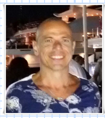

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$$\dot{V}(t) = M\,V(t) \qquad V(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

$$V(t) = e^{Mt}\,V(0) \qquad V(0) = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$M = \frac{1}{2}\begin{pmatrix} 5 & -2 & -1 \\ -1 & 4 & -1 \\ 1 & -2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\frac{1}{2}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$V(t) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} e^{t} & 0 & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{3t} \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{pmatrix} e^{t} & e^{2t} & e^{3t} \\ e^{t} & e^{2t} & -e^{3t} \\ e^{t} & -e^{2t} & e^{3t} \end{pmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} e^{2t} + e^{3t} \\ e^{2t} - e^{3t} \\ e^{3t} - e^{2t} \end{bmatrix}$$

$$M = \frac{1}{2}\begin{pmatrix} 1 & -2 & -1 \\ -1 & 0 & -1 \\ 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -1+2i & -1-2i & 1 \end{pmatrix}\begin{pmatrix} -i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & 1 \end{pmatrix}\frac{1}{4}\begin{pmatrix} 1 & 1-i & -i \\ 1 & 1+i & i \\ 2 & -2 & 0 \end{pmatrix}$$

$$V(t) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ -1+2i & -1-2i & 1 \end{pmatrix}\begin{pmatrix} e^{-it} & 0 & 0 \\ 0 & e^{it} & 0 \\ 0 & 0 & e^{t} \end{pmatrix}\frac{1}{4}\begin{pmatrix} 1 & 1-i & -i \\ 1 & 1+i & i \\ 2 & -2 & 0 \end{pmatrix}\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{pmatrix} e^{t}+\cos t & -e^{t}+\cos t-\sin t & -\sin t \\ -e^{t}+\cos t & e^{t}+\cos t-\sin t & -\sin t \\ e^{t}-\cos t+3\sin t & -e^{t}+\cos t+3\sin t & 2\cos t+\sin t \end{pmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} e^{t}+\cos t \\ -e^{t}+\cos t \\ e^{t}-\cos t+2\sin t \end{bmatrix}$$

$$M = \frac{1}{2}\begin{pmatrix} 5 & -2 & -1 \\ -1 & 4 & -1 \\ 1 & -2 & 3 \end{pmatrix}$$

$$x(t) = e^{2t} + e^{3t}$$
$$y(t) = e^{2t} - e^{3t}$$
$$z(t) = e^{3t} - e^{2t}$$

$$M = \frac{1}{2}\begin{pmatrix} 1 & -2 & -1 \\ -1 & 0 & -1 \\ 3 & 2 & 1 \end{pmatrix}$$

$$x(t) = e^{t} + \cos t$$
$$y(t) = -e^{t} + \cos t$$
$$z(t) = e^{t} - \cos t + 2\sin t$$

$$M = \frac{1}{2}\begin{pmatrix} 6 & -1 & -1 \\ 0 & 5 & -1 \\ 2 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$\exp\left[\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} t\right] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} t + \frac{1}{2!}\begin{pmatrix} \lambda^2 & 2\lambda \\ 0 & \lambda^2 \end{pmatrix} t^2 + \dots = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} e^{\lambda t}$$

$$V(t) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} e^{2t} & te^{2t} & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{3t} \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{pmatrix} e^{2t} & (1+t)e^{2t} & e^{3t} \\ e^{2t} & (1+t)e^{2t} & -e^{3t} \\ e^{2t} & (t-1)e^{2t} & e^{3t} \end{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} (1+t)e^{2t} + e^{3t} \\ (1+t)e^{2t} - e^{3t} \\ (t-1)e^{2t} + e^{3t} \end{bmatrix}$$

$$M = \frac{1}{2}\begin{pmatrix} 7 & -2 & -1 \\ 3 & 2 & -1 \\ 1 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$\exp\left[\begin{pmatrix} \lambda & 1 & 1 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix} t\right] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} \lambda & 1 & 1 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix} t + \begin{pmatrix} \lambda^2 & 2\lambda & 1+2\lambda \\ 0 & \lambda^2 & 2\lambda \\ 0 & 0 & \lambda^2 \end{pmatrix} \frac{t^2}{2!} + \begin{pmatrix} \lambda^3 & 3\lambda^2 & 3\lambda+3\lambda^2 \\ 0 & \lambda^3 & 3\lambda^2 \\ 0 & 0 & \lambda^3 \end{pmatrix} \frac{t^3}{3!} + \dots$$

