An invitation to Study Fuzzy and Generalized Uncertainty Optimization

A Historical and Contemporary View

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MOTIVATION AND OBJECTIVE

- One of the most significant features of human beings is the decision-making of everyday problems.
- The database of the practical problems, in many cases, have approximate and/or imprecise values.
- The goal of this course is to present a brief description how to use fuzzy set and possibility theories in optimization methods.

OUTLINE

- Fuzzy Mathematical Programming – the beginning
- Parametric Approach – the beginning
- Parametric Approach – the evolution

WHERE EVERYTHING BEGAN

INFORMATION AND CONTROL 8, 338–353 (1965)

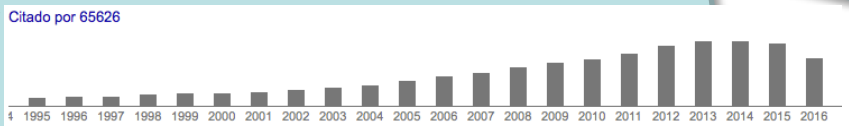
Fuzzy Sets*

L. A. ZADEH

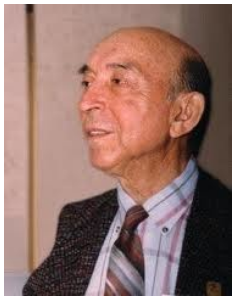
Department of Electrical Engineering and Electronics Research Laboratory, University of California, Berkeley, California

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that the fuzzy sets be disjoint.

1965



Fuzzy sets
LA Zadeh - Information and control, 1965
Citado por 65621 - Artigos relacionados - Todas as 29 versões



Lotfi A. Zadeh

"Fuzzy Sets"
***Information and Control*. 8 (3) 338–353. 1965**

FUZZY MATHEMATICAL PROGRAMMING THE BEGINNING

MANAGEMENT SCIENCE
Vol. 17, No. 4, December, 1970
Printed in U.S.A.

DECISION-MAKING IN A FUZZY ENVIRONMENT*†

R. E. BELLMAN‡ AND L. A. ZADEH§

By decision-making in a fuzzy environment is meant a decision process in which the goals and/or the constraints, but not necessarily the system under control, are fuzzy in nature. This means that the goals and/or the constraints constitute classes of alternatives whose boundaries are not sharply defined.

An example of a fuzzy constraint is: "The cost of A should not be *substantially* higher than α," where α is a specified constant. Similarly, an example of a fuzzy goal is: "x should be in the *vicinity* of x_0," where x_0 is a constant. The italicized words are the sources of fuzziness in these examples.

Fuzzy goals and fuzzy constraints can be defined precisely as fuzzy sets in the space of alternatives. A fuzzy decision, then, may be viewed as an intersection of the given goals and constraints. A maximizing decision is defined as a point in the space of alternatives at which the membership function of a fuzzy decision attains its maximum value.

The use of these concepts is illustrated by examples involving multistage decision processes in which the system under control is either deterministic or stochastic. By using dynamic programming, the determination of a maximizing decision is reduced to the solution of a system of functional equations. A reverse-flow technique is described for the solution of a functional equation arising in connection with a decision process in which the termination time is defined implicitly by the condition that the process stops when the system under control enters a specified set of states in its state space.

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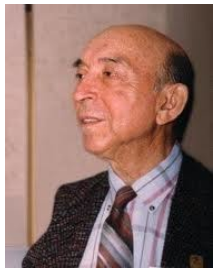
Citado por 7618

Decision-making in a fuzzy environment
RE Bellman, LA Zadeh - Management science, 1970
Citado por 7618 - Artigos relacionados - Todas as 14 versões

