Como criar exercícios de rotação de cônicas

① Escolher resposta final e ângulos de rotação

<u>Resposta</u>: $\frac{(x-1)^2}{9} + \frac{(y-1)^2}{4} = 1$

<u>Ângulos</u>:

$\boxed{\cos\alpha = 1/3}$

$sen^2\alpha + \cos^2\alpha = 1 \rightarrow sen^2\alpha = 1 - \cos^2\alpha \rightarrow sen^2\alpha = 1 - \left(\frac{1}{3}\right)^2 \rightarrow sen^2\alpha = 1 - \frac{1}{9}$

$sen^2\alpha = \frac{9-1}{9} \rightarrow sen^2\alpha = \frac{8}{9} \rightarrow \boxed{sen\,\alpha = \frac{2\sqrt{2}}{3}}$

$tg\,\alpha = \frac{sen\,\alpha}{\cos\alpha} \rightarrow tg\,\alpha = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} \rightarrow \boxed{tg\,\alpha = 2\sqrt{2}}$

② Encontrar as relações entre x, y (rotacionados) e X, Y (s[em] rotação)

$$\underbrace{\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos\alpha & -sen\,\alpha \\ sen\,\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}}_{\text{usamos para resolver exercício}} \longrightarrow \underbrace{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\alpha & sen\,\alpha \\ -sen\,\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix}}_{\text{usamos para montar exercício}}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1/3 & 2\sqrt{2}/3 \\ -2\sqrt{2}/3 & 1/3 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 2\sqrt{2} \\ -2\sqrt{2} & 1 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix}$$

$\boxed{x = \frac{X + 2\sqrt{2}Y}{3}}$ $\boxed{y = \frac{-2\sqrt{2}X + Y}{3}}$

③ Substituir x e y na resposta para encontrar a form[a] inicial do exercício em função de X e Y.

$\frac{(x-1)^2}{9} + \frac{(y-1)^2}{4} = 1$ $x = \frac{X + 2\sqrt{2}Y}{3}$ $y = \frac{-2\sqrt{2}X + Y}{3}$

$\frac{\left(\frac{X+2\sqrt{2}Y}{3} - 1\right)^2}{9} + \frac{\left(\frac{-2\sqrt{2}X+Y}{3} - 1\right)^2}{4} = 1 \rightarrow \left(\frac{X+2\sqrt{2}Y-3}{3}\right)^2 + \left(\frac{-2\sqrt{2}X+Y-3}{3}\right)^2 =$

Resolução

$76x^2-20\sqrt{2}xy+41y^2-24x+108\sqrt{2}x-54y-48\sqrt{2}y=207$

$A=76 \quad B=-20\sqrt{2} \quad C=41$

$$tg\alpha=\frac{C-A\pm\sqrt{(C-A)^2+B^2}}{B} \rightarrow tg\alpha=\frac{41-76\pm\sqrt{(41-76)^2+(-20\sqrt{2})^2}}{-20\sqrt{2}} \rightarrow$$

$$tg\alpha=\frac{-35\pm\sqrt{35^2+800}}{-20\sqrt{2}} \rightarrow 35^2+800=(5.7)^2+8.100=25.49+8.4.25=$$
$$25.49+25.32=25(49+32)=25.81 \rightarrow 5.9=45$$

$$tg\alpha=\frac{-35\pm45}{-20\sqrt{2}} \quad \begin{cases} tg\alpha=\frac{80}{20\sqrt{2}}=2\sqrt{2} \\ tg\alpha=-\frac{10}{20\sqrt{2}}=-\frac{1}{2\sqrt{2}} \end{cases}$$

$$tg\alpha=2\sqrt{2} \rightarrow sen\alpha=\frac{tg\alpha}{\sqrt{1+tg^2\alpha}} \rightarrow sen\alpha=\frac{2\sqrt{2}}{\sqrt{1+8}} \rightarrow sen\alpha=\frac{2\sqrt{2}}{3}$$

$$\cos\alpha=\frac{1}{\sqrt{1+tg^2\alpha}} \rightarrow \cos\alpha=\frac{1}{\sqrt{1+8}} \rightarrow \cos\alpha=\frac{1}{3}$$

$$\begin{pmatrix} X \\ Y \end{pmatrix}=\begin{pmatrix} \cos\alpha & -sen\alpha \\ sen\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} X \\ Y \end{pmatrix}=\frac{1}{3}\begin{pmatrix} 1 & -2\sqrt{2} \\ 2\sqrt{2} & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow$$

