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Estudo de funções do tipo $F(x) = \sqrt{f(x)}/g(x)$

0) domínio $[\ f(x) > 0\ ]$

1) interseções com o eixo x $[\ f(x) = 0\ ]$

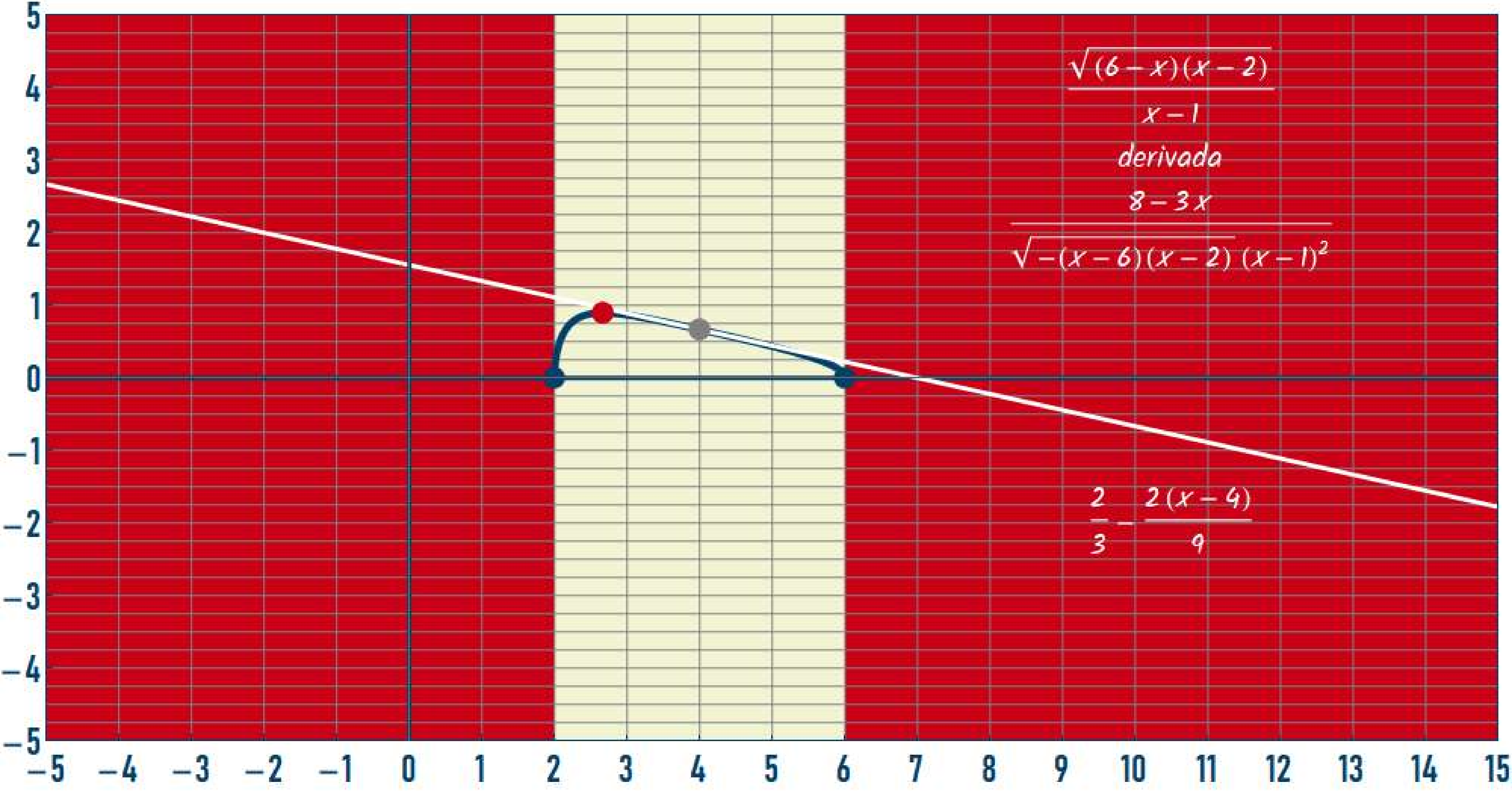
2) interseção com o eixo y $[\ \{\ 0\ ,\ F(0)\ \}\ ]$