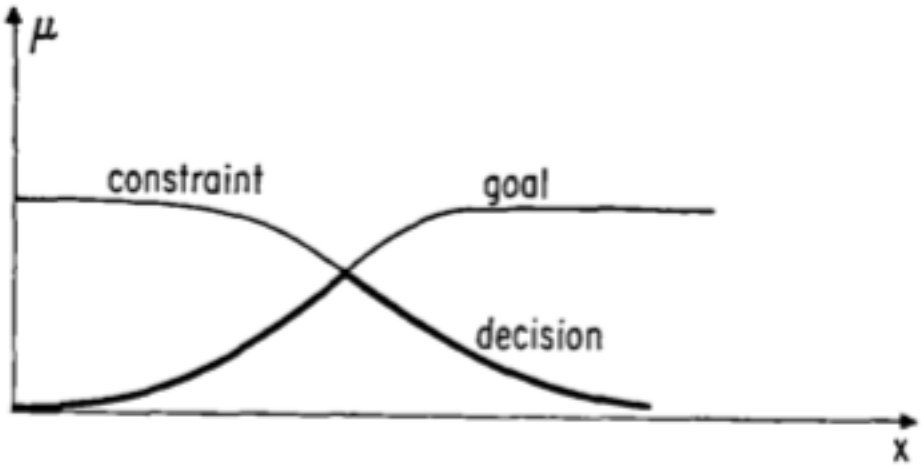


Richard E. Bellman
Lotfi A. Zadeh

"Decision-Making in a Fuzzy Environment"
Management Science. Vol. 17, No. 4,
Application Series (Dec., 1970), pp. B141-B164

DECISION-MAKING IN AN FUZZY ENVIRONMENT

- **Definition**: Assume that we are given a fuzzy goal G and a fuzzy constraint C in a space of alternatives X. Then, G and C combine to form a decision, D, which is a fuzzy set resulting from intersection of G and C.



DECISION-MAKING IN AN FUZZY ENVIRONMENT

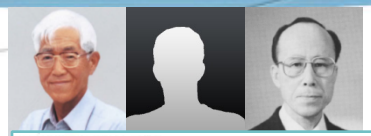
- Suppose that we have n objective functions G_1, …, G_n, and m constraint C_1, …, C_m.
- Then, the resultant decision is the intersection of the given goals, G_1, …, G_n, and the given constraints, C_1, …, C_m. That is,

$$D = G_1 \cap G_2 \cap \cdots \cap G_n \cap C_1 \cap C_2 \cap \cdots \cap C_m$$

- and correspondingly

$$\mu_D = \mu_{G_1} \wedge \mu_{G_2} \wedge \cdots \wedge \mu_{G_n} \wedge \mu_{C_1} \wedge \mu_{C_2} \wedge \cdots \wedge \mu_{C_m}$$

CHRONOLOGY OF METHODS TO FUZZY MATHEMATICAL PROGRAMMING



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Journal of Cybernetics, 1974, 3, 4, pp. 37-46

On Fuzzy-Mathematical Programming

Hideo Tanaka,† Tetsuji Okuda, Kiyoji Asai
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1. Introduction

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Int. J. General Systems
1976, Vol. 2, pp. 209–215

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Printed in Great Britain

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(*Received January 21, 1975; in final form May 29, 1975*)

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INDEX TERMS

Kybernetes
1977, Vol. 6, pp. 197–201

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Printed in Great Britain

ON PROGRAMMING WITH FUZZY CONSTRAINT SETS

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Operations Research Laboratory, Computing Center of the Academy of Sciences of the USSR, Vavilova 40, Moscow 117333 (USSR)

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1965 → 1970 → … → 1974 → 1976 → 1977

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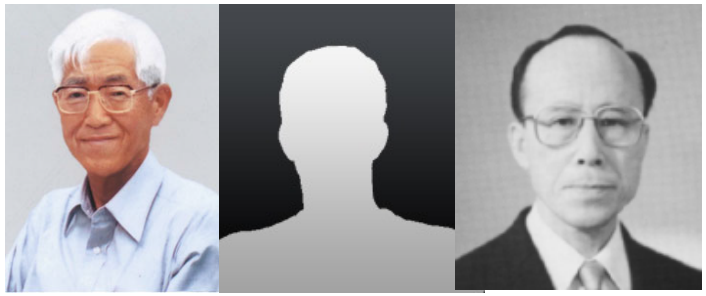
On Fuzzy-Mathematical Programming
www.tandfonline.com/doi/pdf/10.1080/01969727308545912 ▾ Traduzir esta página
de H Tanaka† - 1973 - Citado por 582 - Artigos relacionados
On Fuzzy-Mathematical Programming. Hideo Tanaka,1 Tetsuji Okuda, Kiyoji Asai . Department of Electrical Engineering and Computer Sciences and the ...