$$X=\frac{(x-2\sqrt{2}y)}{3} \qquad Y=\frac{(2\sqrt{2}x+y)}{3}$$

$$76\frac{(x-2\sqrt{2}y)^2}{9}-20\sqrt{2}\frac{(x-2\sqrt{2}y)(2\sqrt{2}x+y)}{9}+41\frac{(2\sqrt{2}x+y)^2}{9}+$$
$$(-24+108\sqrt{2})\frac{(x-2\sqrt{2}y)}{3}-(48\sqrt{2}+54)\frac{(2\sqrt{2}x+y)}{3}=207$$

$$x^2: \frac{76}{9}-\frac{20\sqrt{2}}{9}.2\sqrt{2}+\frac{41}{9}.8=\frac{76-80+328}{9}=\frac{324}{9}=36$$

$$y^2: \frac{76}{9}.8+\frac{20\sqrt{2}}{9}.2\sqrt{2}+\frac{41}{9}=\frac{608+80+41}{9}=\frac{729}{9}=81$$

$$xy: \frac{76}{9}.(-4\sqrt{2})-\frac{20\sqrt{2}}{9}.(1-8)+\frac{41}{9}.4\sqrt{2}=-\frac{4.76.\sqrt{2}}{9}+\frac{140\sqrt{2}}{9}+\frac{164\sqrt{2}}{9}=$$

$$x: \frac{-24+108\sqrt{2}-48\sqrt{2}.2\sqrt{2}-54.2\sqrt{2}}{3}=-\frac{216}{3}=-72$$

$$y: \frac{-24(-2\sqrt{2})-216.2-48\sqrt{2}-54}{3}=-\frac{486}{3}=-162$$

$$36x^2+81y^2-72x-162y=207$$
$$36(x^2-2x)+81(y^2-2y)=207 \rightarrow 36[(x-1)^2-1]+81[(y-1)^2-1]=207 \rightarrow$$
$$36(x-1)^2+81(y-1)^2=207+36+81 \rightarrow \frac{36(x-1)^2}{4.9.9}+\frac{81(y-1)^2}{4.9.9}=\frac{4.9.9}{4.9.9} \rightarrow$$

$$\frac{1}{9}\cdot\frac{(x+2\sqrt{2}y-3)^2}{3^2}+\frac{(-2\sqrt{2}x+y-3)^2}{3^2}\cdot\frac{1}{4}=1 \rightarrow \frac{(x+2\sqrt{2}y-3)^2}{\underset{\hookrightarrow 9.9}{81}}+\frac{(-2\sqrt{2}x+y-}{\underset{\hookrightarrow 4.9}{36}}$$

$$\frac{4(x+2\sqrt{2}y-3)^2}{4.9.9}+\frac{9(-2\sqrt{2}x+y-3)^2}{4.9.9}=\frac{4.9.9}{4.9.9} \rightarrow 4(x+2\sqrt{2}y-3)^2+9(-2\sqrt{2}x+y-3)^2=$$

$$4(x^2+8y^2+9+4\sqrt{2}xy-6x-12\sqrt{2}y)$$

$$+9(8x^2+y^2+9-4\sqrt{2}xy+12\sqrt{2}x-6y)=324$$

$$76x^2+41y^2-20\sqrt{2}xy-24x+108\sqrt{2}x-48\sqrt{2}y-54y=324-36-81 \rightarrow$$

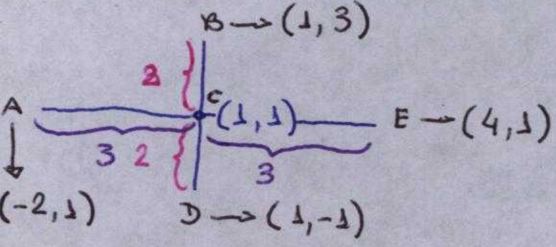
$$\boxed{76x^2-20\sqrt{2}xy+41y^2-24x+108\sqrt{2}x-54y-48\sqrt{2}y=207}$$

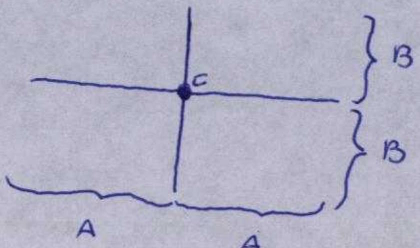
4 Agora é só resolver e graficar!

