO MUDAM O LIMITE !!!

A
B
$A+B$
$A+2B$
$2A+3B$
$3A+5B$
$5A+8B \quad \times 11 = 55A + 88B$
$8A+13B$
$13A+21B$
$21A+34B$

$55A + 88B$

AGORA CONSIDERAMOS

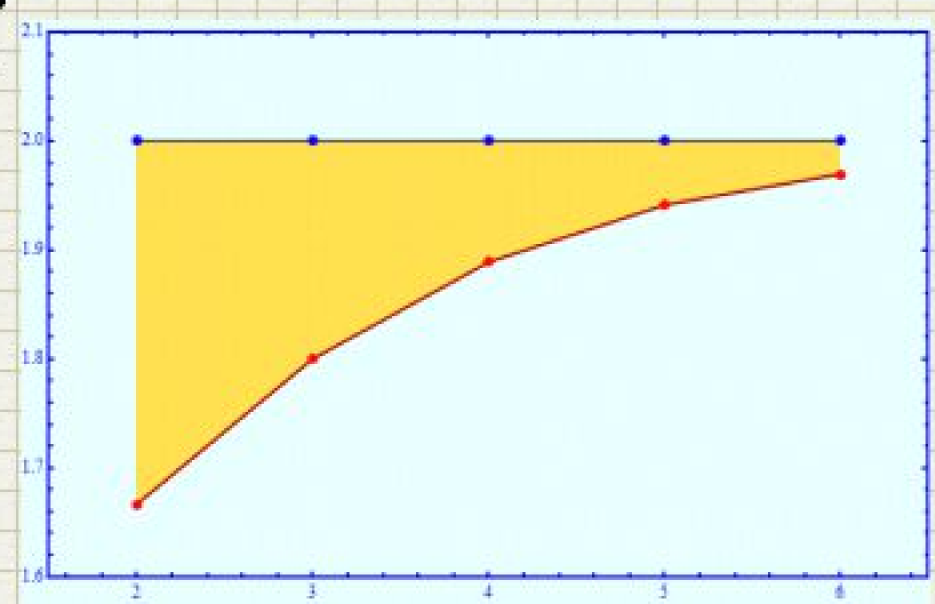
$$S_{n+2} = 3S_{n+1} - 2S_n$$

$$\alpha^2 = 3\alpha - 2$$

$$\alpha = \frac{3 \pm \sqrt{9-8}}{2} = \begin{cases} 2 \\ 1 \end{cases}$$

$$S_n = A\,1^n + B\,2^n = A + B\,2^n$$

$$\lim_{n\to\infty} \frac{\cancel{A} + B\,2^{n+1}}{\cancel{A} + B\,2^n} = 2$$



$S_1 = 3 \quad S_2 = 5 \qquad A + 2B = 3 \qquad B = 1$

$\qquad\qquad\qquad\quad A + 4B = 5 \qquad A = 1$

$$S_n = 1 + 2^n$$

$\{3, 5, 9, 17, 33, 65, \ldots\}$

$$\boxed{g_{n+2} = 2g_{n-1} - g_{n-2}}$$

$\alpha^2 - 2\alpha + 1 = 0 \Rightarrow \alpha = 1$

solução $A(1)^n + B\,n\,(1)^n$

$$\boxed{A + nB}$$

Exemplos:

$A = 0,\ B = 1 \qquad g_n = n$

$A = 1,\ B = 2 \qquad g_n = 2n + 1$

$A = 0,\ B = 2 \qquad g_n = 2n$

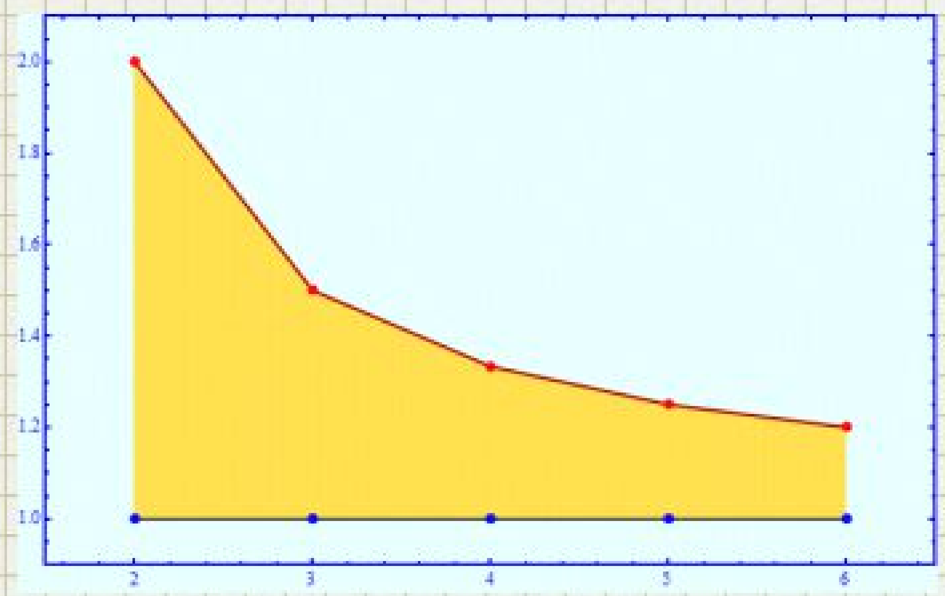
$$\lim_{n \to \infty} \frac{A + (n+1)B}{A + nB} = 1$$

$g_1 = 1 \qquad g_2 = 2$

$A + B = 1 \qquad A + 2B = 2$

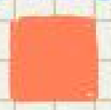
$A = 0,\ B = 1$

$1, 2, 3, \ldots, n, \ldots$



Quantos blocos diferentes de n cm podemos fazer juntando blocos laranjas de 1 cm, azúis de 2 cm e verdes de 2cm?

