$$\sqrt{\frac{x^2-10x+16}{x^2-10x+24}}$$

$\boxed{\text{PRIMEIRO PASSO}}$ DECOMPOR NUMERADOR E DENOMINADOR

PARA PODER FAZER ISSO PRECISAMOS ENCONTRAR OS "ZEROS" DO NUMERADOR E DENOMINADOR

NUM: $x^2-10x+16=0$

$$x = \frac{10 \pm \sqrt{100-64}}{2} = \frac{10 \pm 6}{2} \begin{cases} 2 \\ 8 \end{cases}$$

DEN: $x^2-10x+24=0$

$$x = \frac{10 \pm \sqrt{100-96}}{2} = \frac{10 \pm 2}{2} \begin{cases} 4 \\ 6 \end{cases}$$

ZEROS DA FUNÇÃO
$(2,0)$ $(8,0)$

ASSÍNTOTAS VERTICAIS
$(4^-, ?\infty)$ $(6^-, ?\infty)$
$(4^+, ?\infty)$ $(6^+, ?\infty)$

$$f(x) = x^2-10x+16 = (x-2)(x-8)$$
$$g(x) = x^2-10x+24 = (x-4)(x-6)$$

$$F(x) = \sqrt{\frac{f(x)}{g(x)}} = \sqrt{\frac{x^2-10x+16}{x^2-10x+24}} = \frac{\sqrt{(x-2)(x-8)}}{(x-4)(x-6)}$$

DOMÍNIO DA FUNÇÃO $f(x) \geq 0$
DEVIDO À PRESENÇA DA RAIZ

$(x-2)(x-8) \geq 0$

SABEMOS QUE EM 2 E 8
A FUNÇÃO É ZERO

CONTROLAREMOS O QUE ACONTECE ANTES DE 2
ENTRE 2 E 8 E DEPOIS DE 8 (É SUFICIENTE
PEGAR UM PONTO NOS 3 CASOS)

$x=1$ $(1-2)(1-8)$ A FUNÇÃO É POSITIVA

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$x = 3 \quad (3-2)(3-8)$ A FUNÇÃO É NEGATIVA

$x = 9 \quad (9-2)(9-8)$ A FUNÇÃO É POSITIVA

$+ + + + + \bullet - - - - - - \bullet + + + + + +$

2 ZONA PROIBIDA 8

$\boxed{\text{ASSÍNTOTAS HORIZONTAIS}}$ $x \to \pm\infty$

$$\frac{\sqrt{x^2-10x+16}}{x^2-10x+24} \text{ SE COMPORTA COMO } \frac{\sqrt{x^2}}{x^2}$$

