

Exercícios

① R_1 : { plano passante pelos pontos $(0,0,0)$, $(0,1,0)$, $(2,1,0)$
 { plano paralelo aos vetores $(1,1,2)$, $(2,0,1)$ e passante
 pelo ponto $(1,1,1)$

$$R_2: \begin{cases} x+y=2 \\ x-y+z=d \end{cases}$$

a) Determinar as equações paramétricas das retas.

R₁

$$\begin{aligned} P_A &= (0,0,0) \\ P_B &= (0,1,0) \\ P_C &= (2,1,0) \\ P &= (x,y,z) \end{aligned} \quad \left. \begin{aligned} \vec{v}_A &= (x-0, y-0, z-0) = (x, y, z) \\ \vec{v}_B &= (0-0, 1-0, 0-0) = (0, 1, 0) \\ \vec{v}_C &= (2-0, 1-0, 0-0) = (2, 1, 0) \end{aligned} \right\}$$

	x	y	z	x	y
0	1	0	0	1	
2	1	0	-2	2	1

$$x \cdot 0 + y \cdot 0 + z(-2) = 0 \rightarrow -2z = 0 \quad \Pi_1$$

$$\left. \begin{aligned} \vec{v}_D &= (1,1,2) \\ \vec{v}_E &= (2,0,1) \end{aligned} \right\} \begin{array}{c} \text{x} \quad \text{y} \quad \text{z} \\ 1 \quad 1 \quad 2 \\ 2 \quad 0 \quad 1 \end{array} \begin{array}{c} \text{x} \quad \text{y} \\ 1 \quad 1 \\ 2 \quad 0 \end{array}$$

$$x \cdot 1 + y \cdot 3 + z(-2) = 0 \rightarrow x + 3y - 2z = 0$$

$$\begin{aligned} x + 3y - 2z + a &= 0 \\ P &= (1,1,1) \end{aligned} \quad \left. \begin{aligned} 1 + 3 \cdot 1 - 2 \cdot 1 + a &= 0 \rightarrow 1 + 3 - 2 + a = 0 \rightarrow a + 2 = 0 \rightarrow a = -2 \end{aligned} \right\}$$

$$\Pi_2: x + 3y - 2z = 2$$

$$R_1: \begin{cases} -2z = 0 \rightarrow z = 0 \\ x + 3y - 2z = 2 \end{cases}$$

$$x + 3y - 2 \cdot 0 = 2 \rightarrow x + 3y = 2 \rightarrow \boxed{x = -3y + 2} \quad \boxed{y = t}$$

$$R_1: (-3t + 2, t, 0) = (-3t + 2, t, 0) = (2, 0, 0) + t(-3, 1, 0)$$

$$\boxed{R_1: (2, 0, 0) + t(-3, 1, 0)}$$

R₂

$$\begin{cases} x+y=2 \\ x-y+z=d \end{cases}$$

$$2x+z=d+2 \rightarrow z=\rho \rightarrow 2x=-\rho+d+2 \rightarrow \boxed{x=\frac{d-\rho+2}{2}}$$

$$\begin{cases} x+y=2 \\ -x+y-z=-d \end{cases}$$

$$2y=z-d+2 \rightarrow \boxed{y=\frac{\rho-d+2}{2}}$$

$$R_2: \left(\frac{d}{2} - \frac{\rho}{2} + \frac{2}{2}, \frac{\rho}{2} - \frac{d}{2} + \frac{2}{2}, \rho\right) = \left(\frac{d+2}{2}, \frac{2-d}{2}, 0\right) + \rho \left(-\frac{1}{2}, \frac{1}{2}, 1\right)$$

$$\boxed{R_2: \left(\frac{d+2}{2}, \frac{2-d}{2}, 0\right) + \rho \left(-\frac{1}{2}, \frac{1}{2}, 1\right)}$$

b) Para qual valor de d as retas se encontram?