3) assintotas horizontais $[\ x \to \pm\infty\ ]$

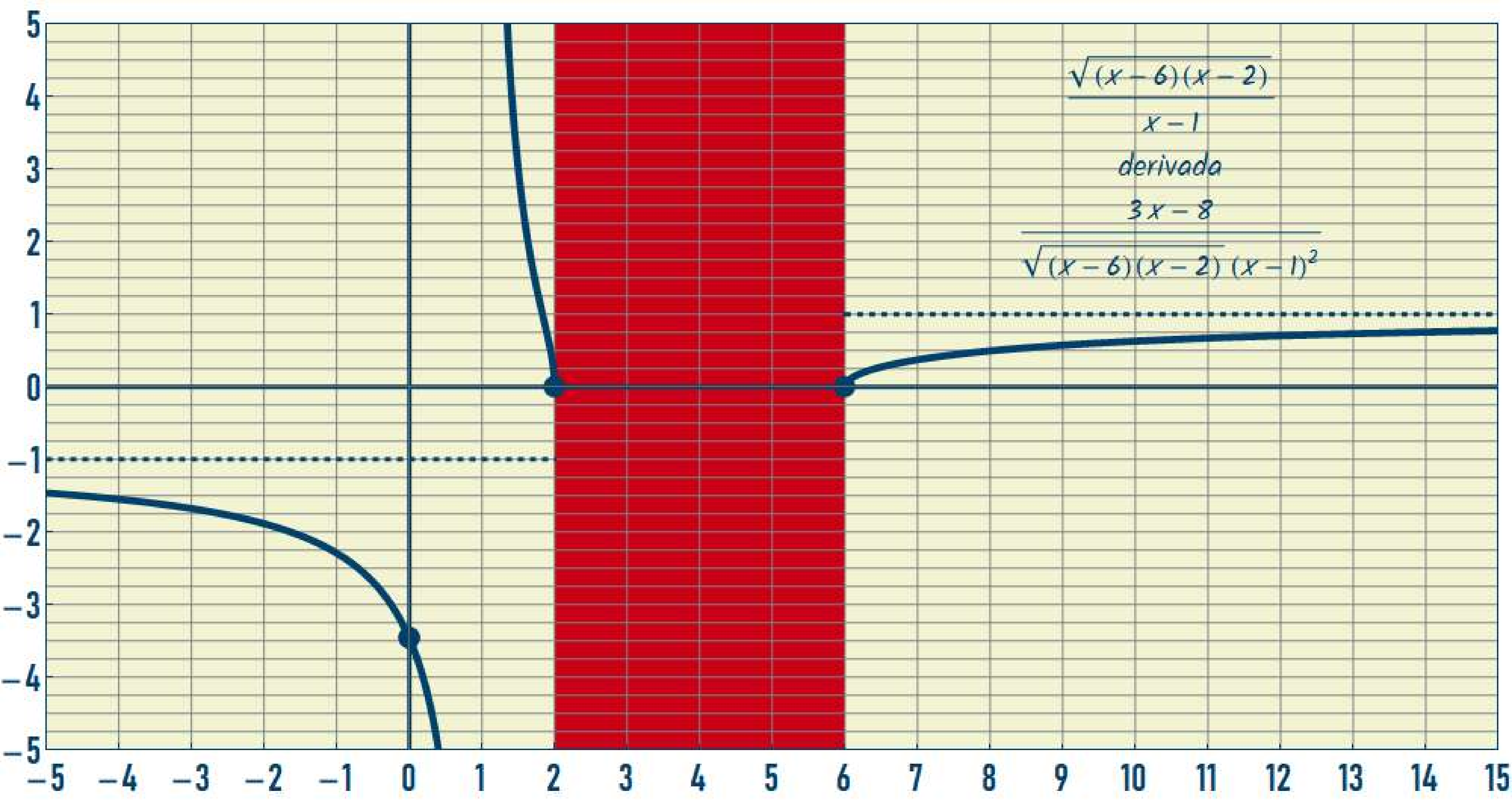
4) assintotas verticais $[\ \{\ a^-,\ ?\ \infty\ \}$ $\{\ a^+,\ ?\ \infty\ \}$

5) max/min $\left[\ F'(x) = \dfrac{f'(x)\,g(x) - 2\,f(x)\,g'(x)}{2\sqrt{f(x)}\ g^2(x)}\ \right]$