Hideo Tanaka
Tetsuji Okuda
Kiyoji Asai

"On Fuzzy Mathematical Programming"
Journal of Cybernetics
1974, 3, 4, pp. 37-46

ON FUZZY-MATHEMATICAL PROGRAMMING

- **Definition 1**: For a in [0,1], an a level set of a constraint C is denoted by C_a and is a nonfuzzy set in x defined by

$$C_\alpha = \{x | \mu_C(x) \geqslant \alpha\}$$

- **Proposition 1**: $\sup_x \mu_D(x) = \sup_\alpha [\alpha \wedge \max_{C_\alpha} \mu_G(x)]$

- **Proposition 2**: $\sup_{\alpha_1 \cdots \alpha_n} \alpha_1 \wedge \cdots \wedge \alpha_n \wedge \max_{C_{\alpha_1}{}^1 \circ \cdots \circ C_{\alpha_n}{}^n} \mu_G(x)$

$$= \sup_\alpha \alpha \wedge \max_{C_\alpha{}^1 \circ \cdots \circ C_\alpha{}^n} \mu_G(x)$$

ON FUZZY-MATHEMATICAL PROGRAMMING

- **Theorem 1**: $\sup_{\alpha} [\alpha \wedge \max_{C_\alpha} f(x)] = \max_{T} f(x)$

  where $T = \{x | f(x) - \mu_C(x) = 0\}$.

- **Theorem 2**: $\inf_{\alpha} [\alpha \vee \max_{C_\alpha} f(x)] = \sup_{\alpha} [\alpha \wedge \max_{C_\alpha} f(x)]$

- **Theorem 3**: If max f(x) is a-continuous and f(x) is strongly fuzy convex, then a^* is unique.

- **Theorem 4**: If the membership function of the set of constraints is strongly fuzzy convex, max f(x) is a-continuous.

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INDEX TERMS Fuzzy set, fuzzy systems, decision making, fuzzy optimal decisions, fuzzy linear programming, fuzzy solutions.

Hans J. Zimmermann

DESCRIPTION AND OPTIMIZATION OF FUZZY SYSTEMS
www.tandfonline.com/doi/pdf/10.1080/03081077608547470 - Traduzir esta página
de HJ ZIMMERMANN - 1976 - Citado por 1012 - Artigos relacionados
After a brief **description** of the basic theory of **fuzzy** sets, implications to **systems** theory and decision making are indicated. **Fuzzy** set theory is then applied to.

"Description and optimization of fuzzy systems"
International Journal of General Systems
1976, vol 2, pp. 209-215

DESCRIPTION AND OPTIMIZATION OF FUZZY SYSTEMS

- Let be a standard linear programming problem with fuzzy order relation;
- By using the conceptions describe in Bellman & Zadeh, the objective function is transformed into a constraint;
- Each constraint has a maximal allowance tolerance and it is transformed into a classical constraint function.

DESCRIPTION AND OPTIMIZATION OF FUZZY SYSTEMS

- By using the maximal allowance tolerance, we can define a membership function for each constraint;
- After, we can do an intersection of all this membership functions and we define the membership function to set of constraints;
- Finally, we optimize the level cut *a* in order to maximize it.

FUZZY MATHEMATICAL PROGRAMMING THE BEGINNING

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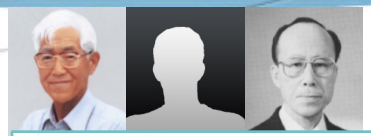
ON PROGRAMMING WITH FUZZY CONSTRAINT SETS: Kybernetes ...
www.emeraldinsight.com/doi/pdf/10.1108/eb005453 - Traduzir esta página
de SA ORLOVSKY - 1977 - Citado por 87 - Artigos relacionados
Abstract: Two solution concepts for a FMP problem are suggested. The first one makes use of level **sets** of the **fuzzy set** of feasible alternatives. The second ...