$$1 + \lambda t + \lambda^2 t^2/2! + \lambda^3 t^3/3! + \dots = e^{\lambda t}$$

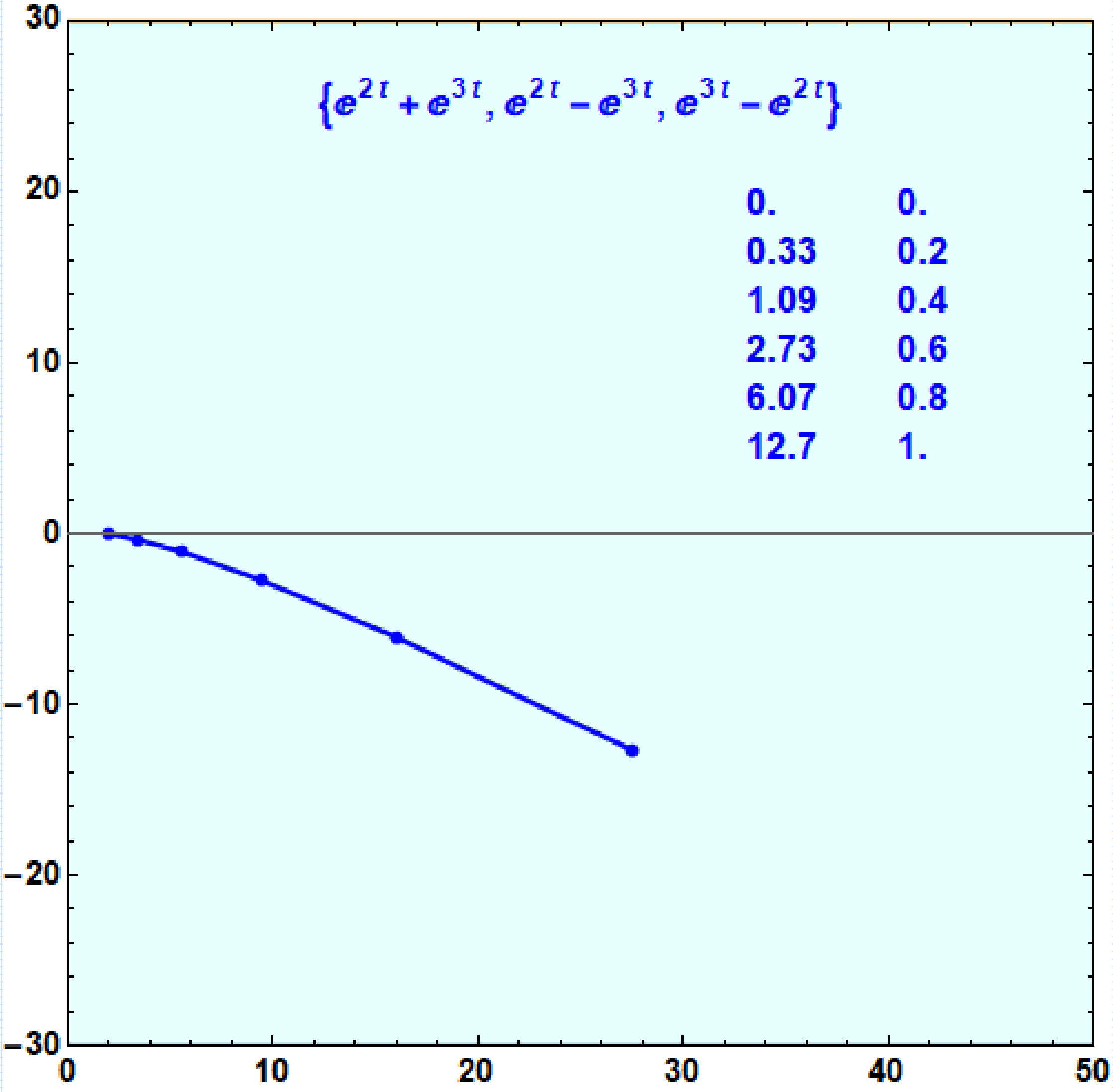
$$0 + t + \lambda t^2 + \lambda^2 t^3/2! + \dots = t e^{\lambda t}$$