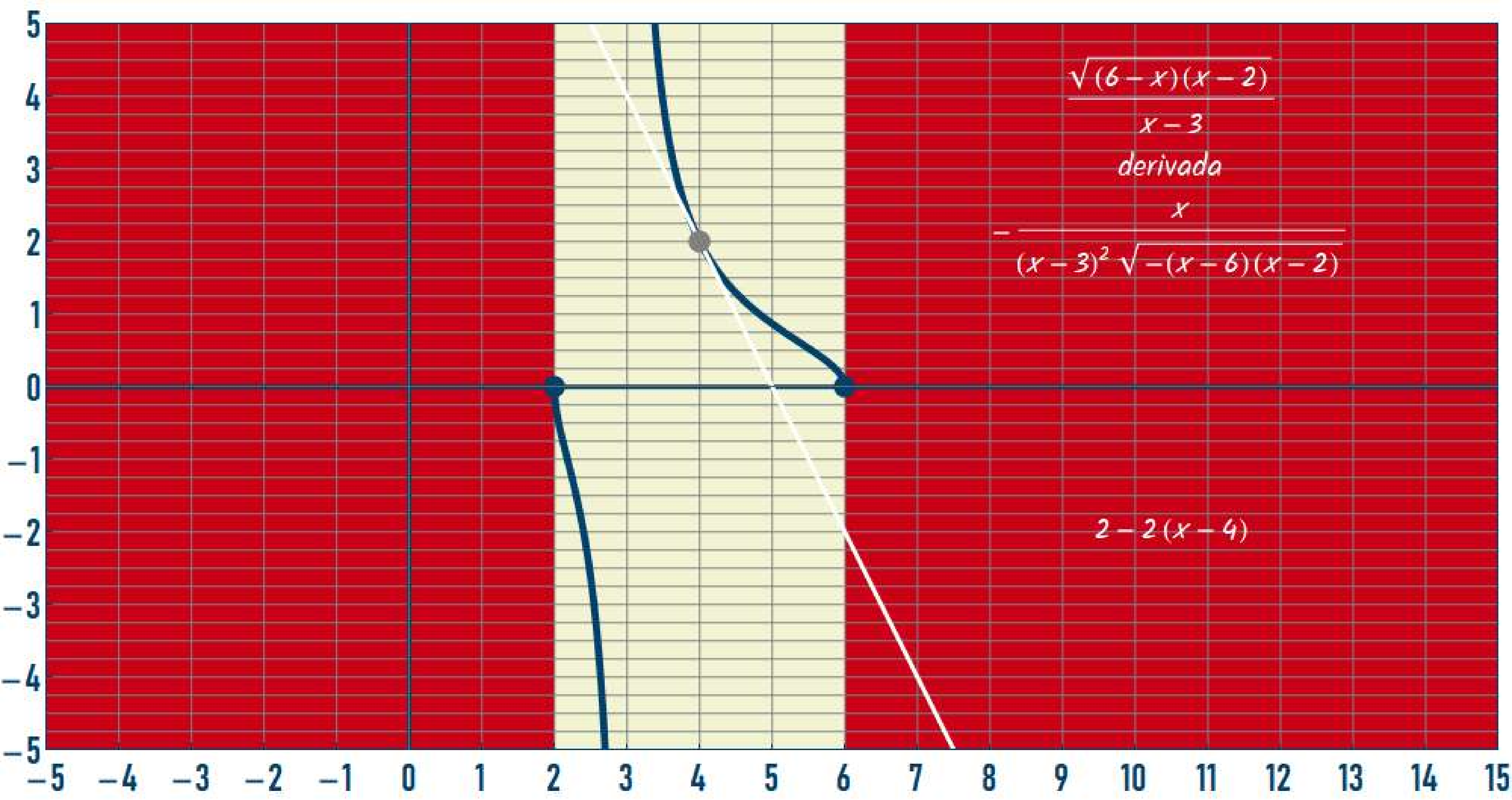
6) reta tangente ao ponto (x0 , y0) $[\ y = F'(x0)\,(\ x - x0\ ) + y0\ ]$

