Tópico 3

Relative Risk (RR)
Odds Ratio (OR)
Hazard Ratio (HR)

Grupo A, Grupo B, Evento Sim, Evento Não

tempo 0 Grupo A = 200 TODOS EVENTO NÃO
Grupo B = 300

AGORA PREPARAMOS UMA TABELA

tempo 1

	S	N	T	OR	RR
A	10	190	200	$\frac{10/190}{10/290} = 1.53$	$\frac{10/200}{10/300} = 1.50$
B	10	290	300		

2

	S	N	T	OR	RR
A	12	178	190	$\frac{12/178}{14/276} = 1.33$	$\frac{12/190}{14/290} = 1.31$
B	14	276	290		

3

	S	N	T	OR	RR
A	10	168	178	$\frac{10/168}{10/266} = 1.58$	$\frac{10/178}{10/276} = 1.55$
B	10	266	276		

4

	S	N	T	OR	RR
A	10	158	168	$\frac{10/158}{12/254} = 1.34$	$\frac{10/168}{12/266} = 1.32$
B	12	254	266		

5

	S	N	T	OR	RR
A	8	150	158	$\frac{8/150}{14/240} = 0.91$	$\frac{8/158}{14/254} = 0.90$
B	14	240	254		

1-5

	S	N	T	OR	RR
A	50	150	200	$\frac{50/150}{60/240} = \frac{4}{3}$	$\frac{50/200}{60/300} = \frac{5}{4} \rightarrow 1.25$
B	60	240	300		

Hazard Ratio $\frac{1}{5}(1.50 + 1.31 + 1.55 + 1.32 + 0.9) \sim 1.32$

	E1	E2	T
G1	a	b	a+d
G2	c	d	c+d

ODDS RATIO $\frac{a/b}{c/d}$

RELATIVE RISK $\frac{a/(a+b)}{c/(c+d)}$

$\boxed{\text{Odds Ratio - Relative Risk - Hazard Ratio}}$

Lançando 5 dados qual a probabilidade de obter 3 números 1 (chamaremos 2,3,4,5,6 de zero)

11100 número de possibilidades $\binom{5}{3} = \frac{5!}{3!\,2!}$

11100 01110
11010 01101
11001 01011
10110 00111
10101 "10" ✓
10011

probabilidade $\binom{5}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 \longrightarrow$ binomial $\binom{n}{k}p^k(1-p)^{n-k}$

PROPRIEDADE DA BINOMIAL $\boxed{\sum_0^n{}_k \binom{n}{k} p^k (1-p)^{n-k} = 1}$

indução $n=0$ $\binom{0}{0}p^0(1-p)^0 = 1$ ✓

verdadeiro para n usaremos $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

$$\frac{(n+1)!}{(n+1-k)!\,k!} = \frac{n!}{k!(n-k)!} + \frac{n!}{(n-k+1)!(k-1)!}$$

$$= \left(\underbrace{\frac{n-k+1}{n+1} + \frac{k}{n+1}}_{1\ ✓}\right)\binom{n+1}{k}$$

$$\sum_0^{n+1}{}_k \binom{n+1}{k} p^k (1-p)^{n+1-k}$$

$$= \sum_0^{n+1}{}_k \binom{n}{k} p^k (1-p)^{n+1-k} + \sum_0^{n+1}{}_k \binom{n}{k-1} p^k (1-p)^{n+1-k}$$

$$= \underbrace{\sum_0^{n}{}_k \binom{n}{k} p^k (1-p)^{n-k+1}}_{1-p} + \underbrace{\sum_0^{n}{}_r \binom{n}{r} p^{r+1} (1-p)^{n-r}}_{p} = 1$$

$$\boxed{E(X) = \sum_0^n{}_k k\binom{n}{k} p^k (1-p)^{n-k} = np}$$

$$k\binom{n}{k} = n\binom{n-1}{k-1} = \frac{n!}{(k-1)!\,(n-k)!} = k\binom{n}{k} ✓$$

$$E(X) = \sum_0^n{}_k n\binom{n-1}{k-1} p^k (1-p)^{n-k} = n\sum_0^{n-1}{}_r \binom{n-1}{r} p^{r+1} (1-p)^{n-r-1} = np$$

