

Comparação de estimadores, sob a ótica frequentista, no modelo de Bernoulli

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- $X_i | \theta \stackrel{i.i.d.}{\sim} \text{Bernoulli}(\theta), i = 1, 2, \dots, n.$
- Priori: $\theta \sim \text{Beta}(a, b).$
- EMV: $\hat{\theta} = \bar{X}.$
- Posteriori: $\theta | \mathbf{x} \sim \text{Beta}(n\bar{x} + a, n(1 - \bar{x}) + b).$
- Esperança e moda a posterioris:
 - $\hat{\theta}_{EAP} = \frac{n\bar{x} + a}{n + a + b}$
 - $\hat{\theta}_{MAP} = \frac{n\bar{x} + a - 1}{n + a + b - 2}$
- Variância e desvio-padrão à posteriori:
 - $\mathcal{V}(\theta | \mathbf{x}) = \frac{(n\bar{x} + a)(n(1 - \bar{x}) + b)}{(n + a + b)^2(n + a + b + 1)}.$
 - $DP(\theta | \mathbf{x}) = \sqrt{\frac{(n\bar{x} + a)(n(1 - \bar{x}) + b)}{(n + a + b)^2(n + a + b + 1)}}$

Medidas de precisão frequentistas dos estimadores

■ Estimador de MV.

- $\mathcal{E}(\hat{\theta}_{MV}) = \theta.$

- $\mathcal{V}(\hat{\theta}_{MV}) = \frac{\theta(1-\theta)}{n}.$

- $B(\hat{\theta}_{MV}) = 0.$

- $EQM(\hat{\theta}_{MV}) = \frac{\theta(1-\theta)}{n}.$

■ EAP($\hat{\theta}_{EAP}$).

- $\mathcal{E}(\hat{\theta}_{EAP}) = \frac{n\theta+a}{n+a+b}.$

- $\mathcal{V}(\hat{\theta}_{EAP}) = \frac{n\theta(1-\theta)}{(n+a+b)^2}.$

- $B(\hat{\theta}_{EAP}) = \frac{a-\theta(a+b)}{n+a+b}.$

- $EQM(\hat{\theta}_{EAP}) = \frac{n\theta(1-\theta)+[a-\theta(a+b)]^2}{(n+a+b)^2}.$

Medidas de precisão frequentistas dos estimadores (cont.)

■ $EAP(\hat{\theta}_{MAP})$.

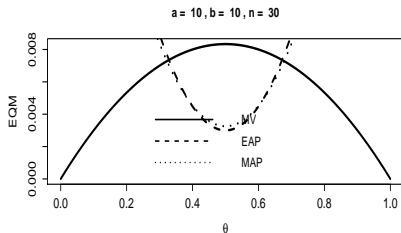
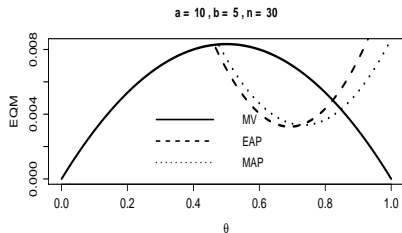
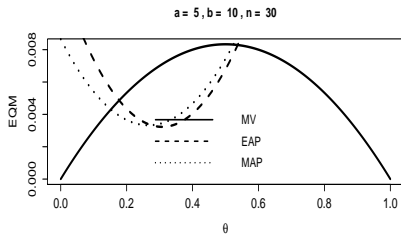
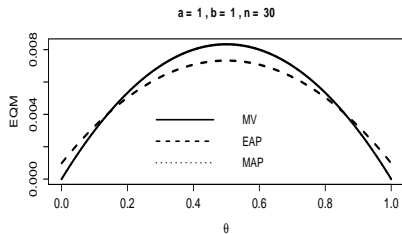
$$\blacksquare \mathcal{E}(\hat{\theta}_{MAP}) = \frac{n\theta + a - 1}{n + a + b - 2}.$$

$$\blacksquare \mathcal{V}(\hat{\theta}_{MAP}) = \frac{n\theta(1-\theta)}{(n+a+b-2)^2}.$$