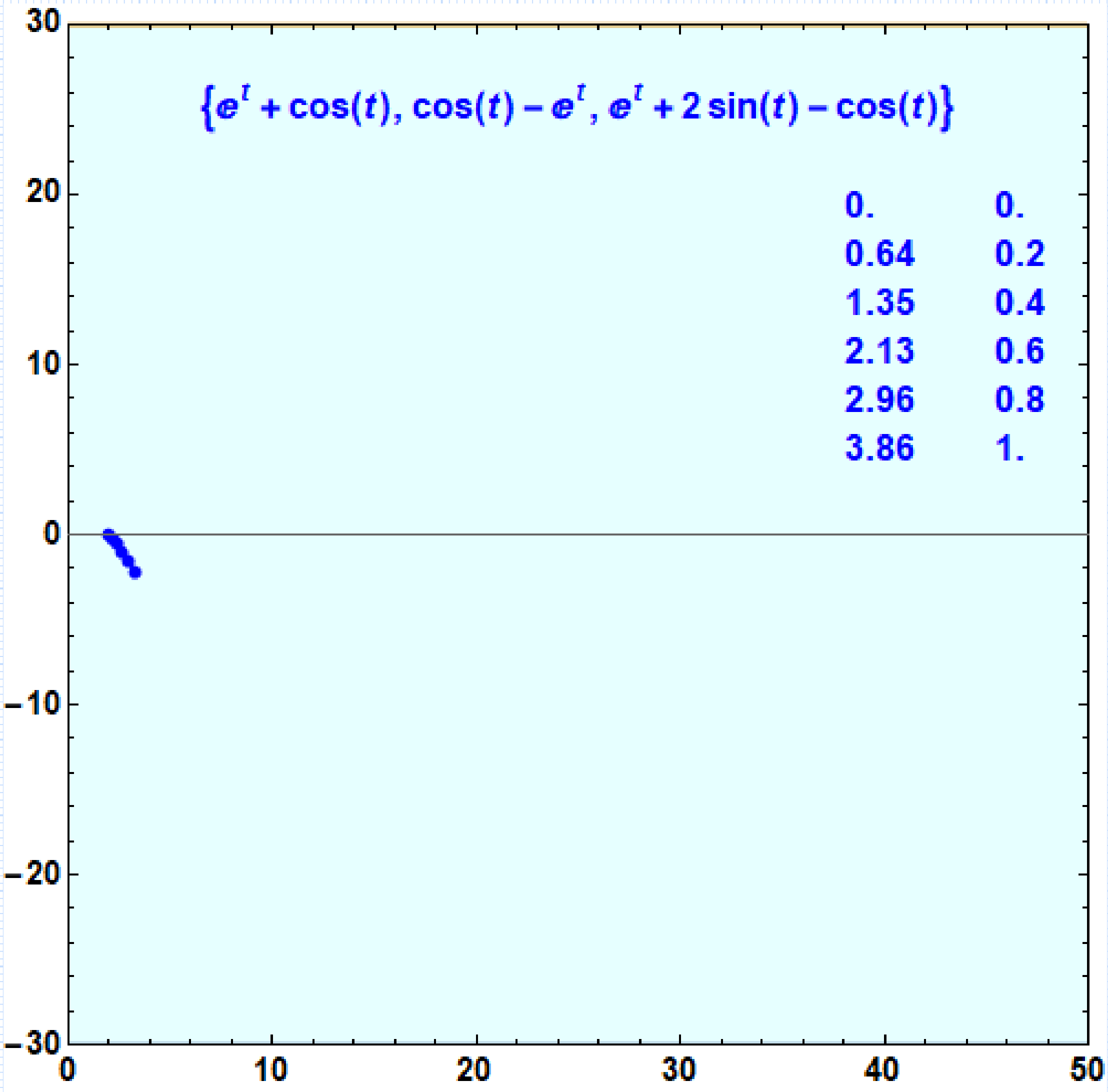
$$t + \frac{t^2}{2!} + \lambda t^2 + \lambda \frac{t^3}{2!} + \lambda^2 \frac{t^3}{2!} + \dots = \left(t + \frac{t^2}{2}\right) e^{\lambda t}$$