$$\boxed{E(X^2) = \sum_0^n{}_k k^2\binom{n}{k} p^k (1-p)^{n-k} = n^2p^2 - np^2 + np}$$

$$k(k-1)\binom{n}{k} = n(n-1)\binom{n-2}{k-2} = \frac{n!}{(k-2)!\,(n-k)!} = k(k-1)\binom{n}{k} ✓$$

$$\sum_0^n k\,[k(k-1)+k]\binom{n}{k}p^k(1-p)^{n-k} = n(n-1)\sum_0^n k\binom{n-2}{k-2}p^k(1-p)^{n-k} + np$$

$$= n(n-1)\underbrace{\sum_0^{n-2} r\binom{n-2}{r}p^{r+2}(1-p)^{n-r-2}}_{p^2} + np \quad \checkmark$$

$$SE(X) = \sqrt{E(X^2)-[E(X)]^2} = \sqrt{\cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}} = \sqrt{np(1-p)}$$

$$SE(X/n) = \sqrt{\frac{E(X^2)-[E(X)]^2}{n^2}} = \sqrt{\frac{p(1-p)}{n}}$$

Lançando 120 moedas valor esperado de caras seria $np = 60$

120 dados valor esperado de "1" seria $np = 20$

qual o intervalo de confiânsa 95% $\pm 1.96\, SE(X) \sim \pm 2\, SE(X)$

$60 \pm 2\sqrt{120\,\frac{1}{2}\,\frac{1}{2}} \sim 60 \pm 11 \quad (49, 71)$ 95% das vezes

$20 \pm 2\sqrt{120\,\frac{1}{6}\,\frac{5}{6}} \sim 20 \pm 8 \quad (12, 28)$ 95% das vezes

Falando de percentual?

$\frac{1}{2} \pm 2\sqrt{\frac{1}{4}\,\frac{1}{120}} \sim 50\% \pm 9\% \quad (41\%, 59\%)$

$\frac{1}{6} \pm 2\sqrt{\frac{5}{36}\,\frac{1}{120}} \sim 16.7\% \pm 6.8\% \quad (9.9\%, 23.5\%)$

$$Var(\theta) = \left(\frac{\partial\theta}{\partial a}\right)^2 Var(a) + \left(\frac{\partial\theta}{\partial b}\right)^2 Var(b) + \left(\frac{\partial\theta}{\partial c}\right)^2 Var(c) + \left(\frac{\partial\theta}{\partial d}\right)^2 Var(d)$$

$$SE(\theta) = \sqrt{Var(\theta)}$$

$$\theta = \ln OR = \ln\frac{a/b}{c/d} = \ln a - \ln b + \ln c - \ln d$$

$$Var(\theta) = \left(\frac{1}{a}\right)^2 a + \left(-\frac{1}{b}\right)^2 b + \left(\frac{1}{c}\right)^2 c + \left(-\frac{1}{d}\right)^2 d$$

$$\ln OR \pm 1.96\sqrt{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{b}} \qquad \left[OR\, e^{-1.96\sqrt{\dots}},\ OR\, e^{1.96\sqrt{\dots}}\right]$$

$$\theta = \ln RR = \ln \frac{a(c+d)}{c(a+b)} = \ln a - \ln(a+b) + \ln c - \ln(c+d)$$

$$\operatorname{Var}\theta = \left(\frac{1}{a} - \frac{1}{a+b}\right)^2 a + \left(-\frac{1}{a+b}\right)^2 b + \left(\frac{1}{c} - \frac{1}{c+d}\right)^2 c + \left(-\frac{1}{c+d}\right)^2 d$$

$$= \frac{b^2}{a(a+b)^2} + \frac{b}{(a+b)^2} + \frac{d^2}{c(c+d)^2} + \frac{d}{(c+d)^2}$$

$$= \frac{b}{a(a+b)} + \frac{d}{c(c+d)} \qquad SE(\theta) = \sqrt{\frac{b}{a(a+b)} + \frac{d}{c(c+d)}}$$

$$\left[RR\, e^{-1.96\sqrt{\dots}},\ RR\, e^{1.96\sqrt{\dots}}\right]$$

Analizaremos dados reais de vacinas!