

Funções geradoras e séries finitas

USAREMOS O CONCEITO DE FUNÇÃO GERADORA PARA OBTER A SOMA DE SÉRIES FINITAS

CALCULAREMOS COMO EXEMPLOS A SOMA DAS SEGUINTE SÉRIES

$$1 + 2 + 3 + \dots + n$$

$$1 + 2^2 + 3^2 + \dots + n^2$$

$$1 + 2^3 + 3^3 + \dots + n^3$$

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CALCULAREMOS COMO EXEMPLOS A SOMA DAS SEQUENTES SÉRIES

$$1 + 2 + 3 + \dots + n$$

$$1 + 2^2 + 3^2 + \dots + n^2$$

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EXPLICAREMOS ANTES A IDÉIA: CONSIDERAMOS A SÉRIE INFINITA

$$\sum_0^{\infty} c_n x^n$$

$$\begin{aligned} \sum_0^{\infty} x^n \cdot \sum_0^{\infty} c_n x^n &= (1 + x + x^2 + x^3 + \dots) (c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots) \\ &= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots \\ &\quad c_0 x + c_1 x^2 + c_2 x^3 + \dots \\ &\quad + c_0 x^2 + c_1 x^3 + \dots \\ &\quad + c_0 x^3 + \dots \\ &= c_0 + (c_0 + c_1) x + (c_0 + c_1 + c_2) x^2 + \\ &\quad (c_0 + c_1 + c_2 + c_3) x^3 + \dots \\ &= \sum_0^{\infty} x^n \left(\sum_0^n c_k \right) x^n \end{aligned}$$

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O COEFICIENTE DO TERMO x^n DARÁ COMO RESULTADO

$$\sum_0^n c_k$$

O coeficiente do termo x^n dará como resultado

$$\sum_{0 \leq k \leq n} C_k$$

$$\frac{1}{1-x} = \sum_0^{\infty} x^n$$

$$\sum_0^{\infty} x^n x = x \left(\sum_0^{\infty} x^n \right)' = x \left(\frac{1}{1-x} \right)' = \frac{x}{(1-x)^2}$$

$$\sum_0^{\infty} x^n x^2 = x \left(\sum_0^{\infty} x^n x \right)' = x \left[\frac{x}{(1-x)^2} \right]' = \frac{x(1+x)}{(1-x)^3}$$

$$\sum_0^{\infty} x^n x^3 = x \left(\sum_0^{\infty} x^n x^2 \right)' = x \left[\frac{x(1+x)}{(1-x)^3} \right]' = \frac{x(x^2 + 4x + 1)}{(1-x)^4}$$

O COEFICIENTE DO TERMO x^n DARÁ COMO RESULTADO

$$\sum_{0 \leq k \leq n} C_k$$

$$\frac{1}{1-x} = \sum_0^{\infty} x^n$$

$$\sum_0^{\infty} x x^n = x \left(\sum_0^{\infty} x^n \right)' = x \left(\frac{1}{1-x} \right)' = \frac{x}{(1-x)^2}$$

$$\sum_0^{\infty} x x^2 x^n = x \left(\sum_0^{\infty} x x^n \right)' = x \left[\frac{x}{(1-x)^2} \right]' = \frac{x(1+x)}{(1-x)^3}$$

$$\sum_0^{\infty} x x^3 x^n = x \left(\sum_0^{\infty} x x^2 x^n \right)' = x \left[\frac{x(1+x)}{(1-x)^3} \right]' = \frac{x(x^2+4x+1)}{(1-x)^4}$$

AGORA MULTIPLICAMOS PARA $\frac{1}{1-x}$ QUE REPRESENTA A SÉRIE $(1+x+x^2+\dots)$
E CALCULAMOS O COEFICIENTE DE x^2

O COEFICIENTE DO TERMO x^n DARÁ COMO RESULTADO

$$\sum_{0 \leq k \leq n} C_k$$

$$\frac{1}{1-x} = \sum_0^{\infty} x^n$$

$$\sum_0^{\infty} x x^n = x \left(\sum_0^{\infty} x^n \right)' = x \left(\frac{1}{1-x} \right)' = \frac{x}{(1-x)^2}$$

$$\sum_0^{\infty} x x^2 x^n = x \left(\sum_0^{\infty} x x^n \right)' = x \left[\frac{x}{(1-x)^2} \right]' = \frac{x(1+x)}{(1-x)^3}$$

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E CALCULAMOS O COEFICIENTE DE x^2

O COEFICIENTE DO TERMO x^n DARÁ COMO RESULTADO

$$\sum_{0 \leq k \leq n} C_k$$

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E CALCULAMOS O COEFICIENTE DE x^n

$$1+2+3+\dots+n$$

$$\frac{1}{1-x} \cdot \frac{x}{(1-x)^2} = \frac{x}{(1-x)^3}$$

$$x \sum_0^{\infty} \binom{n+2}{2} x^n$$

$$\hookrightarrow \binom{n-1+2}{2} = \frac{(n+1)n}{2}$$

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

O COEFICIENTE DO TERMO x^n DARÁ COMO RESULTADO

$$\sum_{0 \leq k \leq n} C_k$$

$$\frac{1}{1-x} = \sum_0^{\infty} x^n$$

$$\sum_0^{\infty} n x^n = x \left(\sum_0^{\infty} x^n \right)' = x \left(\frac{1}{1-x} \right)' = \frac{x}{(1-x)^2}$$

