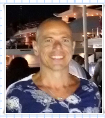

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SCHRÖDINGER EQUATION ($\hbar=1$)

$$H = -\frac{1}{2m}\partial_z^2 + V(z) \qquad V(z) = \begin{cases} 0 & 0<z<d \\ \infty & z<0 \text{ AND } z>d \end{cases}$$

$\gamma = z/d$ $\qquad$ $k = pd$

SOLUTION: $e^{ik\gamma} + R\, e^{-ik\gamma}$

$\gamma=0 \qquad 1+R=0 \Rightarrow R=-1$

$\gamma=1 \qquad e^{ik}+R\,e^{-ik}=0$

$$\Downarrow$$

$$e^{ik}-e^{-ik}=0 \Rightarrow \sin k = 0 \qquad \boxed{k=n\pi}$$

$$E_n = p_n^2/2m = (k_n/d)^2/2m = n^2\pi^2/2md^2$$

WAVE FUNCTIONS $\varphi_n(\gamma) = N \sin(n\pi\gamma)$

$$\int_0^1 d\gamma\, \sin(n\pi\gamma)\sin(m\pi\gamma) = \frac{1}{2}\int_0^1 d\gamma\, \{\cos[(n-m)\pi\gamma] - \cos[(n+m)\pi\gamma]\}$$

$$= \frac{1}{2}\left\{\frac{\sin[(n-m)\pi\gamma]}{(n-m)\pi} - \frac{\sin[(n+m)\pi\gamma]}{(n+m)\pi}\right\}_0^1$$

$n\neq m$ "0" $\qquad$ $n=m$ "$1/2$"

$$\boxed{E_n = n^2\pi^2/2md^2 \qquad \varphi_n(\gamma) = \sqrt{2}\,\sin(n\pi\gamma) \qquad \int_0^1 d\gamma\, \varphi_n(\gamma)\varphi_m(\gamma) = \delta_{nm}}$$

$$(H+\lambda\tilde{V})(\varphi_n+\lambda\tilde{\varphi}_n) = (E_n+\lambda\tilde{E}_n)(\varphi_n+\lambda\tilde{\varphi}_n)$$

FIRST ORDER CORRECTIONS

$$\underline{H\varphi_n} + \lambda H\tilde{\varphi}_n + \lambda\tilde{V} = \underline{E_n\varphi_n} + \lambda E_n\tilde{\varphi}_n + \lambda\tilde{E}_n\varphi_n$$

$$[H(\gamma) - E_n]\,\tilde{\varphi}_n(\gamma) = [\tilde{E}_n - \tilde{V}(\gamma)]\,\varphi_n(\gamma)$$

$$\int_0^1 d\gamma\, \varphi_n(\gamma) H(\gamma)\tilde{\varphi}_n(\gamma) = \int_0^1 d\gamma\, \varphi_n(\gamma)\left[-\frac{1}{2m}\partial_\gamma^2 + V(\gamma)\right]\tilde{\varphi}_n(\gamma)$$

$$= -\frac{1}{2m}\int_0^1 d\gamma\, \varphi_n(\gamma)\tilde{\varphi}_n''(\gamma) + \int_0^1 d\gamma\, V(\gamma)\varphi_n(\gamma)\tilde{\varphi}_n(\gamma) \qquad \tilde{\varphi}_n(\gamma) = \sum_s c_{ns}\varphi_s(\gamma)$$

$$= -\frac{1}{2m}\left[\cancel{\varphi_n(\gamma)\tilde{\varphi}_n'(\gamma)}\right]_0^1 + \frac{1}{2m}\int_0^1 d\gamma\, \varphi_n'(\gamma)\tilde{\varphi}_n'(\gamma) + \int_0^1 d\gamma\, V(\gamma)\varphi_n(\gamma)\hat{\varphi}_n(\gamma)$$

$$= \frac{1}{2m}\left[\cancel{\varphi_n'(\gamma)\tilde{\varphi}_n(\gamma)}\right]_0^1 - \frac{1}{2m}\int_0^1 d\gamma\, \varphi_n''(\gamma)\hat{\varphi}_n(\gamma) + \int_0^1 d\gamma\, V(\gamma)\varphi_n(\gamma)\hat{\varphi}_n(\gamma)$$

$$\int_0^1 d\gamma\, \varphi_n(\gamma)\, H(\gamma)\tilde{\varphi}_n(\gamma) = \int_0^1 d\gamma\, \underbrace{[H(\gamma)\varphi_n(\gamma)]}_{E_n\varphi_n(\gamma)}\, \tilde{\varphi}_n(\gamma)$$

