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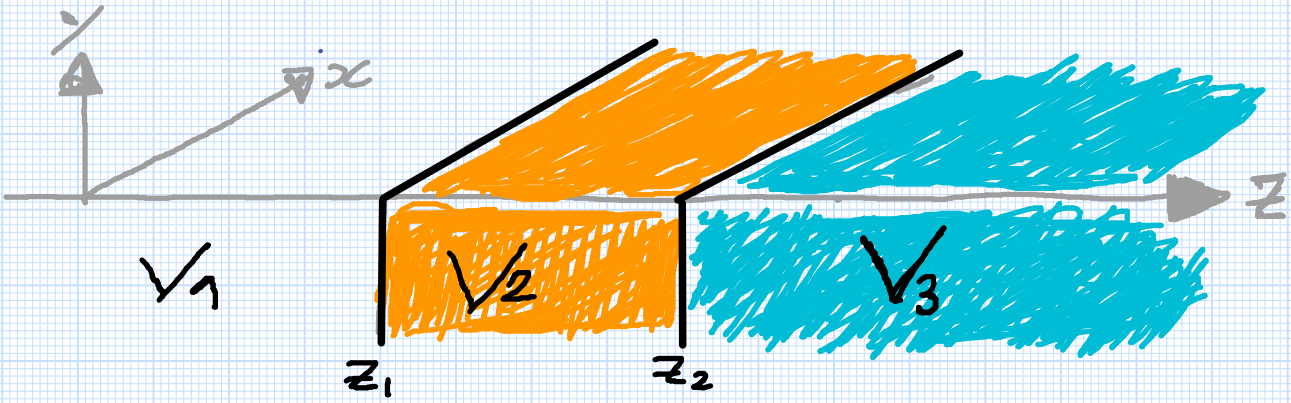


$$e^{ip_1 z} + R_{123} e^{-ip_1 z} \Big|_{z_1} \quad A_{12} e^{ip_2 z} + B_{12} e^{-ip_2 z} \Big|_{z_2} \quad T_{123}\, e^{ip_3 z}$$

$$e^{ip_1 z_1} + R_{123} e^{-ip_1 z_1} = A_{12} e^{ip_2 z_1} + B_{12} e^{-ip_2 z_1} \qquad A_{12} e^{ip_2 z_2} + B_{12} e^{-ip_2 z_2} = T_{123} e^{ip_3 z_2}$$

$$ip_1 \left(e^{ip_1 z_1} - R_{123} e^{-ip_1 z_1}\right) = ip_2 \left(A_{12} e^{ip_2 z_1} - B_{12} e^{-ip_2 z_1}\right) \qquad ip_2 \left(A_{12} e^{ip_2 z_2} - B_{12} e^{-ip_2 z_2}\right) = ip_3 T_{123} e^{ip_3 z_2}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} e^{ip_1 z_1} & 0 \\ 0 & e^{-ip_1 z_1} \end{pmatrix} \begin{pmatrix} 1 \\ R_{123} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{p_2}{p_1} & -\frac{p_2}{p_1} \end{pmatrix} \begin{pmatrix} e^{ip_2 z_1} & 0 \\ 0 & e^{-ip_2 z_1} \end{pmatrix} \begin{pmatrix} A_{12} \\ B_{12} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} e^{ip_2 z_2} & 0 \\ 0 & e^{-ip_2 z_2} \end{pmatrix} \begin{pmatrix} A_{12} \\ B_{12} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{p_3}{p_2} \end{pmatrix} T_{123} e^{ip_3 z_2}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \frac{p_2}{p_1} & -\frac{p_2}{p_1} \end{pmatrix} = \begin{pmatrix} \frac{p_1+p_2}{2p_1} & \frac{p_1-p_2}{2p_1} \\ \frac{p_1-p_2}{2p_1} & \frac{p_1+p_2}{2p_1} \end{pmatrix} = \frac{1}{t_{12}} \begin{pmatrix} 1 & r_{12} \\ r_{12} & 1 \end{pmatrix}$$

$$\begin{pmatrix} e^{ip_1 z_1} \\ R_{123} e^{-ip_1 z_1} \end{pmatrix} = \frac{1}{t_{12}} \begin{pmatrix} 1 & r_{12} \\ r_{12} & 1 \end{pmatrix} \begin{pmatrix} e^{-ip_2 d_2} & 0 \\ 0 & e^{ip_2 d_2} \end{pmatrix} \frac{1}{t_{23}} \begin{pmatrix} 1 \\ r_{23} \end{pmatrix} T_{123} e^{ip_3 z_2}$$

$$\begin{pmatrix} 1 \\ R_{123} e^{-2ip_1 z_1} \end{pmatrix} = \frac{1}{t_{12}} \begin{pmatrix} 1 & r_{12} \\ r_{12} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{2ip_2 d_2} \end{pmatrix} \frac{1}{t_{23}} \begin{pmatrix} 1 \\ r_{23} \end{pmatrix} T_{123} e^{i(p_3 z_2 - p_2 d_2 - p_1 z_1)}$$

$$= \frac{1}{t_{12} t_{23}} \begin{pmatrix} 1 & r_{12} e^{2ip_2 d_2} \\ r_{12} & e^{2ip_2 d_2} \end{pmatrix} \begin{pmatrix} 1 \\ r_{23} \end{pmatrix} T_{123} e^{i(p_3 z_2 - p_2 d_2 - p_1 z_1)}$$

$$= \begin{pmatrix} 1 + r_{12} r_{23} e^{2ip_2 d_2} \\ r_{12} + r_{23} e^{2ip_2 d_2} \end{pmatrix} \frac{T_{123}}{t_{12} t_{23}} e^{i(p_3 z_2 - p_2 d_2 - p_1 z_1)}$$