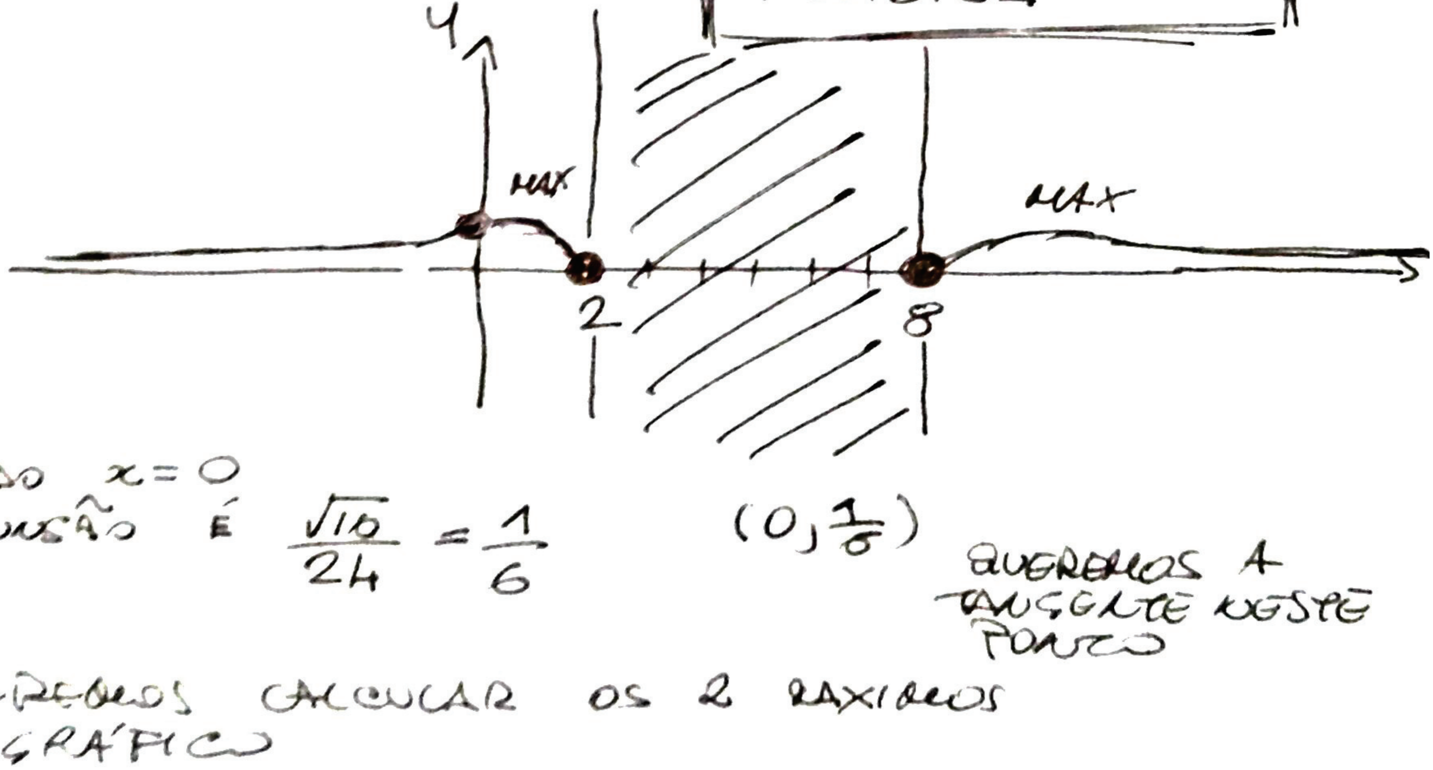
$x \to +\infty \qquad \sqrt{x^2}/x^2 \longrightarrow 0^+$

$x \to -\infty \qquad \sqrt{x^2}/x^2 \longrightarrow 0^+$

$\boxed{(+\infty, 0^+) \quad (-\infty, 0^-)}$

$\boxed{\text{ASSÍNTOTAS VERTICAIS}}$

$\boxed{\text{4 E 6 ESTÃO NA ZONA PROIBIDA}}$



QUANDO $x = 0$

A FUNÇÃO É $\frac{\sqrt{16}}{24} = \frac{1}{6}$ $\quad (0, \frac{1}{6})$

QUEREMOS A TANGENTE NESTE PONTO

QUEREMOS CALCULAR OS 2 MAXIMOS DO GRÁFICO

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$$F'(x) = \frac{f'(x)\, g(x) - 2 f(x)\, g'(x)}{2\sqrt{f(x)}\; g^2(x)}$$

$f'(x) = 2x - 10$ $\qquad f(x) = x^2 - 10x + 16$

$g'(x) = 2x - 10$ $\qquad g(x) = x^2 - 10x + 24$

num

$(2x-10)(x^2-10x+24) - 2(2x-10)(x^2-10x+16)$

$(2x-10)\,[x^2-10x+24-2x^2+20x-32]$

$2(x-5)(-x^2+10x-8)$

$$F'(x) = -\frac{(x-5)(x^2-10x+8)}{\sqrt{x^2-10x+16}\;(x^2-10x+24)^2}$$

ZEROS DE $F'(x)$ $\qquad x = 5$ (NA ZONA PROIBIDA)

$x^2 - 10x + 8 = 0$ $\qquad \frac{10 \pm \sqrt{100-32}}{2} = \frac{10 \pm 2\sqrt{17}}{2}$

$\boxed{5 \pm \sqrt{17}}$

TANGENTE EM $\left(0, \frac{1}{6}\right)$

$$y = F'(0)(x-0) + \frac{1}{6}$$

$$F'(0) = -\frac{(-5)\,8}{\sqrt{16}\,(24)^2} = \frac{10}{2^2\, 12^2} = \frac{5}{288}$$

$$\boxed{y = \frac{5}{288}x + \frac{1}{6}}$$

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$$\frac{x^2-10x+16}{x\,(x^2-10x+24)} = \frac{(x-2)(x-8)}{x\,(x-4)(x-6)}$$

