# Exercício de aula 5/03/15

**Fórmulas**

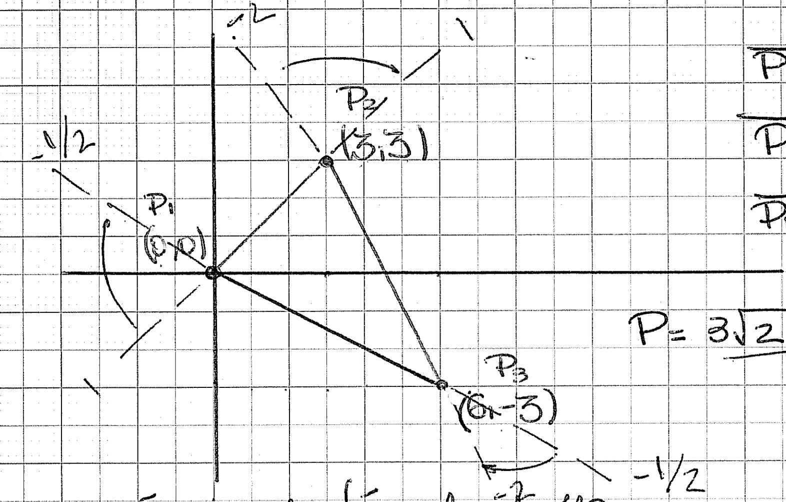
$$P = \frac{\overline{P_1P_2} + \overline{P_2P_3} + \overline{P_1P_3}}{2}$$

fórmula de Heron: $\text{Área} = \sqrt{P\,(P-\overline{P_1P_2})(P-\overline{P_2P_3})(P-\overline{P_1P_3})}$

$$A = \frac{1}{2}\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}$$

$$\boxed{tg\,\alpha = \frac{a_1 - a_2}{1 + a_2 . a_1}}$$

**EXERCÍCIO** 1 Explicativo




$$\overline{P_1P_2} = \sqrt{(3-0)^2 + (3-0)^2} = 3\sqrt{2}$$

$$\overline{P_2P_3} = \sqrt{(-3-3)^2 + (6-3)^2} = 3\sqrt{5}$$

$$\overline{P_1P_3} = \sqrt{(-3-0)^2 + (6-0)^2} = 3.\sqrt{5}$$

$$P = \frac{3\sqrt{2} + 3\sqrt{5} + 3\sqrt{5}}{2} = \frac{3}{2}(\sqrt{2} + 2\sqrt{5})$$

→ área pela fórmula de Heron:

$$A = \sqrt{\frac{3}{2}(\sqrt{2}+2\sqrt{5})\left(\frac{3}{2}(\sqrt{2}+2\sqrt{5}) - (3\sqrt{2})\right)\left(\frac{3}{2}(\sqrt{2}+2\sqrt{5}) - 3\sqrt{5}\right)^2}$$

$$A = \sqrt{\frac{3}{2}(\sqrt{2}+2\sqrt{5}).(-\sqrt{2}+2\sqrt{5}).\left(\frac{3}{2}\sqrt{2}\right)^2} \Rightarrow A = \frac{9}{4}\sqrt{(20-2).2} = \frac{27}{2}$$

→ área pelo determinante:

$$A = \frac{1}{2}\begin{vmatrix} 3 & 3 \\ 6 & -3 \end{vmatrix} = \frac{27}{2}$$

→ tangente

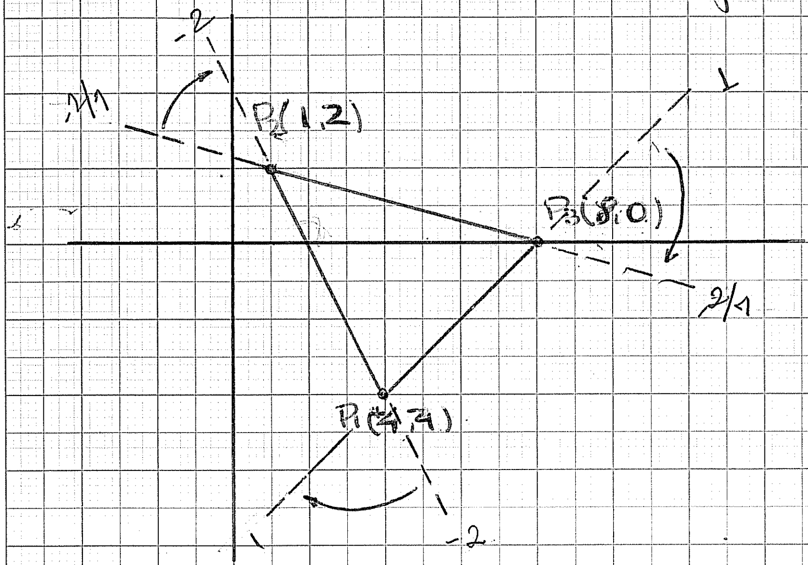
$a_{12} = 1$ $\quad a_{23} = -2$ $\quad a_{13} = -1/2$