$n = 1$



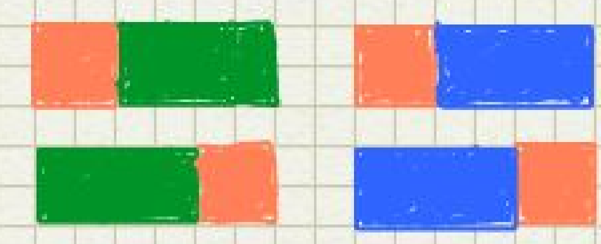
TEREMOS 1 POSSIBILIDADE

$n = 2$

3 POSSIBILIDADES

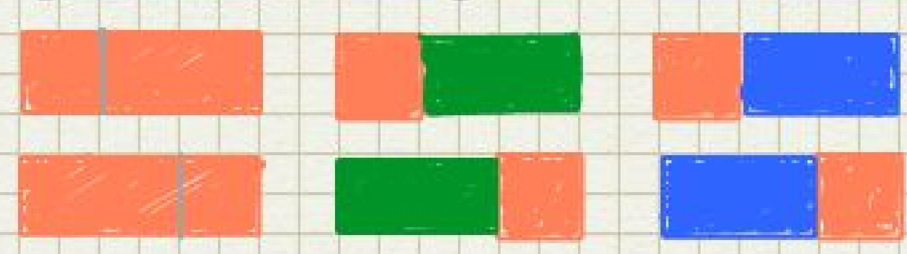
$n = 3$

PODEMOS USAR O RESULTADO DE $n = 1$
E UTILIZAR OS BLOCOS AZÚIS E VERDES



4 POSSIBILIDADES $= 4C_1$

PODEMOS USAR O RESULTADO DE $n = 2$
E UTILIZAR O BLOCO LARANJA

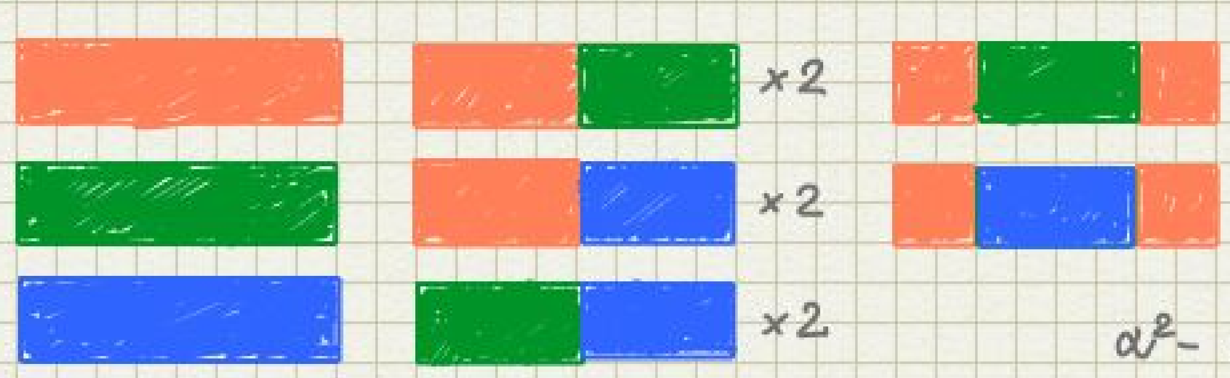


6 POSSIBILIDADES $= 2C_2$

EVITANDO AS REPETIÇÕES SERIAM $(4C_1 + 2C_2)/2$

$$C_3 = C_2 + 2C_1 = 5$$

TESTAMOS $C_4 = C_3 + 2C_2 = 11$



$$\boxed{C_n = C_{n-1} + 2C_{n-2}}$$

$\alpha^2 - \alpha - 2 = 0 \Rightarrow \boxed{\alpha = \{-1, 2\}}$

$c_n = A(-1)^n + B\,2^n$

$$\begin{aligned} -A + 2B &= 1 \\ A + 4B &= 3 \end{aligned} \Rightarrow 6B = 4 \Rightarrow B = \frac{2}{3} \Rightarrow A = \frac{1}{3}$$

$$\boxed{c_n = \frac{(-1)^n + 2^{n+1}}{3}} \qquad \lim_{n\to\infty} \frac{c_{n+1}}{c_n} \sim \lim_{n\to\infty} \frac{2^{n+2}}{2^{n+1}} = 2$$

$\underbrace{\{1, 3, 5, 11, 21, 43, \ldots\}}_{c_n}$ $\qquad$ $\underbrace{\left\{3, \frac{5}{3}, \frac{11}{5}, \frac{21}{11}, \frac{43}{21}, \ldots\right\}}_{c_{n+1}/c_n}$