$$\int_0^1 d\gamma\, \varphi_n(\gamma)\, [H(\gamma)\tilde{\varphi}_n(\gamma) - E_n\tilde{\varphi}_n(\gamma)] = 0 \Rightarrow \int_0^1 d\gamma\, \varphi_n(\gamma)\, [\tilde{E}_n - \tilde{V}(\gamma)]\, \varphi_n(\gamma)$$

$$\Downarrow$$

ENERGY CORRECTIONS TO E_n

$$\frac{n^2\pi^2}{2md^2} + \lambda \tilde{E}_n$$

$$\boxed{\tilde{E}_n = \int_0^1 d\gamma\, \tilde{V}(\gamma)\, \varphi_n^2(\gamma)}$$

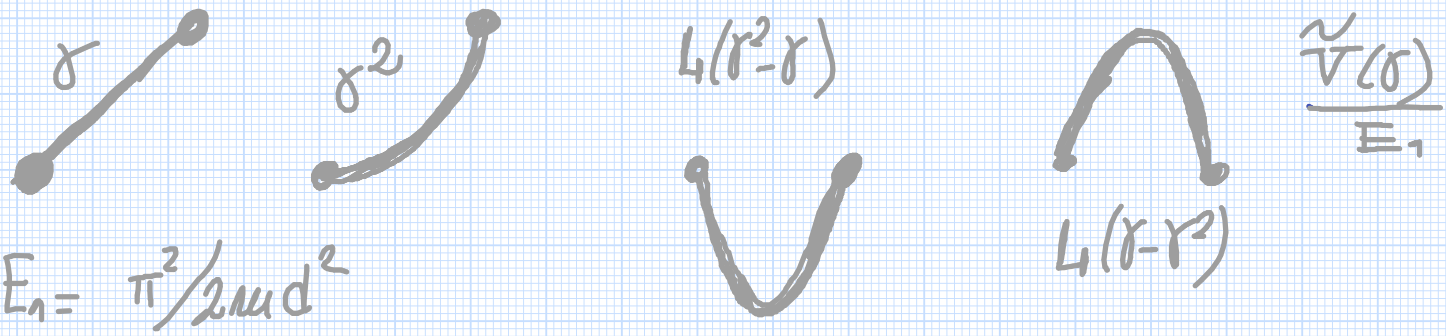
$$\int_0^1 d\gamma\, \varphi_m(\gamma)\tilde{\varphi}_n(\gamma) = \sum_s c_{ns} \int_0^1 d\gamma\, \varphi_m(\gamma)\varphi_s(\gamma) = c_{nm}$$

$$\int_0^1 d\gamma\, \varphi_m(\gamma)[H(\gamma) - E_n]\tilde{\varphi}_n(\gamma) = \int_0^1 d\gamma\, \varphi_m(\gamma)[\cancel{\tilde{E}_n} - \tilde{V}(\gamma)]\varphi_n(\gamma) \qquad m \neq n$$

$$(E_m - E_n)\int_0^1 d\gamma\, \varphi_m(\gamma)\tilde{\varphi}_n(\gamma) = -\int_0^1 d\gamma\, \varphi_m(\gamma)\varphi_n(\gamma)\tilde{V}(\gamma)$$

$$c_{nm} = \frac{\int_0^1 d\gamma\, \varphi_m(\gamma)\varphi_n(\gamma)\tilde{V}(\gamma)}{E_n - E_m}$$

$$\boxed{\tilde{\varphi}_n(\gamma) = \sum_m \frac{\int_0^1 d\gamma\, \varphi_m(\gamma)\varphi_n(\gamma)\tilde{V}(\gamma)}{E_n - E_m}\, \varphi_m(\gamma) \qquad m \neq n}$$



$$E_1 = \pi^2/2md^2$$

$$\tilde{V}(\gamma) = \{\gamma, \gamma^2, 4(\gamma^2 - \gamma), 4(\gamma - \gamma^2)\}\, E_1$$

$$\tilde{E}_n = E_1 \int_0^1 d\gamma\, \{\gamma, \gamma^2, 4(\gamma^2 - \gamma), 4(\gamma - \gamma^2)\}\, \sin^2(n\pi\gamma)$$