$$R_1: (-3t+2, t, 0)$$

$$R_2: \left(\frac{d+2}{2} - \frac{\rho}{2}, \frac{2-d}{2} + \frac{\rho}{2}, \rho\right)$$

$$\boxed{\rho=0}$$

$$\frac{2-d}{2} + \frac{\rho}{2} = t \rightarrow \boxed{t = \frac{2-d}{2}}$$

$$-3t+2 = \frac{d+2}{2} - \frac{\rho}{2} \rightarrow -3\left(\frac{2-d}{2}\right) + \frac{4}{2} = \frac{d+2}{2} \rightarrow 3d-6+4 = d+2 \rightarrow$$

$$3d-d = 2+6-4 \rightarrow 2d=4 \rightarrow \boxed{d=2}$$

c) Qual o ponto de intersecção?

$$t = \frac{2-d}{2} \rightarrow t = \frac{2-2}{2} \rightarrow \boxed{t=0}$$

$$PE: (-3t+2, t, 0) = (-3 \cdot 0 + 2, 0, 0) = (2, 0, 0)$$

$$PE: \left(\frac{d+2}{2} - \frac{\rho}{2}, \frac{2-d}{2} + \frac{\rho}{2}, \rho\right) = \left(\frac{2+2}{2} - 0, \frac{2-2}{2} + 0, 0\right) = (2, 0, 0)$$

$$\boxed{PE = (2, 0, 0)}$$

$$\textcircled{2} R_1: \begin{cases} 3x+y-2z=3 \\ 2x+y+z=2 \end{cases}$$

$$R_2: \begin{cases} x+y-z=3 \\ 3x-y+z=1 \end{cases}$$

Determinar a equação do plano paralelo às retas dadas e que passa pela ponto de interseção entre a reta R_1 e o eixo x .

$$R_1: \begin{cases} 3x+y-2z=3 \\ -2x-y-z=-2 \end{cases}$$

$$x-3z=1 \rightarrow \boxed{x=3z+1}$$

$$y=2-2x-z \rightarrow y=2-6z-2-z \rightarrow \boxed{y=-7z}$$

$$\boxed{z=t}$$

$$R_1: (3t+1, -7t, t) = (1, 0, 0) + t(3, -7, 1)$$

$$\boxed{R_1: (1, 0, 0) + t(3, -7, 1)}$$

\vec{v}_1

$$R_2: \begin{cases} x+y-z=3 \\ 3x-y+z=1 \end{cases}$$

$$4x=4 \rightarrow \boxed{x=1}$$

$$y=-x+z+3 \rightarrow y=-1+z+3 \rightarrow \boxed{y=z+2}$$

$$\boxed{z=s}$$

$$R_2: (1, s+2, s) = (1, 2, 0) + s(0, 1, 1)$$

$$\boxed{R_2: (1, 2, 0) + s(0, 1, 1)}$$

\vec{v}_2

$$\begin{matrix} \vec{v}_1 = (3, -7, 1) \\ \vec{v}_2 = (0, 1, 1) \end{matrix} \left\{ \begin{array}{c} \textcircled{x} \quad \textcircled{y} \quad \textcircled{z} \quad x \quad y \\ 3 \quad -7 \quad 1 \quad 3 \quad -7 \\ 0 \quad 1 \quad 1 \quad 0 \quad 1 \end{array} \right.$$

$$x(-8) + y(-3) + z \cdot 3 = 0 \rightarrow -8x - 3y + 3z = 0$$

$$\boxed{\Pi: -8x - 3y + 3z + d = 0}$$

$$R_1: (3t+1, -7t, t) \left. \begin{matrix} y=0 \\ z=0 \end{matrix} \right\} \rightarrow t=0 \rightarrow (3 \cdot 0 + 1, -7 \cdot 0, 0) = \boxed{(1, 0, 0)}$$

corta eixo x

$$-8 \cdot 1 - 3 \cdot 0 + 3 \cdot 0 + d = 0 \rightarrow d - 8 = 0 \rightarrow \boxed{d=8}$$

$$\boxed{\Pi: -8x - 3y + 3z + 8 = 0}$$