Como calcular a distância entre retas reversas

$r_1: (1,2,3) + t(1,0,1)$
$r_2: (1,1,3) + r(1,1,0)$

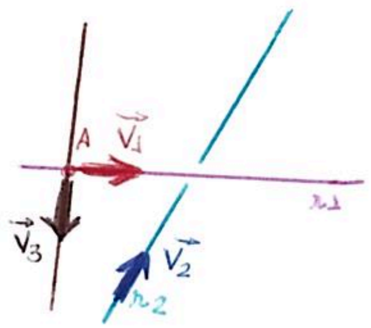
① Fazemos o produto vetorial entre $\vec{V_1}$ e $\vec{V_2}$, obtendo $\vec{V_3}$. Com isso, podemos escrever a r_3, perpendicular a r_1.

$\vec{V_1} = (1,0,1)$
$\vec{V_2} = (1,1,0)$

$$\begin{matrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{matrix}$$

$(0-1, 1-0, 1-0) \rightarrow \boxed{\vec{V_3} = (-1,1,1)}$

$\boxed{r_3 = (1,2,3) + p(-1,1,1)}$



Repare que r_1 e r_2 não se cruzam, pois estão em planos diferentes e paralelos.

② Montamos r_4, paralela à r_3, somando r_1 a um novo parâmetro, mas mantendo o vetor de $\vec{V_3}$. Após isso, descobrimos o ponto de encontro entre a r_4 e a r_2 e r_4 e r_1.

$r_4 = [(1,2,3) + t(1,0,1)] + w(-1,1,1) = (t+1, 2, t+3) + (-w, w, w) = (t+1-w, 2+w, t+3+w)$

$r_2 = (1,1,3) + r(1,1,0) = (r+1, r+1, 3)$

$t+3+w = 3 \rightarrow t = 3-3-w \rightarrow \boxed{t = -w}$

$2+w = r+1 \rightarrow \boxed{r = w+1}$

$t+1-w = r+1 \rightarrow -w+1-w = w+1+1 \rightarrow 3w = -1 \rightarrow \boxed{w = -1/3}$

$r = -\frac{1}{3} + 1 \rightarrow \boxed{r = \frac{2}{3}}$

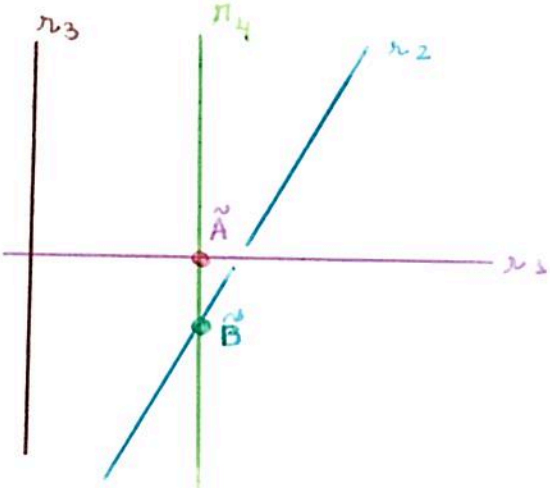
$\boxed{t = \frac{1}{3}}$

$\tilde{B} = (r+1, r+1, 3) = \left(\frac{2}{3}+1, \frac{2}{3}+1, 3\right) = \left(\frac{5}{3}, \frac{5}{3}, 3\right)$

$\boxed{\tilde{B} = (5/3, 5/3, 3)}$

$\tilde{A} = (t+1, 2, t+3) = \left(\frac{1}{3}+1, 2, \frac{1}{3}+3\right) = \left(\frac{4}{3}, 2, \frac{10}{3}\right)$

$\boxed{\tilde{A} = (4/3, 2, 10/3)}$



③ Descubra o valor da distância entre $\tilde{A}$ e $\tilde{B}$.

$d = |\tilde{B}\tilde{A}| = \sqrt{(5/3 - 4/3)^2 + (5/3 - 2)^2 + (3 - 10/3)^2} = \sqrt{(1/3)^2 + (-1/3)^2 + (-1/3)^2} =$

$\sqrt{1/9 + 1/9 + 1/9} = \sqrt{3/9} = \frac{\sqrt{3}}{3}$

$\boxed{d = \frac{\sqrt{3}}{3}}$