S. A. Orlovsky

"On programming with fuzzy constraint sets"
Kybernetes
1977, vol 6, pp. 197-201

CHRONOLOGY OF METHODS TO FUZZY MATHEMATICAL PROGRAMMING



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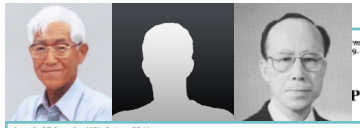
1965 → 1970 → … → 1974 → 1976 → 1977

GOAL TO METHODS TO FUZZY MATHEMATICAL PROGRAMMING

FUZZY PROBLEM

OPTIMAL SOLUTION (classical solution)

CHRONOLOGY OF METHODS TO FUZZY MATHEMATICAL PROGRAMMING
to be continue …



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Fuzzy Information and Decision Processes
M.M. Gupta and E. Sanchez (eds.)
© North-Holland Publishing Company, 1982

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FUZZY MATHEMATICAL PROGRAMMING

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Facultad de Ciencias. Universidad de Granada
Granada. Spain

This paper presents a new method to solve the Fuzzy Mathematical Programming (FMP) problem. The α-cuts of the constraints fuzzy set and the α-cuts of the objective are considered for a FMP problem. One can obtain a mathematical programming problem for each α-cut, these particular problems are then solved and thus the fuzzy family of solutions defines the fuzzy solution to the original FMP problem. It is proved that the solutions calculated by other methods are obtained as particular values of this fuzzy solution. In this way, a new formulation to the sensitivity analysis in the FMP problem is given. Finally, a new approach to the fuzzy objective is made.

Fuzzy Sets and Systems 11 (1983) 243–251
North-Holland

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THE USE OF PARAMETRIC PROGRAMMING IN FUZZY LINEAR PROGRAMMING

rsity, Wrocław, Poland

tification of a complete fuzzy decision (not only the programming by use of the parametric programming n that this fact can be useful in the Zimmermann gramming. The presented remarks are illustrated by

Multiple objective linear programming, Parametric

1974 → 1976 → 1977 → … → 1982 → 1983

FUZZY MATHEMATICAL PROGRAMMING

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Stefan CHANAS
Institute of Management, Technical University, Wrocław, Poland

Received February 1982
Revised March 1983

In this paper the possibility of the identification of a complete fuzzy decision (not only the maximizing alternative) in fuzzy linear programming by use of the parametric programming technique is presented. Also, it is shown that this fact can be useful in the Zimmermann approach to multiple objective linear programming. The presented remarks are illustrated by some numerical examples.

Keywords: Fuzzy linear programming, Multiple objective linear programming, Parametric programming.

The use of parametric programming in fuzzy linear ... - ScienceDirect
www.sciencedirect.com/science/article/pii/S0165011483800839 ▾ Traduzir esta página
de S Chanas - 1983 - Citado por 242 - Artigos relacionados
8 de mar de 2005 - In this paper the possibility of the identification of a complete **fuzzy** decision (not only the maximizing alternative) in **fuzzy linear programming** by ...

Stefan Chanas

The use of parametric programming in fuzzy linear programming"
Fuzzy Sets and Systems
11 (1983) 243–251

FUZZY MATHEMATICAL PROGRAMMING PARAMETRIC APPRAOCH

Fuzzy Information and Decision Processes
M.M. Gupta and E. Sanchez (eds.)
© North-Holland Publishing Company, 1982

FUZZY MATHEMATICAL PROGRAMMING

J. L. Verdegay

Departamento de Estadística Matemática
Facultad de Ciencias. Universidad de Granada
Granada. Spain

This paper presents a new method to solve the Fuzzy Mathematical Programming (FMP) problem. The α-cuts of the constraints fuzzy set and the α - cuts of the objective are considered for a FMP problem. One can obtain a mathematical programming problem for each α-cut, these particular problems are then solved and thus the fuzzy family of solutions defines the fuzzy solution to the original FMP problem. It is proved that the solutions calculated by other methods are obtained as particular values of this fuzzy solution. In this way, a new formulation to the sensitivity analysis in the FMP problem is given. Finally, a new approach to the fuzzy objective is made.