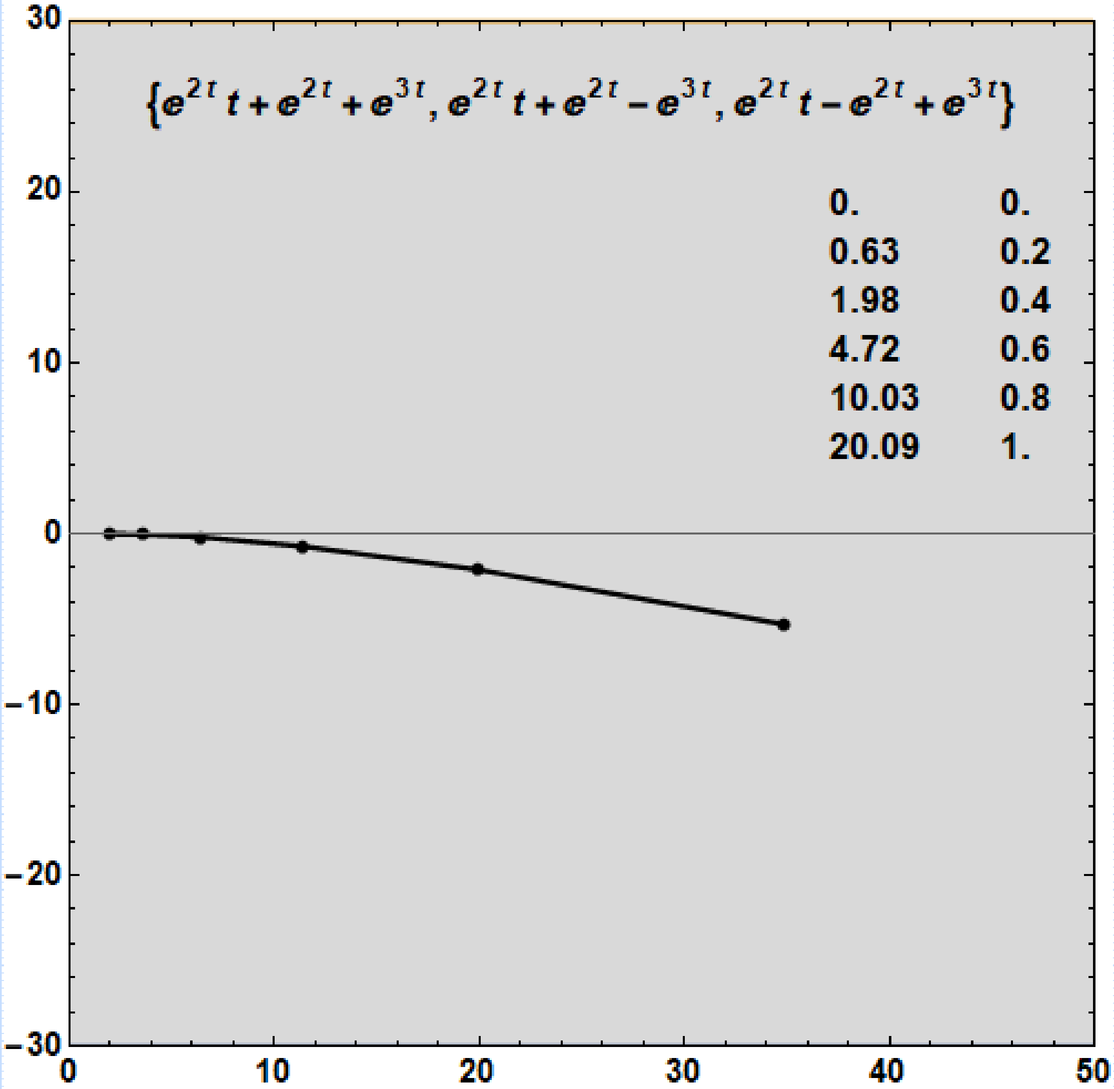
$$V(t) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & t & t+t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{2t}$$