ZEROS DA FUNÇÃO: $\boxed{(2,0)\ \ (8,0)}$

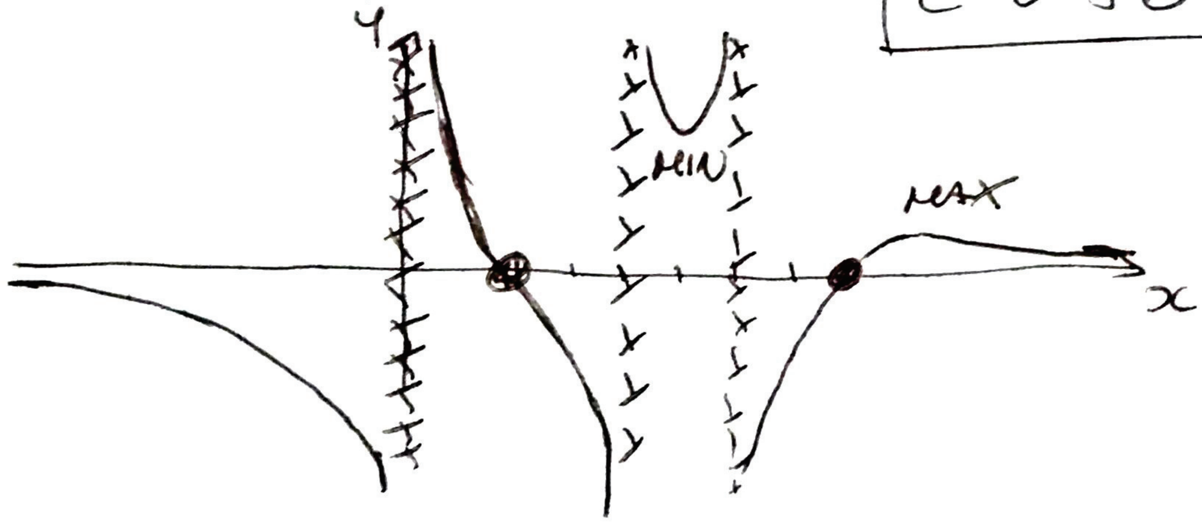
ASSÍNTOTAS VERTICAIS: $(0^{\pm}, ?\infty)\,(4^{\pm}, ?\infty)\,(6^{\pm}, ?\infty)$

$0^-\ (-0.1)$	$\frac{-\ -}{-\ -\ -}$	$= -$	$(0^-, -\infty)$
$0^+\ (+0.1)$	$\frac{-\ -}{+\ -\ -}$	$= +$	$(0^+, +\infty)$
$4^-\ (3.9)$	$\frac{+\ -}{+\ -\ -}$	$= -$	$(4^-, -\infty)$
$4^+\ (4.1)$	$\frac{+\ -}{+\ +\ -}$	$= +$	$(4^+, +\infty)$
$6^-\ (5.9)$	$\frac{+\ -}{+\ +\ -}$	$= +$	$(6^-, +\infty)$
$6^+\ (6.1)$	$\frac{+\ -}{+\ +\ +}$	$= -$	$(6^+, -\infty)$

ASSÍNTOTAS HORIZONTAIS $x \to \pm\infty$

A FUNÇÃO SE COMPORTA COMO x^2/x^3

$$\boxed{\begin{matrix}(+\infty, 0^+)\\(-\infty, 0^-)\end{matrix}}$$



TANGENTE EM $x=1$

$$F(1) = \frac{1-10+16}{1\,(1-10+24)} = \frac{7}{15}$$

$$y = F'(1)\,(x-1) + \frac{7}{15}$$

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$$F'(x) = [f'(x)g(x) - f(x)g'(x)]/g^2(x)$$

$f(x) = x^2 - 10x + 16$ $\qquad$ $f'(x) = 2x - 10$

$g(x) = x^3 - 10x^2 + 24x$ $\qquad$ $g'(x) = 3x^2 - 20x + 24$

num:

$$(2x-10)(x^3-10x^2+24x) - (3x^2-20x+24)(x^2-10x+1$$

$$2x^4 \; \cancel{-20x^3} \; \cancel{+48x^2}$$
$$\cancel{-10x^3} + 100x^2 \; \cancel{-240x}$$
$$-3x^4 \; \cancel{+30x^3} \; \cancel{-48x^2}$$
$$+20x^3 - 200x^2 + 320x$$
$$-24x^2 \; \cancel{+240x} - 384$$

$$-x^4 + 20x^3 - 124x^2 + 320x - 384$$

$$F'(x) = - \frac{x^4 - 20x^3 + 124x^2 - 320x + 384}{x^2(x-4)^2(x-6)^2}$$

$x=1$ $\qquad$ $$F'(1) = - \frac{1 - 20 + 124 - 320 + 384}{1^2\,(-3)^2\,(-5)^2}$$

