Nonsmooth Fuzzy Optimization

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Abstract. This presentation has as main goal to provide the introduction of our study about fuzzy optimization problems whose objective function is a nonsmooth fuzzy-interval-valued function. In this context, necessary and sufficient optimal conditions are the main expected results, and in order to achieve this goal, some necessary results about nonsmooth fuzzy analysis will be provided previously. Moreover, some examples, which show the applicability of the theory developed in this study, will be presented.

Keywords: Fuzzy-interval-valued functions, fuzzy optimization, nonsmooth optimal conditions

1 Extended Abstract

Optimization problems whose objective function is a fuzzy-interval valued function, have been studied under different approaches [1], [4], [5], [9], [10]. During the last years occurred significant advances in obtaining optimal conditions for fuzzy optimization problems, since initially the results were given to objective functions that satisfy the property of being fuzzy convex and differentiable, whose concept of differentiability is very restricted, and in [2] was used pseudo-invexity concepts and $gH-$differentiability, which is a concept more general than the concepts used in the articles cited above, to provide optimal conditions for a larger class of problems than the classes studied in the articles cited above. Seeking to expand the class of optimization problems approached in [2] the following two problems, which are the base of this research, were formulated: **(1)** Given a unrestricted fuzzy optimization problem **(UFO)** $\min_{t\in U} \hat{F}(t)$, where, $U \subset \mathbb{R}^n$ is a nonempty convex set, $\mathcal{F}(\mathbb{R})$ is the space of fuzzy intervals and $\hat{F} : U \longrightarrow \mathcal{F}(\mathbb{R})$ is the objective function that is not $gH-$differentiable. Which assumptions about $\hat{F}$ are required to provide optimal conditions for **(UFO)**?
(2) Given a restricted fuzzy optimization problem

$$\begin{aligned} \textbf{(RFO)} \quad & \min \hat{F}(t) \\ & \text{subject to } g_i(x) \leq 0,\ i = 1, \cdots, m; \\ & t \in U \subset \mathbb{R}^n, \end{aligned}$$

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where $U \subset \mathbb{R}^n$ is a nonempty open set, $\hat{F} : U \longrightarrow \mathbb{R}_{\mathcal{F}}$ is not $gH-$differentiable and g_i, $i = 1, \cdots, m$ are functions whose domain is U. Which assumptions about $\hat{F}$ and about the set of restrictions are required to provide optimal conditions for **(RFO)**?

The main goal of this research is to answer **(1)** and **(2)**. For this end, this research moves towards relating the results founded in articles that associate fuzzy optimization problems with vector optimization problems [8], [10] to results given in nonsmooth vector optimization [3], [6], [7].

It is important emphasize that, even using some fuzzy concept of differentiability, we have not found any results in the literature associated with optimal conditions of second order for fuzzy optimization problems. For this reason, this project is a great first step in that direction.

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