$$\int_0^1 d\gamma\, \gamma \sin^2(n\pi\gamma) = \int_0^1 d\gamma\, \gamma\, \frac{1-\cos(2n\pi\gamma)}{2} = \frac{1}{4} - \frac{1}{2}\int_0^1 d\gamma\, \gamma \cos(2n\pi\gamma) = \frac{1}{4} - \frac{1}{2}\int_0^1 d\gamma\, \gamma \left[\frac{\sin(2n\pi\gamma)}{2n\pi}\right]'$$

$$= \frac{1}{4} + \frac{1}{4n\pi}\int_0^1 d\gamma \sin(2n\pi\gamma) = \frac{1}{4} - \frac{1}{4n\pi}\left[\frac{\cos(2n\pi\gamma)}{2n\pi}\right]_0^1 = \frac{1}{4}$$

$$E_n = \frac{\pi^2}{2md^2}\left(n^2 + \frac{\lambda}{4}\right)$$

$$\boxed{\int_0^1 d\gamma\, \gamma \sin^2(n\pi\gamma) = \frac{1}{4}}$$

$$\int_0^1 d\gamma\, \gamma^2 \sin^2(\pi\gamma) = \frac{1}{6} - \frac{1}{4\pi^2} \qquad \int_0^1 d\gamma\, \gamma^2 \sin^2(2\pi\gamma) = \frac{1}{6} - \frac{1}{16\pi^2}$$

$$\int_0^1 d\gamma\, \gamma^2 \sin^2(3\pi\gamma) = \frac{1}{6} - \frac{1}{36\pi^2} \qquad \boxed{\int_0^1 d\gamma\, \gamma^2 \sin^2(n\pi\gamma) = \frac{1}{6} - \frac{1}{4n^2\pi^2}}$$

γ : $1/4$

γ^2 : $1/6 - 1/4n^2\pi^2$

$4(\gamma^2-\gamma)$: $2/3 - \frac{1}{n^2\pi^2} - 1 = -\frac{1}{3} - \frac{1}{n^2\pi^2}$

$4(\gamma-\gamma^2)$: $\frac{1}{3} + \frac{1}{n^2\pi^2}$

$$\boxed{\frac{\pi^2}{2md^2}\left[\begin{array}{ccc} & 1/4 & \gamma \\ n^2+\lambda & 1/6 - 1/4n^2\pi^2 & \gamma^2 \\ & -1/3 - 1/n^2\pi^2 & 4(\gamma^2-\gamma) \\ & 1/3 + 1/n^2\pi^2 & 4(\gamma-\gamma^2) \end{array}\right]}$$

$$\boxed{\tilde{\varphi}_n(\gamma) = \sum_m \frac{\int_0^1 d\gamma\, \varphi_m(\gamma)\varphi_n(\gamma)\tilde{V}(\gamma)}{E_n - E_m}\varphi_m(\gamma) \qquad m \neq n}$$

$$\int_0^1 d\gamma \{\gamma, \gamma^2\} \sin(n\pi x)\sin(m\pi x) \Leftarrow \quad n \neq m$$

$$\int_0^1 d\gamma\, \frac{\gamma}{2}\{\cos[(n-m)\pi x] - \cos[(n+m)\pi x]\}$$

$$\int_0^1 d\gamma\, \gamma\cos(a\gamma) = \int_0^1 d\gamma\, \gamma\left[\frac{\sin(a\gamma)}{a}\right]' = -\int_0^1 d\gamma\, \frac{\sin(a\gamma)}{a} = \frac{\cos(a)-1}{a^2}$$

$$\frac{1}{2}\left\{\frac{\cos[(n-m)\pi]-1}{(n-m)^2\pi^2}-\frac{\cos[(n+m)\pi]-1}{(n+m)^2\pi^2}\right\}$$

$$\frac{1}{2\pi^2}\left\{\underbrace{\frac{1}{(n+m)^2}-\frac{1}{(n-m)^2}}+\frac{\cos[(n-m)\pi]}{(n-m)^2}-\frac{\cos[(n+m)\pi]}{(n+m)^2}\right\}$$

$$\frac{1}{2\pi^2}\left\{\frac{4nm}{(n^2-m^2)^2}+\frac{\cos(n\pi)\cos(m\pi)}{(n-m)^2}-\frac{\cos(n\pi)\cos(m\pi)}{(n+m)^2}\right\}$$

$$\frac{1}{2\pi^2}\frac{4nm}{(n^2-m^2)^2}\left\{1+(-1)^n(-1)^m\right\}$$

$$\frac{2nm}{(n^2-m^2)^2\pi^2}$$

2 $n+m$ EVEN

0 $n+m$ ODD

TO BE CHECKED !!!