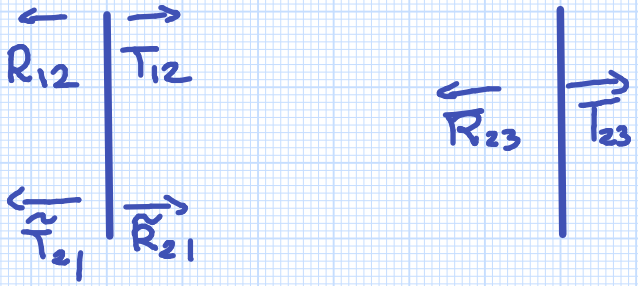
$$T_{123} = \frac{t_{12}\, t_{23}}{1 + r_{12} r_{23} e^{2i p_2 d_2}} e^{i(p_1 z_1 + p_2 d_2 - p_3 z_2)}$$

$$R_{123} = \frac{r_{12} + r_{23} e^{2i p_2 d_2}}{1 + r_{12} r_{23} e^{2i p_2 d_2}} e^{2i p_1 z_1}$$

IF $p_1, p_3 \in \mathbb{R}$ $\qquad |R_{123}|^2 + \frac{p_3}{p_1} |T_{123}|^2 = 1$

TWO CASE $p_2 \in \mathbb{R}$, $p_2 = i|p_2| \in i\,\mathbb{R}$ $\qquad$ TO PROOF

ALTERNATIVE WAY :



$$R_{123} = R_{12} + T_{12} R_{23} \tilde{T}_{21} + T_{12} R_{23} \tilde{R}_{21} R_{23} \tilde{T}_{21} + T_{12} R_{23} (\tilde{R}_{21} R_{23})^2 \tilde{T}_{21} + \dots$$

$$= R_{12} + T_{12} R_{23} \tilde{T}_{21} / (1 - \tilde{R}_{21} R_{23})$$

$$T_{123} = T_{12} T_{23} + T_{12} R_{23} \tilde{R}_{21} T_{23} + T_{12} (R_{23} \tilde{R}_{21})^2 T_{23} + \dots$$

$$= T_{12} T_{23} / (1 - \tilde{R}_{21} R_{23})$$

$R_{12} = r_{12} e^{2ip_1 z_1}$ $\quad \tilde{R}_{21} = r_{21} e^{-2ip_2 z_1}$ $\quad R_{23} = r_{23} e^{2ip_2 z_2}$ $\quad \tilde{R}_{21} R_{23} = r_{21} r_{23} e^{2ip_2 d_2}$

$$1 - \tilde{R}_{21} R_{23} = 1 + r_{12} r_{23} e^{2ip_2 d_2}$$

$T_{12} T_{23} = t_{12} e^{i(p_1 - p_2) z_1} t_{23} e^{i(p_2 - p_3) z_2}$

$$T_{123} = \frac{t_{12} t_{23}}{1 + r_{12} r_{23} e^{2ip_2 d_2}} e^{i(p_1 z_1 + p_2 d_2 - p_3 z_2)}$$

$R_{12} - R_{12} \tilde{R}_{21} R_{23} + T_{12} R_{23} \tilde{T}_{21}$ $\qquad$ AVAM $\quad R_{123}$

$R_{12} + R_{23} (T_{12} \tilde{T}_{21} - R_{12} \tilde{R}_{21})$

$$\frac{2p_1}{p_1+p_2} \frac{2p_2}{p_2+p_1} e^{i(p_1 - p_2) z_1} \overline{e^{i(p_2 - p_1) z_1}} - r_{12} e^{2ip_1 z_1} r_{21} e^{-2ip_2 z_1}$$

$$\left[\frac{4 p_1 p_2}{(p_1+p_2)^2} + \frac{(p_1 - p_2)^2}{(p_1+p_2)^2} \right] e^{2i(p_1 - p_2) z_1}$$

$$r_{12} e^{2ip_1 z_1} + r_{23} e^{2ip_2 z_2} e^{2i(p_1 - p_2) z_1}$$

$$(r_{12} + r_{23} e^{2ip_2 d_2}) e^{2ip_1 z_1}$$

$$R_{123} = \frac{r_{12} + r_{23} e^{2ip_2 d_2}}{1 + r_{12} r_{23} e^{2ip_2 d_2}} e^{2ip_1 z_1}$$