$$tg\,\alpha_1 = \frac{a_{12} - a_{13}}{1 + a_{12}.a_{13}} = \frac{1-(-1/2)}{1+1.(-1/2)} = 3 \qquad tg\,\alpha_2 = \frac{a_{23} - a_{12}}{1 + a_{23}.a_{12}} = \frac{-2-1}{1+(-2).1} = 3$$

$$tg\,\alpha_3 = \frac{a_{13} - a_{23}}{1 + a_{13}\,a_{23}} = \frac{-1/2+2}{1+(-1/2)(-2)} = \frac{3}{4}$$

controle: $tg\,\alpha_1 . tg\,\alpha_2 . tg\,\alpha_3 = tg\,\alpha_1 + tg\,\alpha_2 + tg\,\alpha_3$

Tendo o exercício 1 como base, faça os seguintes:




1. ÁREA

2. TANGENTE

250

200

**Resolução**

$$\frac{2-4}{1-4} = \frac{-2}{-3} = \frac{2}{3}$$

(1) ÁREA $P_1(+4,-4)$ $P_2(1,2)$ $P_3(8,0)$

por determinante: $$A = \frac{1}{2}\begin{vmatrix} x_2-x_1 & y_2-y_1 \\ x_3-x_1 & y_3-y_1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -3 & 6 \\ 4 & 4 \end{vmatrix} = 12$$

(2) TANGENTE $[a = \Delta y / \Delta x]$

150

$$a_{12} = \frac{6}{-3} = -2 \qquad a_{23} = \frac{-2}{7} = -\frac{2}{7} \qquad a_{13} = \frac{4}{4} = 1$$

1,89

$$tg\,\alpha_1 = \frac{-2-1}{1+(-2)(1)} = \frac{-3}{1-2} = \frac{-3}{-1} = 3$$

$$tg\,\alpha_2 = \frac{-\frac{2}{7} - (-2)}{1+(-2/7)(-2)} = \frac{\frac{12}{\cancel{7}}}{\frac{11}{\cancel{7}}} = \frac{12}{11}$$

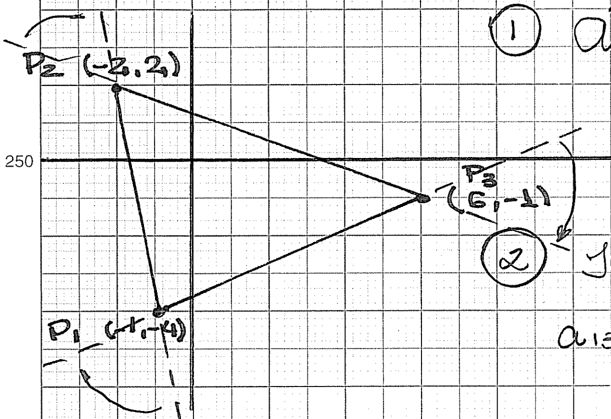
$$tg\,\alpha_3 = \frac{1-(-2/7)}{1+(-2/7)(1)} = \frac{\frac{9}{\cancel{7}}}{\frac{5}{\cancel{7}}} = \frac{9}{5}$$

50

0

0 50 100 150

$P_1(-1,-4) \quad P_2(-2,2) \quad P_3(6,-1)$




① Área: $A = \frac{1}{2}\begin{vmatrix} -2-(-1) & 2-(-4) \\ 6-(-1) & -1-(-4) \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -1 & 6 \\ 7 & 3 \end{vmatrix}$

$A = \frac{1}{2}|-45| \quad A = \frac{45}{2}$

② Tangente: $a_{12} = \frac{2+4}{-2+1} = -6 \quad a_{23} = \frac{-1-2}{6+2} = \frac{-3}{8}$

$a_{13} = \frac{-1+4}{6+1} = \frac{3}{7}$

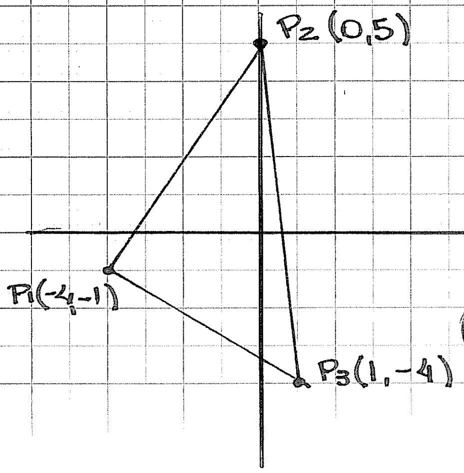
$tg\,\alpha_1 = \frac{-6-\frac{3}{7}}{1+(-6)\left(\frac{3}{7}\right)} = \frac{\frac{-45}{7}}{\frac{-11}{7}} = \frac{45}{11}$

$tg\,\alpha_3 = \frac{\frac{3}{7}+\frac{3}{8}}{1+\left(\frac{3}{7}\right)\left(\frac{-3}{8}\right)} = \frac{\frac{45}{56}}{\frac{47}{56}} = \frac{45}{47}$