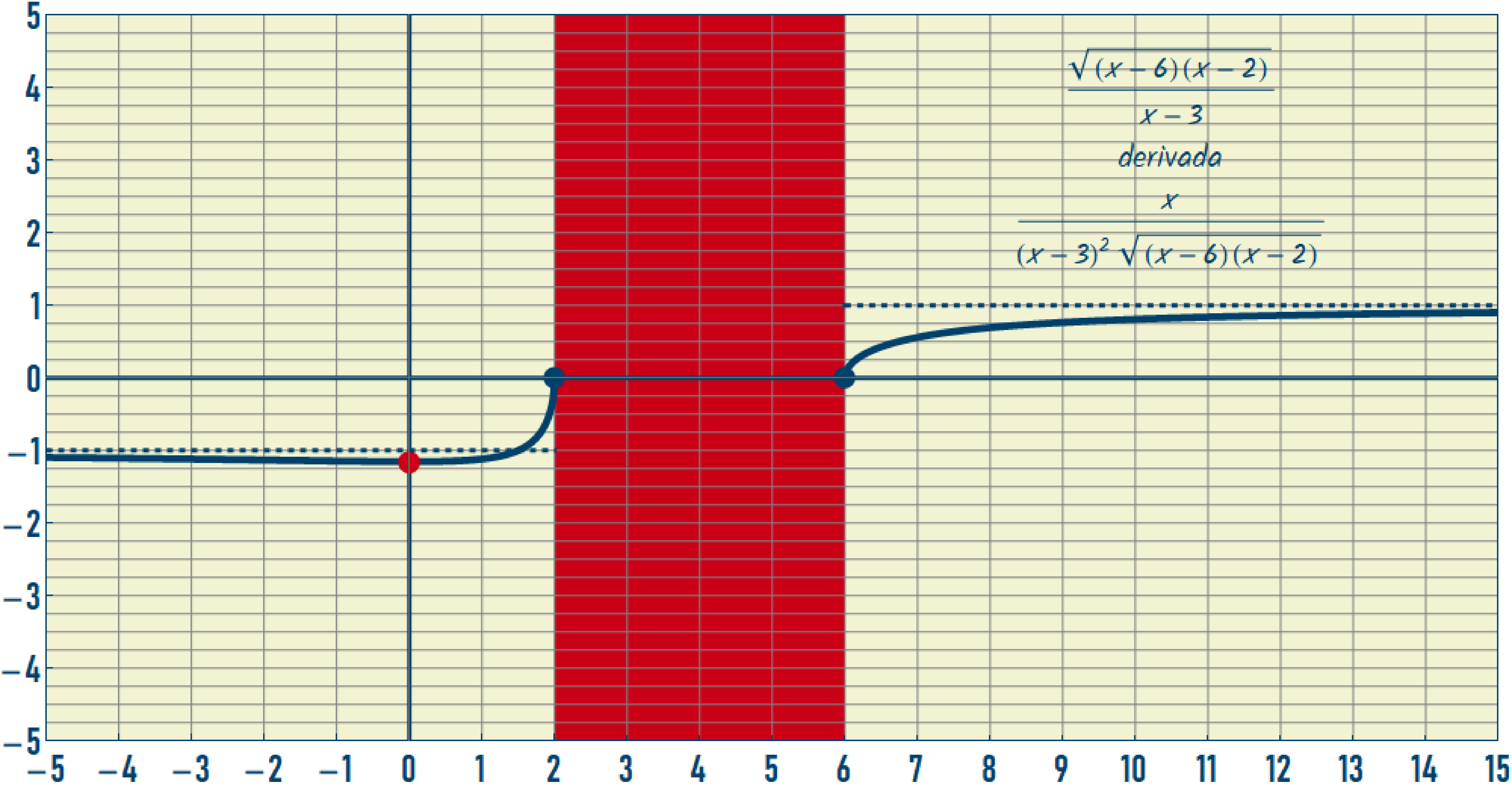


$\frac{\sqrt{(6-x)(x-2)}}{x-1}$
derivada
$\frac{8-3x}{\sqrt{-(x-6)(x-2)}\,(x-1)^2}$
$\frac{2}{3}-\frac{2(x-4)}{9}$



$\frac{\sqrt{(x-6)(x-2)}}{x-1}$
derivada
$\frac{3x-8}{\sqrt{(x-6)(x-2)}\,(x-1)^2}$


$\frac{\sqrt{(6-x)(x-2)}}{x-3}$
derivada
$-\frac{x}{(x-3)^2\sqrt{-(x-6)(x-2)}}$
$2-2(x-4)$
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5 −4 −3 −2 −1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15



$\dfrac{\sqrt{(x-6)(x-2)}}{x-3}$
derivada
$\dfrac{x}{(x-3)^2\sqrt{(x-6)(x-2)}}$
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

$$\frac{\sqrt{(x-2)(6-x)}}{x-1}$$

$- \; - \; - \; - \; + \; + \; + \; + \; - \; - \; - \; -$

$-1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5 \;\; 6 \;\; 7 \;\; 8 \;\; 9$