$$\blacksquare B(\hat{\theta}_{MAP}) = \frac{a-1-\theta(a+b-2)}{n+a+b-2}.$$

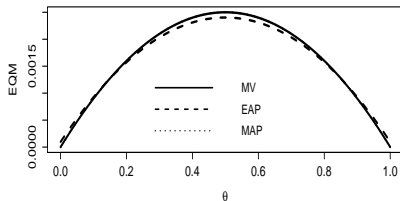
$$\blacksquare EQM(\hat{\theta}_{MAP}) = \frac{n\theta(1-\theta) + [a-1-\theta(a+b-2)]^2}{(n+a+b-2)^2}.$$

Comparação dos EQM's ($n = 30$)

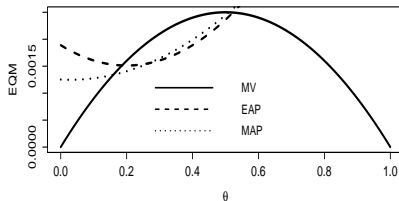


Comparação dos EQM's ($n = 100$)

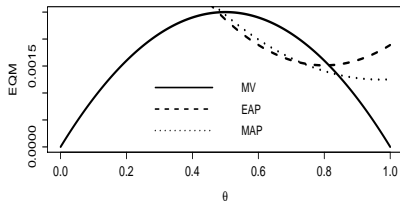
$a = 1, b = 1, n = 100$



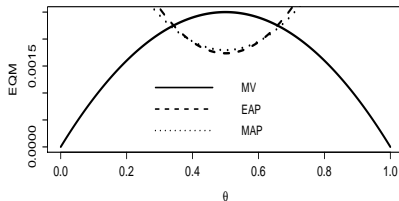
$a = 5, b = 10, n = 100$



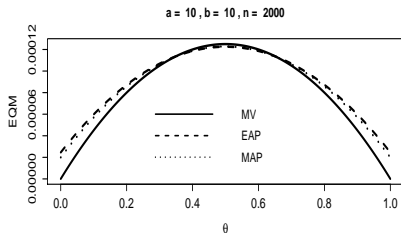
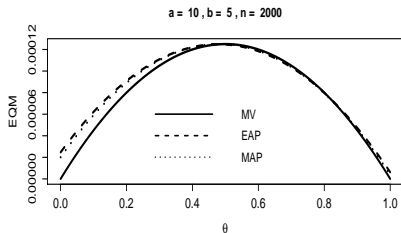
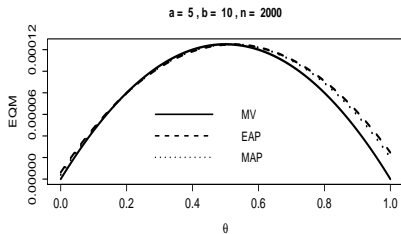
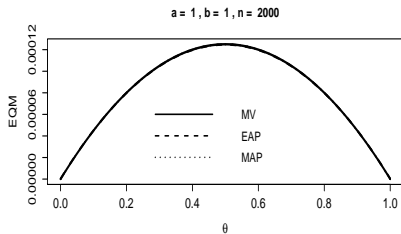
$a = 10, b = 5, n = 100$



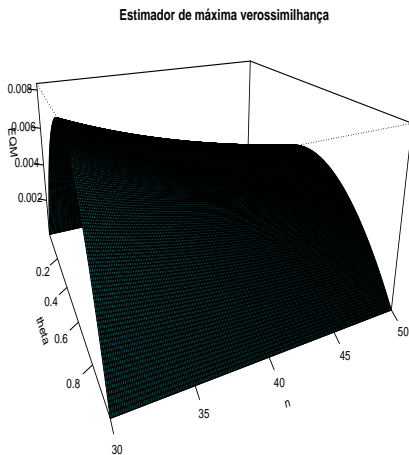
$a = 10, b = 10, n = 100$



Comparação dos EQM's ($n = 2000$)