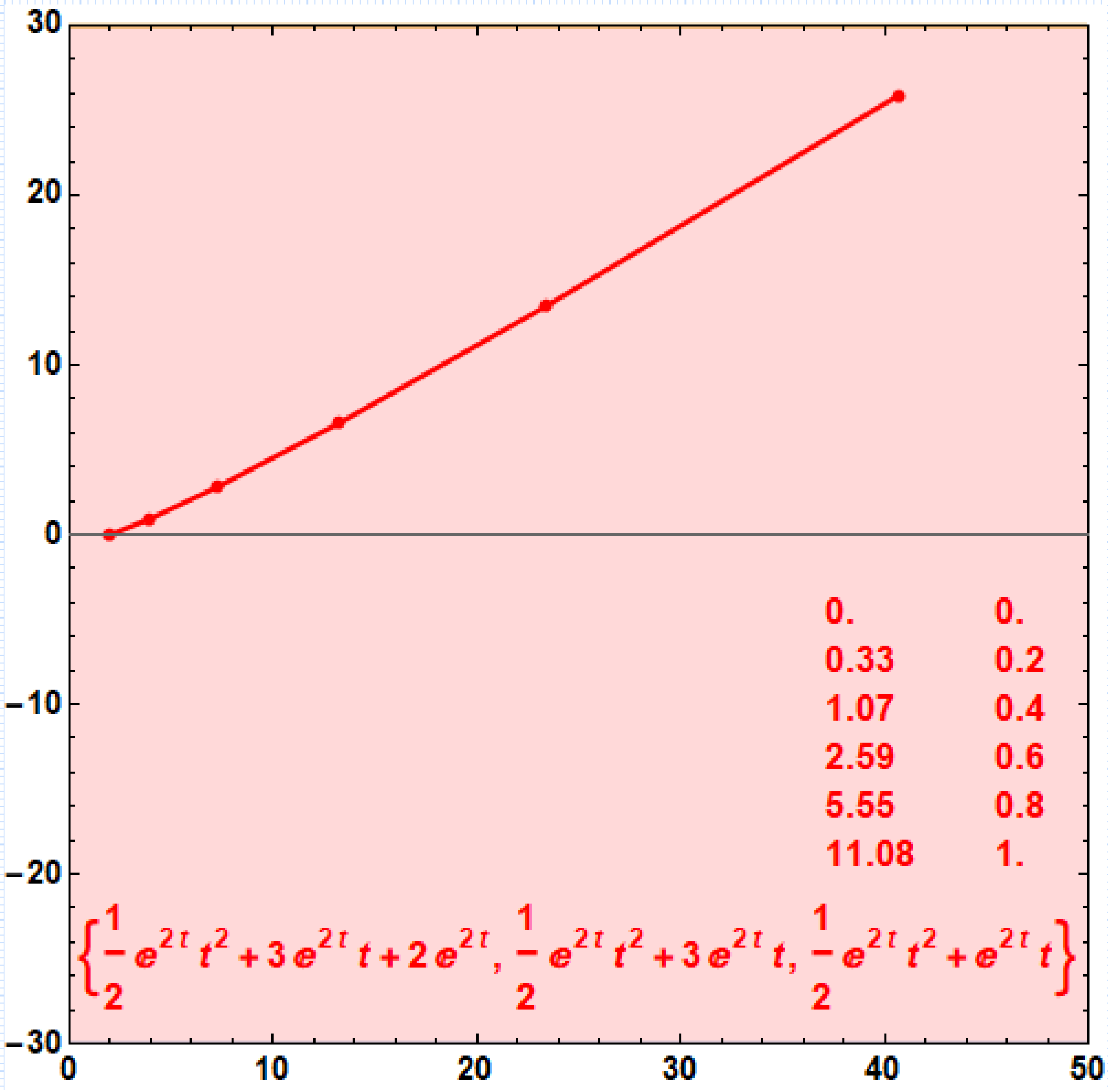
$$= \begin{pmatrix} 1 & 1+t & 1+2t+t^2/2 \\ 1 & 1+t & 2t+t^2/2-1 \\ 1 & t-1 & 1+t^2/2 \end{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = e^{2t}$$

$$= \begin{bmatrix} 2+3t+t^2/2 \\ 3t+t^2/2 \\ t+t^2/2 \end{bmatrix} e^{2t}$$


{e^{2t} + e^{3t}, e^{2t} − e^{3t}, e^{3t} − e^{2t}}
0. 0.
0.33 0.2
1.09 0.4
2.73 0.6
6.07 0.8
12.7 1.
30
20
10
0
−10
−20
−30
0
10
20
30
40
50


{e^t + cos(t), cos(t) − e^t, e^t + 2 sin(t) − cos(t)}
0. 0.
0.64 0.2
1.35 0.4
2.13 0.6
2.96 0.8
3.86 1.
30
20
10
0
−10
−20
−30
0
10
20
30
40
50


$\{e^{2t} t + e^{2t} + e^{3t}, e^{2t} t + e^{2t} - e^{3t}, e^{2t} t - e^{2t} + e^{3t}\}$
0. 0.
0.63 0.2
1.98 0.4
4.72 0.6
10.03 0.8
20.09 1.
30
20
10
0
−10
−20
−30
0
10
20
30
40
50


0. 0.
0.33 0.2
1.07 0.4
2.59 0.6
5.55 0.8
11.08 1.
$\{\frac{1}{2} e^{2t} t^2 + 3 e^{2t} t + 2 e^{2t}, \frac{1}{2} e^{2t} t^2 + 3 e^{2t} t, \frac{1}{2} e^{2t} t^2 + e^{2t} t\}$
30
20
10
0
−10
−20
−30
0
10
20
30
40
50