Como graficar

$$\frac{(x-1)^2}{9}+\frac{(y-1)^2}{4}=1$$

$$C=(x_0,y_0)\rightarrow C=(1,1)$$





$$\left.\begin{array}{l} A=(-2,1)\\ B=(1,3)\\ C=(1,1)\\ D=(1,-1)\\ E=(4,1)\end{array}\right\} x,y$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}\cos\alpha & -\operatorname{sen}\alpha\\ \operatorname{sen}\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}\rightarrow\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{3}\begin{pmatrix}1 & -2\sqrt{2}\\ 2\sqrt{2} & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

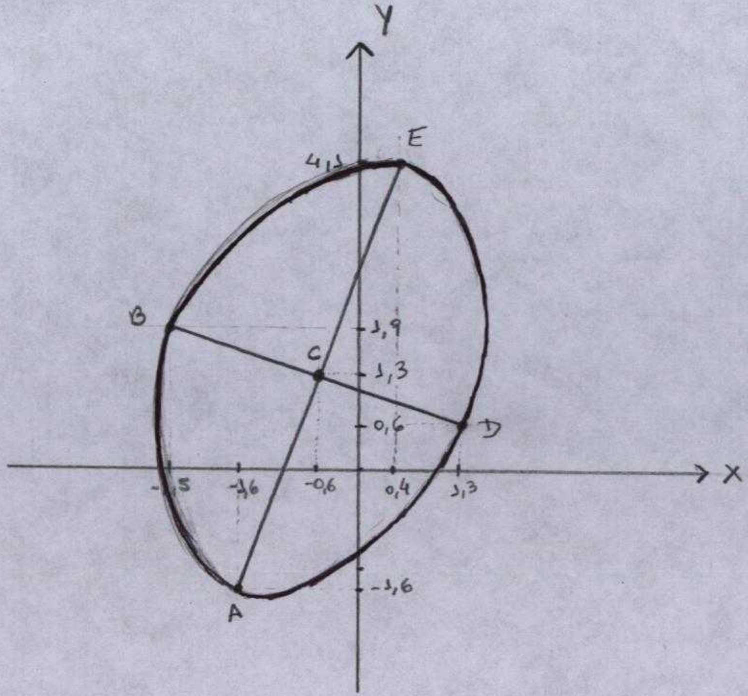
$$A=\frac{1}{3}\begin{pmatrix}1 & -2\sqrt{2}\\ 2\sqrt{2} & 1\end{pmatrix}\begin{pmatrix}-2\\1\end{pmatrix}\rightarrow A=\begin{pmatrix}-2-2\sqrt{2}\\-4\sqrt{2}+1\end{pmatrix}\cdot\frac{1}{3}\rightarrow A\cong\begin{pmatrix}-1{,}6\\-1{,}6\end{pmatrix}$$

$$B=\frac{1}{3}\begin{pmatrix}1 & -2\sqrt{2}\\ 2\sqrt{2} & 1\end{pmatrix}\begin{pmatrix}1\\3\end{pmatrix}\rightarrow B=\begin{pmatrix}1-6\sqrt{2}\\2\sqrt{2}+3\end{pmatrix}\cdot\frac{1}{3}\rightarrow B\cong\begin{pmatrix}-2{,}5\\1{,}9\end{pmatrix}$$

$$C=\frac{1}{3}\begin{pmatrix}1 & -2\sqrt{2}\\ 2\sqrt{2} & 1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\rightarrow C=\begin{pmatrix}1-2\sqrt{2}\\2\sqrt{2}+1\end{pmatrix}\cdot\frac{1}{3}\rightarrow C\cong\begin{pmatrix}-0{,}6\\1{,}3\end{pmatrix}$$

$$D=\frac{1}{3}\begin{pmatrix}1 & -2\sqrt{2}\\ 2\sqrt{2} & 1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}\rightarrow D=\begin{pmatrix}1+2\sqrt{2}\\2\sqrt{2}-1\end{pmatrix}\cdot\frac{1}{3}\rightarrow D\cong\begin{pmatrix}1{,}3\\0{,}6\end{pmatrix}$$

$$E=\frac{1}{3}\begin{pmatrix}1 & -2\sqrt{2}\\ 2\sqrt{2} & 1\end{pmatrix}\begin{pmatrix}4\\1\end{pmatrix}\rightarrow E=\begin{pmatrix}4-2\sqrt{2}\\8\sqrt{2}+1\end{pmatrix}\cdot\frac{1}{3}\rightarrow E\cong\begin{pmatrix}0{,}4\\4{,}1\end{pmatrix}$$



Y
X
E
4,1
B
1,9
C
1,3
0,6
D
-1,5
-1,6
-0,6
0,4
1,3
-1,6
A