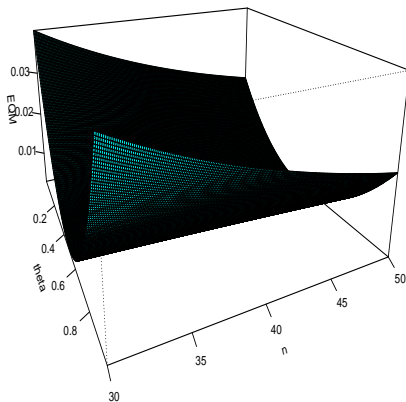


Comportamento do EQM em função de θ e n



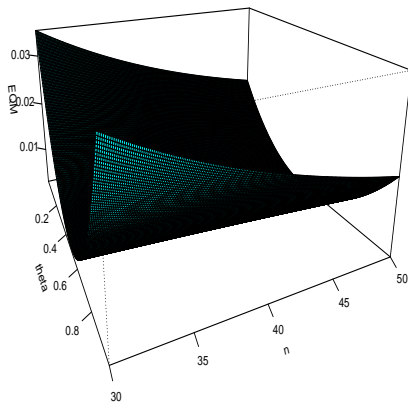
Comportamento do EQM em função de θ e n

Esperança à posteriori, $a = 10$, $b = 10$

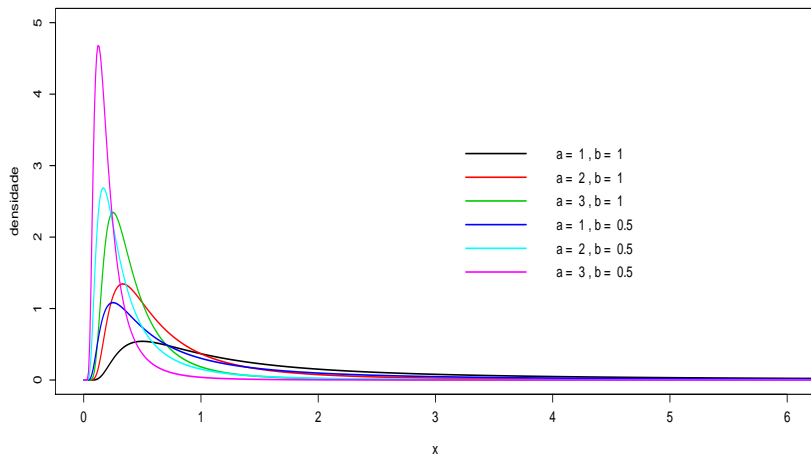


Comportamento do EQM em função de θ e n

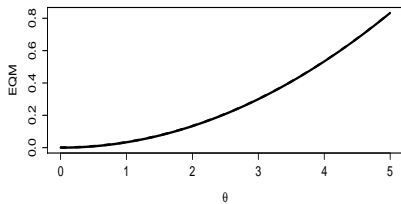
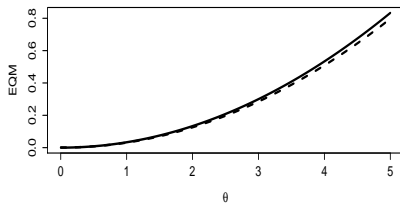
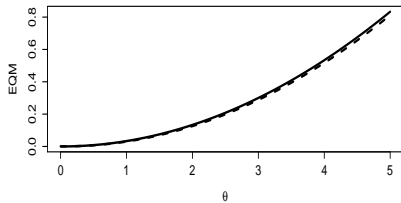
Moda à posteriori, $a = 10$, $b = 10$



densidades da distribuição inversa gama



- Comparação entre os estimador de máxima verossimilhança e o EAP.
- $EQM(\hat{\theta}_{MV}) = \frac{\theta^2}{n}$.
- $EQM(\hat{\theta}_{EAP}) = \frac{n\theta^2 + [b - \theta(a-1)]^2}{(n+a-1)^2}$

$a = 1, b = 1, n = 30$  $a = 2, b = 1, n = 30$  $a = 3, b = 1, n = 30$  $a = 3, b = 0.5, n = 30$ 