$$= - \frac{169}{225}$$

Tangente $\qquad$ $$y = - \frac{169}{225}(x-1) + \frac{7}{15}$$

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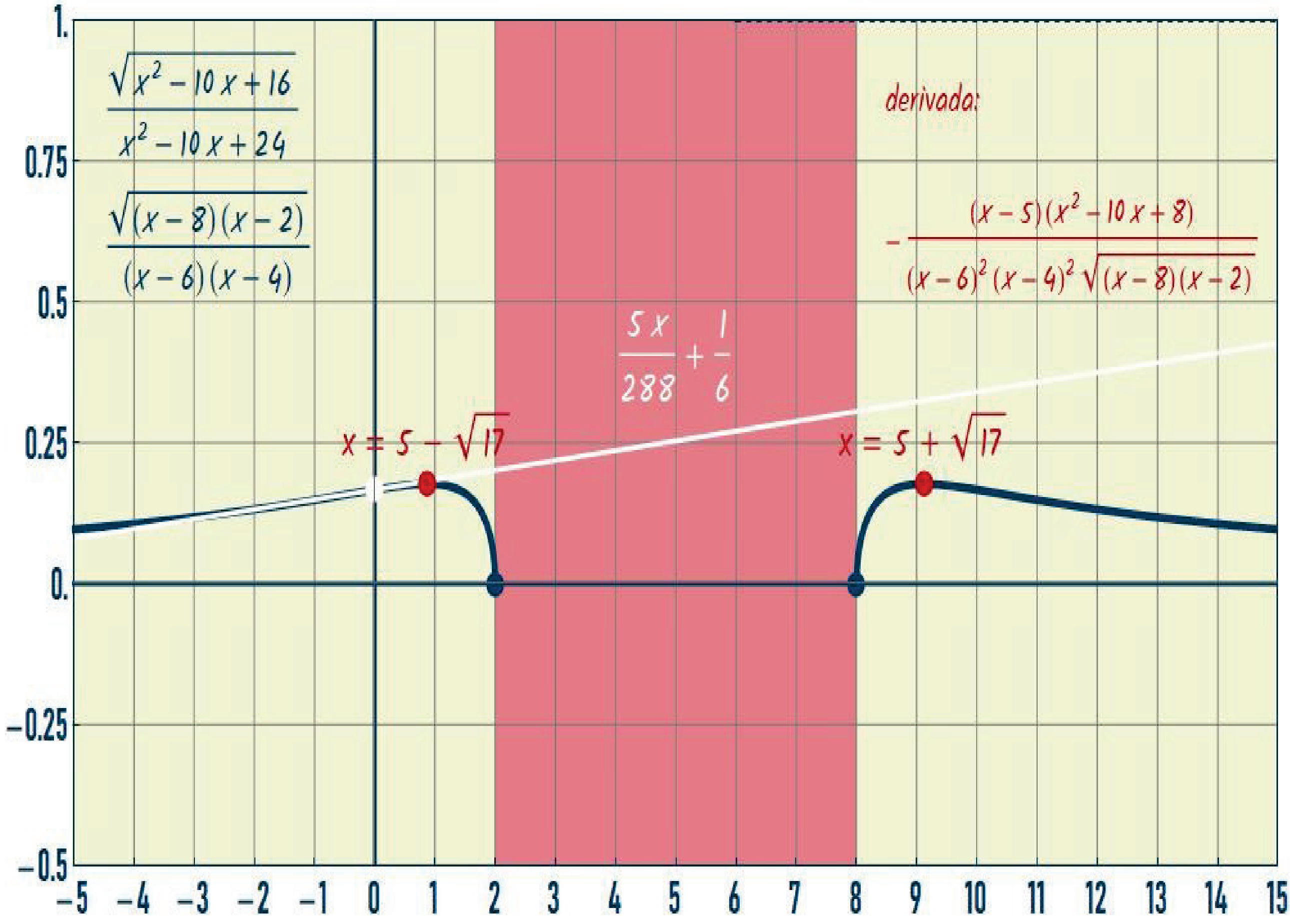
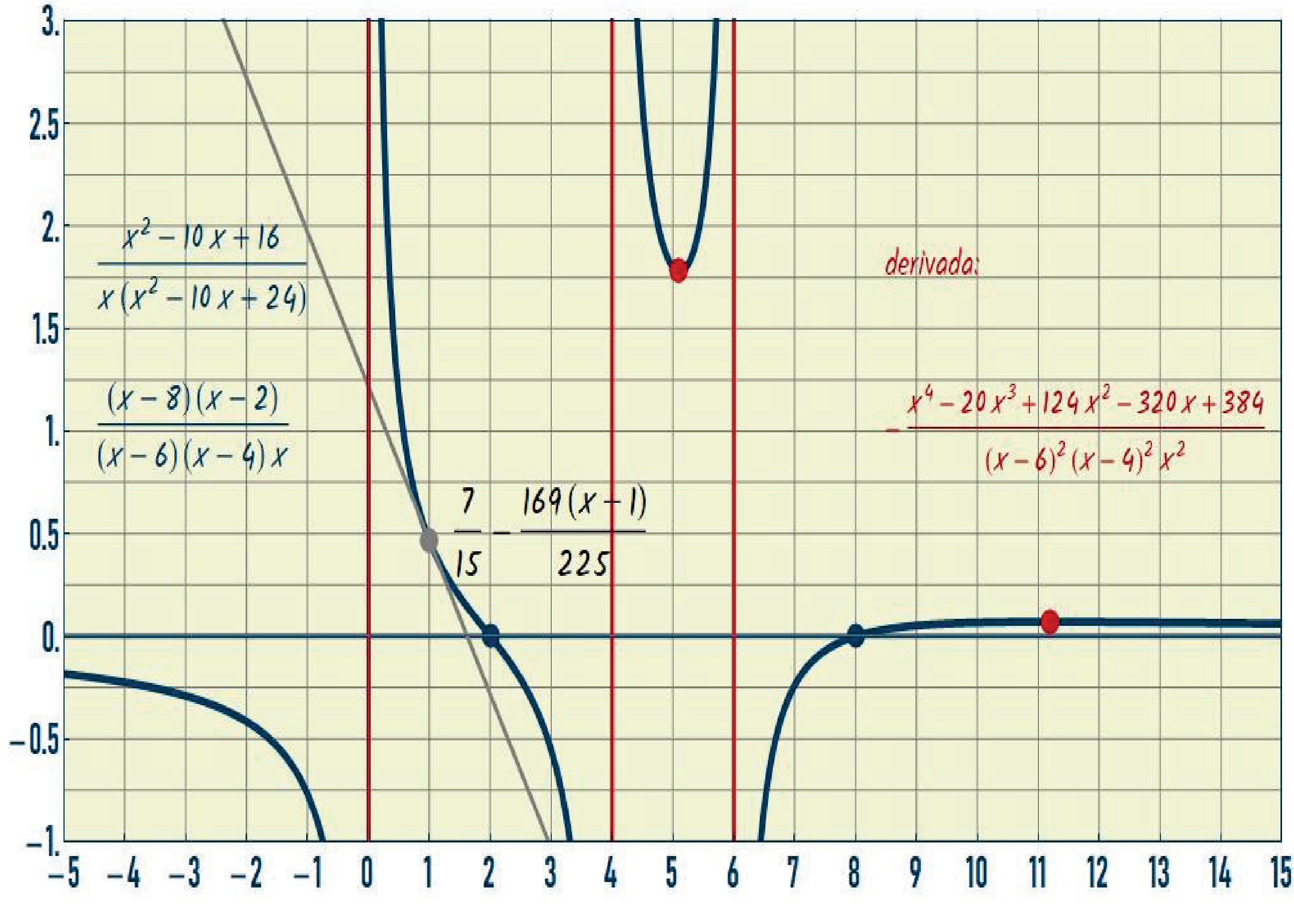


$\frac{\sqrt{x^2-10x+16}}{x^2-10x+24}$
$\frac{\sqrt{(x-8)(x-2)}}{(x-6)(x-4)}$
derivada:
$-\frac{(x-5)(x^2-10x+8)}{(x-6)^2(x-4)^2\sqrt{(x-8)(x-2)}}$
$\frac{5x}{288}+\frac{1}{6}$
$x=5-\sqrt{17}$
$x=5+\sqrt{17}$



$\frac{x^2-10x+16}{x(x^2-10x+24)}$
$\frac{(x-8)(x-2)}{(x-6)(x-4)x}$
derivada:
$-\frac{x^4-20x^3+124x^2-320x+384}{(x-6)^2(x-4)^2x^2}$
$\frac{7}{15}-\frac{169(x-1)}{225}$