1) ZEROS DA FUNÇÃO $f(x) = 0 \Rightarrow (2,0) \; (6,0)$

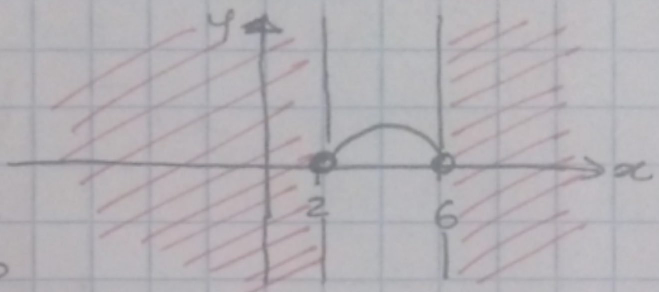
2) INTERSEÇÃO COM O EIXO Y $x = 0$ FORA DO DOMÍNIO

3) ASSÍNTOTAS HORIZONTAIS $x \to \pm\infty$

FORA DO DOMÍNIO

4) ASSÍNTOTAS VERTICAIS $x = 1$ FORA DO DOMÍNIO

TENTAMOS ESBOÇAR O GRÁFICO



OBSERVAR QUE O DENOMINADOR $x-1$ É POSITIVO NO DOMÍNIO

CALCULAREMOS AGORA O PONTO DE MAX

5) $F'(x) = \left[f'(x) g(x) - 2 f(x) g'(x) \right] / 2\sqrt{f(x)}\; g^2(x)$

$f(x) = -x^2 + 8x - 12 \qquad f'(x) = -2x + 8$

$g(x) = x - 1 \qquad g'(x) = 1$

$$F'(x) = \frac{(-2x+8)(x-1) - 2(-x^2+8x-12)\,1}{2\sqrt{(x-2)(6-x)}\;(x-1)^2}$$

NUM: $-2x^2 + 2x$

$+8x - 8$

$+2x^2 - 16x + 24$

$/\!/ \quad -6x + 16$

$$F'(x) = \frac{8-3x}{\sqrt{(x-2)(6-x)}\;(x-1)^2}$$

$\boxed{\text{MAX} \quad x = 8/3}$

6) TANGENTE NO PONTO DE ABSCISSA $x_0 = 4$

$y = F'(x_0)(x - x_0) + F(x_0)$

$F(x_0) = 2/3$

$F'(x_0) = \frac{8-12}{2 \cdot 9} = -2/9$

$$\boxed{y = -\frac{2}{9}(x-4) + \frac{2}{3}}$$

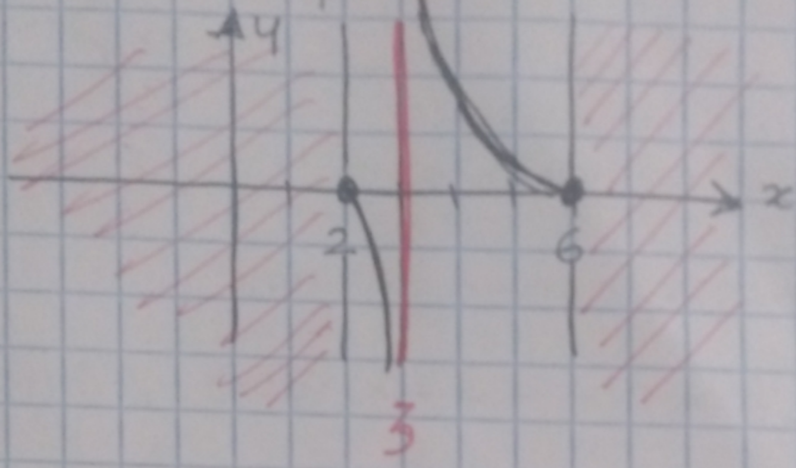
$$\frac{\sqrt{(x-2)(6-x)}}{x-3}$$

1) $(2,0)$ $(6,0)$
2) FORA DO DOMÍNIO