$tg\,\alpha_2 = \frac{-\frac{3}{8}+6}{1+(-6)\left(\frac{-3}{8}\right)} = \frac{\frac{45}{8}}{\frac{26}{8}} = \frac{45}{26}$

$P_1(-4,-1) \quad P_2(0,5) \quad P_3(1,-4)$

$-\frac{12}{18}$




① Área: $A = \frac{1}{2}\begin{vmatrix} 0-(-4) & 5-(-1) \\ 1-(-4) & -4-(-1) \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 4 & 6 \\ 5 & -3 \end{vmatrix}$

$A = \frac{1}{2}|-30| = 15$

② Tangente

$a_{12} = \frac{6}{0+4} = \frac{3}{2} \quad a_{23} = \frac{-9}{1} = -9 \quad a_{13} = \frac{-3}{5}$

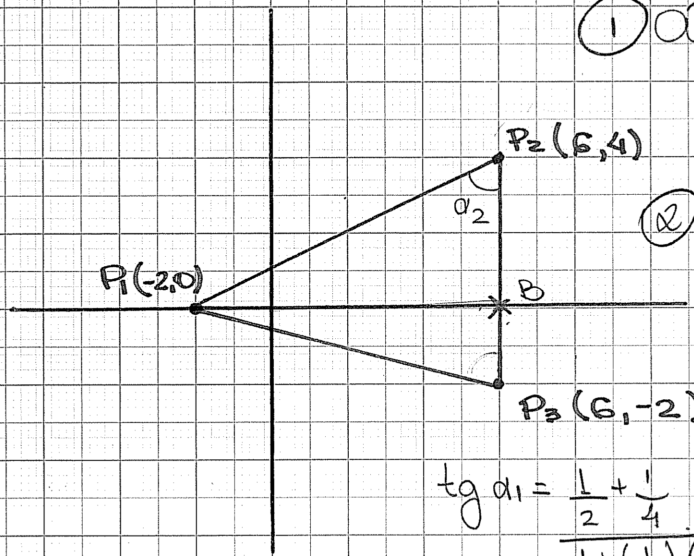
$tg\,\alpha_1 = \frac{\frac{3}{2}+\frac{3}{5}}{1+\left(\frac{3}{2}\right)\left(\frac{-3}{5}\right)} = \frac{\frac{21}{10}}{\frac{1}{10}} = 21$

$tg\,\alpha_2 = \frac{-9-\frac{3}{2}}{1+(-9)\left(\frac{3}{2}\right)} = \frac{\frac{-21}{2}}{\frac{-25}{2}} = \frac{21}{25}$

$tg\,\alpha_3 = \frac{-\frac{3}{5}+9}{1+\left(-\frac{3}{5}\right)(-9)} = \frac{\frac{42}{5}}{\frac{23}{5}} = \frac{42}{23}$

$P_1(-2,0) \quad P_2(6,4) \quad P_3(6,-2)$

① Área: $A = \frac{1}{2}\begin{vmatrix} 8 & 4 \\ 8 & -2 \end{vmatrix} = \frac{1}{2}|48| = 24$




16
32
48

② Tangente..

$a_{12} = \frac{4}{8} = \frac{1}{2} \qquad a_{23} = \to \infty$

$a_{13} = \frac{-2}{8} = -\frac{1}{4}$

$tg\,\alpha_1 = \frac{\frac{1}{2} + \frac{1}{4}}{1 + \left(\frac{1}{2}\right)\left(\frac{-1}{4}\right)} = \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{6}{7}$

como $a_{23} \to \infty$ temos que usar outra forma para descobrir $tg\,\alpha_2$ e $tg\,\alpha_3$.

Usemos a linha do eixo x como referencia e usemos do Teorema de Pitágoras

$\Delta P_2P_1B \quad tg\,\alpha_2 = \frac{8}{4} = 2$

$\Delta P_3P_1B \quad tg\,\alpha_3 = \frac{8}{2} = 4$

Usando apenas os pontos.

$P_1(5,5) \qquad P_2(3,-1) \qquad P_3(0,0)$

① Área

$A = \frac{1}{2}\begin{vmatrix} -2 & -6 \\ -5 & -5 \end{vmatrix} \qquad A = \frac{1}{2}|30| = 15$

② Tangentes:

$a_{12} = \frac{-6}{-2} = 3 \qquad a_{23} = \frac{+1}{-3} = \frac{-1}{3} \qquad a_{13} = \frac{-5}{-5} = 1$

$tg\,\alpha_1 = \frac{3-1}{1+(3)(1)} = \frac{2}{4} = \frac{1}{2}$

$tg\,\alpha_3 = \frac{1 + \frac{1}{3}}{1+\left(\frac{-1}{3}\right)(1)} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2$

$tg\,\alpha_2 = \frac{\frac{-1}{3} - 1}{1 + \left(\frac{-1}{3}\right)(1)} = \frac{-\frac{4}{3}}{+\frac{2}{3}} = -2$