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E CALCULAMOS O COEFICIENTE DE x^n

$$1+2+3+\dots+n$$

$$\frac{1}{1-x} \frac{x}{(1-x)^2} = \frac{x}{(1-x)^3}$$

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O COEFICIENTE DO TERMO x^n DARÁ COMO RESULTADO

$$\sum_{0 \leq k \leq n} C_k$$

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E CALCULAMOS O COEFICIENTE DE x^n

$$1+2+3+\dots+n$$

$$\frac{1}{1-x} \frac{x}{(1-x)^2} = \frac{x}{(1-x)^3}$$

$$x \sum_0^{\infty} \binom{n+2}{2} x^n$$

$$\binom{n-1+2}{2} = \frac{(n+1)n}{2}$$

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2$$

$$\frac{1}{1-x} \cdot \frac{x(1+x)}{(1-x)^3} = \frac{x(1+x)}{(1-x)^4}$$

$$(x+x^2) \sum_0^{\infty} x \binom{x+3}{3} x^2$$

COEFICIENTE
DE x^2

$$\binom{n-1+3}{3} + \binom{n-2+3}{3} = \frac{(n+2)(n+1)n}{6} + \frac{(n+1)n(n-1)}{6}$$

$$= \frac{n+1}{6} (n^2 + 2n + n^2 - n)$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2$$

$$\frac{1}{1-x} \cdot \frac{x(1+x)}{(1-x)^3} = \frac{x(1+x)}{(1-x)^4}$$

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$$1^2 + 2^2 + 3^2 + \dots + n^2$$

$$\frac{1}{1-x}$$

$$\frac{x(1+x)}{(1-x)^3} =$$

$$\frac{x(1+x)}{(1-x)^4}$$

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DE x^2

$$\binom{n-1+3}{3} + \binom{n-2+3}{3} = \frac{(n+2)(n+1)n}{6} + \frac{(n+1)n(n-1)}{6}$$

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$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2$$

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DE x^2

$$\binom{n-1+3}{3} + \binom{n-2+3}{3} = \frac{(n+2)(n+1)n}{6} + \frac{(n+1)n(n-1)}{6}$$

$$= \frac{n+1}{6} (n^2 + 2n + n^2 - n)$$

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$$1^2 + 2^2 + 3^2 + \dots + n^2$$

$$\frac{1}{1-x}$$

$$\frac{x(1+x)}{(1-x)^3} =$$

$$\frac{x(1+x)}{(1-x)^4}$$

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COEFICIENTE
DE x^2

$$\binom{n-1+3}{3} + \binom{n-2+3}{3} = \frac{(n+2)(n+1)n}{6} + \frac{(n+1)n(n-1)}{6}$$

$$= \frac{n+1}{6} (n^2 + 2n + n^2 - n)$$

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$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2$$

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$$\frac{x(1+x)}{(1-x)^3} =$$

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COEFICIENTE
DE x^2

$$\binom{x-1+3}{3} + \binom{x-2+3}{3} = \frac{(x+2)(x+1)x + (x+1)x(x-1)}{6}$$

$$= \frac{x+1}{6} (x^2 + 2x + x^2 - x)$$

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$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

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$$\frac{x(1+x)}{(1-x)^3} =$$

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COEFICIENTE
DE x^2

$$\binom{n-1+3}{3} + \binom{n-2+3}{3} = \frac{(n+2)(n+1)n + (n+1)n(n-1)}{6}$$

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$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^2 + 2^2 + 3^2 + \dots + m^2$$

$$\frac{1}{1-x}$$

$$\frac{x(1+x)}{(1-x)^3} =$$

$$\frac{x(1+x)}{(1-x)^4}$$

$$(x+x^2) \sum_0^{\infty} x \binom{x+3}{3} x^x$$

COEFFICIENTE
DE x^m

$$\binom{m-1+3}{3} + \binom{m-2+3}{3} = \frac{(m+2)(m+1)m + (m+1)m(m-1)}{6}$$

$$= \frac{m+1}{6} (m^2 + 2m + m^2 - m)$$

$$1^2 + 2^2 + 3^2 + \dots + m^2 = \frac{m(m+1)(2m+1)}{6}$$

$$= \frac{m(m+1)(2m+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3$$

$$\frac{1}{1-x} \frac{x(x^2+4x+1)}{(1-x)^4} = \frac{x(x^2+4x+1)}{(1-x)^5}$$

$$(x^3+4x^2+x) \sum_0^{\infty} \binom{x+4}{4} x^n$$

$$\binom{n-3+4}{4} + 4 \binom{n-2+4}{4} + \binom{n-1+4}{4}$$

$$\frac{(n+1)n(n-1)(n-2) + 4(n+2)(n+1)n(n-1) + (n+3)(n+2)(n+1)n}{24}$$

$$\frac{(n+1)n}{24} (n^2-3n+2 + 4n^2+4n-8 + n^2+5n+6)$$

$$6n^2+6n$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

$$1^3 + 2^3 + 3^3 + \dots + n^3$$

$$\frac{1}{1-x} \cdot \frac{x(x^2+4x+1)}{(1-x)^4} = \frac{x(x^2+4x+1)}{(1-x)^5}$$

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