3) FORA DO DOMÍNIO

4) $(3^-, ?\infty)$ $(3^+, ?\infty)$

3^- (2.9) $\frac{+}{-} = -$ $(3^-, -\infty)$

3^+ (3.1) $\frac{+}{+} = +$ $(3^+, +\infty)$



5) $f(x) = -x^2 + 8x - 12$ $f'(x) = -2x + 8$

$g(x) = x - 3$ $g'(x) = 1$

$$F'(x) = \frac{(-2x+8)(x-3) - 2(-x^2+8x-12)\,1}{2\sqrt{(x-2)(6-x)}\,(x-3)^2}$$

NUM $-2x^2 + 6x$

$+8x - 24$

$2x^2 - 16x + 24$

$// \quad -2x \quad //$

$$F'(x) = -\frac{x}{\sqrt{(x-2)(6-x)}\,(x-3)^2}$$

$\boxed{x = 0 \text{ FORA DO DOMÍNIO}}$

6) TANGENTE NO PONTO DE ABSCISSA $x_0 = 4$

$$y = -\frac{4}{2 \cdot 1^2}(x-4) + \frac{2}{1}$$

$$\boxed{y = -2(x-4) + 2}$$

AGORA TROCAREMOS A FUNÇÃO $f(x)$
DE $(x-2)(6-x)$ A $(x-2)(6-x)$

AGORA O DOMÍNIO SERÁ
NA ZONA COMPLEMENTAR

$+++ \circ_{2} ---- \circ_{6} +++$

$$\frac{\sqrt{(x-2)(x-6)}}{x-1}$$

1) $(2,0)$ $(6,0)$

2) $x=0$ $(0, -2\sqrt{3})$

$\frac{\sqrt{12}}{-1} = -2\sqrt{3}$

3) ASSÍNTOTAS HORIZONTAIS

$$\lim_{x\to\pm\infty} \frac{\sqrt{x^2-8x+12}}{x-1}$$

$x=10$ $\frac{\sqrt{100-80+12}}{10-1} = \frac{\sqrt{32}}{9} < 1$

$\boxed{(+\infty, 1^-)}$

$x=-10$ $\frac{\sqrt{100+80+12}}{-10-1} = -\frac{\sqrt{192}}{11}$ >1

$\boxed{(-\infty, -1^-)}$

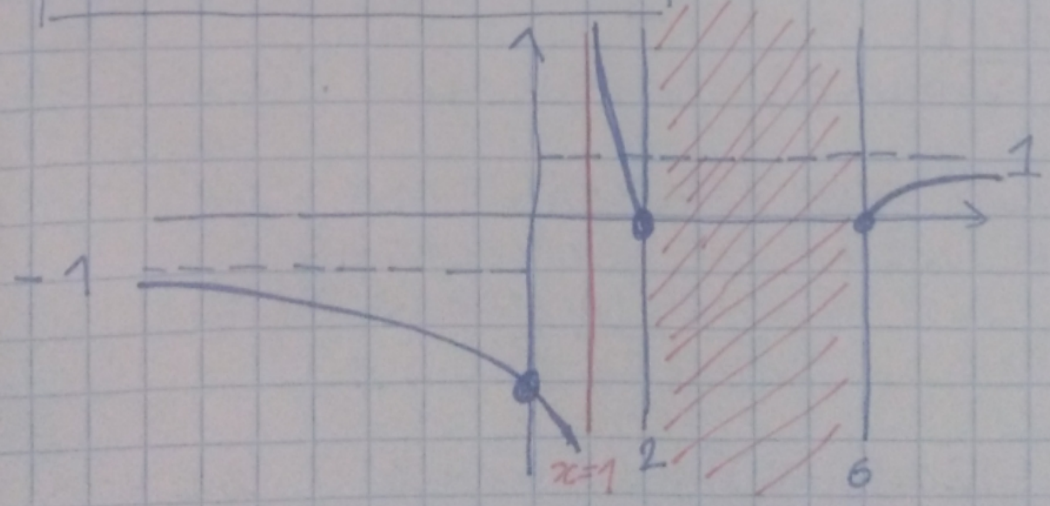
4) ASSÍNTOTAS VERTICAIS

$$\frac{\sqrt{(x-2)(x-6)}}{x-1}$$

1^- (0.9) $\frac{+}{-} = -$

1^+ (1.1) $\frac{+}{+} = +$

$\boxed{(1^-, -\infty) \ (1^+, +\infty)}$



$$\frac{\sqrt{(x-2)(x-6)}}{x-3}$$

1) $(2,0)$ $(6,0)$

2) $x=0$ $\left(0, -\frac{2}{3}\sqrt{3}\right)$

$$\frac{\sqrt{12}}{-3} = -\frac{2}{3}\sqrt{3}$$

4) ASSINTOTAS VERTICAIS FORA DO DOMÍNIO

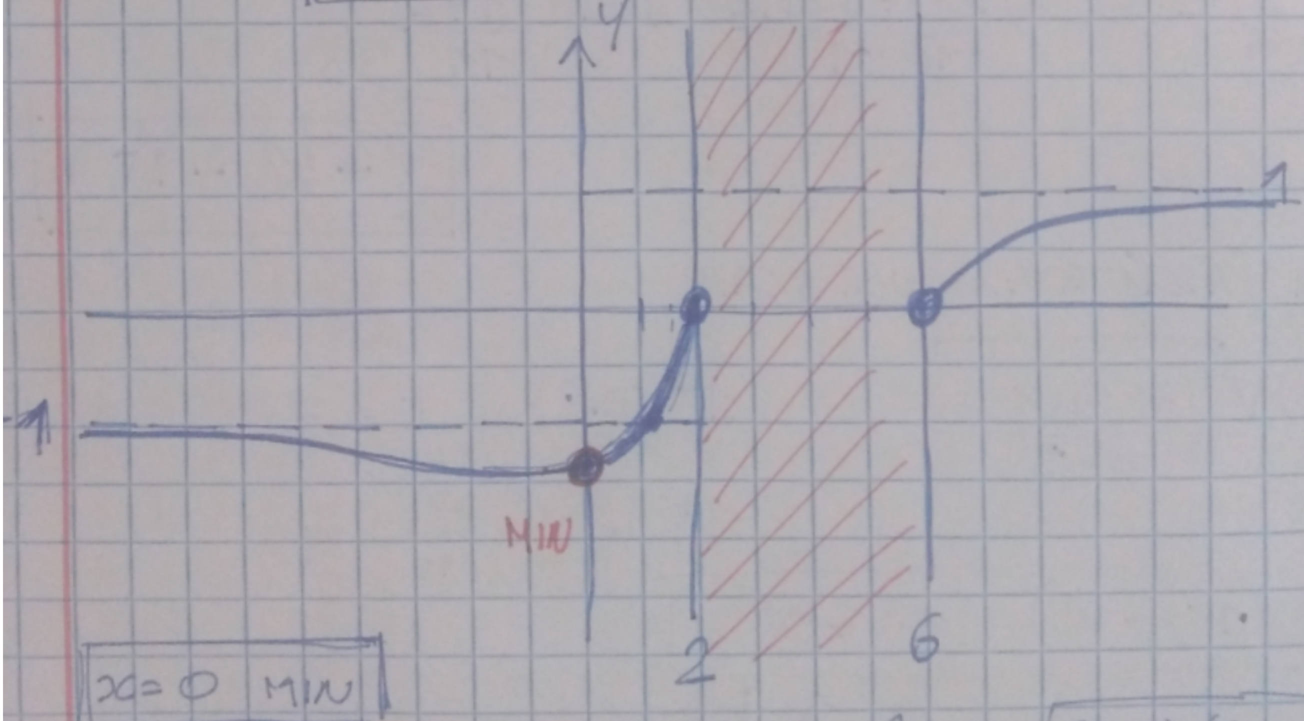
3) ASSÍNTOTAS HORIZONTAIS $x \to \pm\infty$

$$\frac{\sqrt{x^2-8x+12}}{x-3} \qquad x=10 \quad \frac{\sqrt{100-80+12}}{10-3} = \frac{\sqrt{32}}{7} < 1$$

$$\boxed{(+\infty, 1^-)}$$

$$x=-10 \quad \frac{\sqrt{100+80+12}}{-10-3} = -\frac{\sqrt{192}}{13} > 1$$

$$\boxed{(-\infty, -1^+)}$$



$\boxed{x=0 \text{ MIN}}$

VER DERIVADA DA FUNÇÃO $\frac{\sqrt{(x-2)(6-x)}}{x-3}$

LEMBRAMOS QUE A FUNÇÃO PODE CRUZAR ASSÍNTOTAS HORIZONTAIS NESTE CASO PODEMOS ACHAR O PONTO RESOLVENDO $\frac{\sqrt{(x-2)(x-6)}}{x-3} = -1$

$x^2-8x+12 = (3-x)^2 = x^2-6x+9 \qquad 2x=3$

$$\boxed{x = 3/2}$$