Citado por 250

Fuzzy mathematical programming
JL Verdegay - Fuzzy information and decision processes, 1982
Citado por 250 - Artigos relacionados

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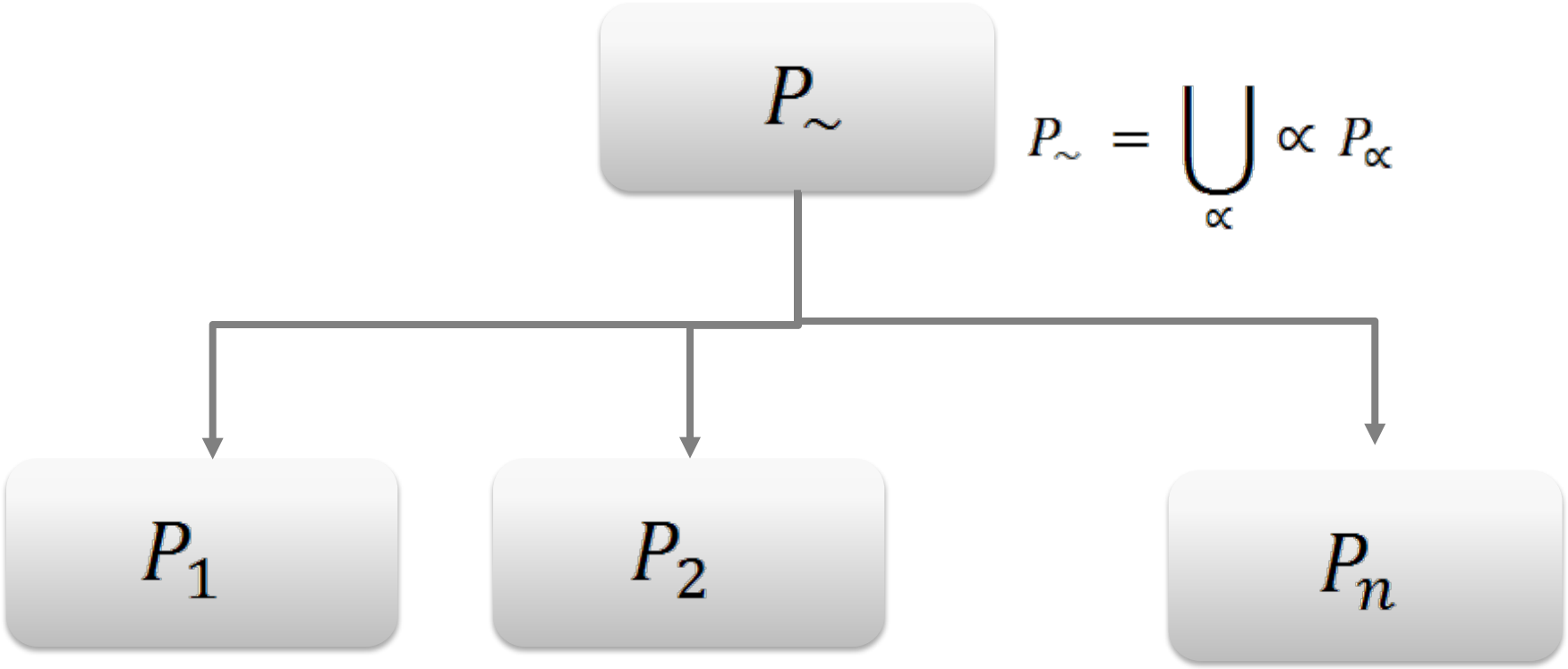
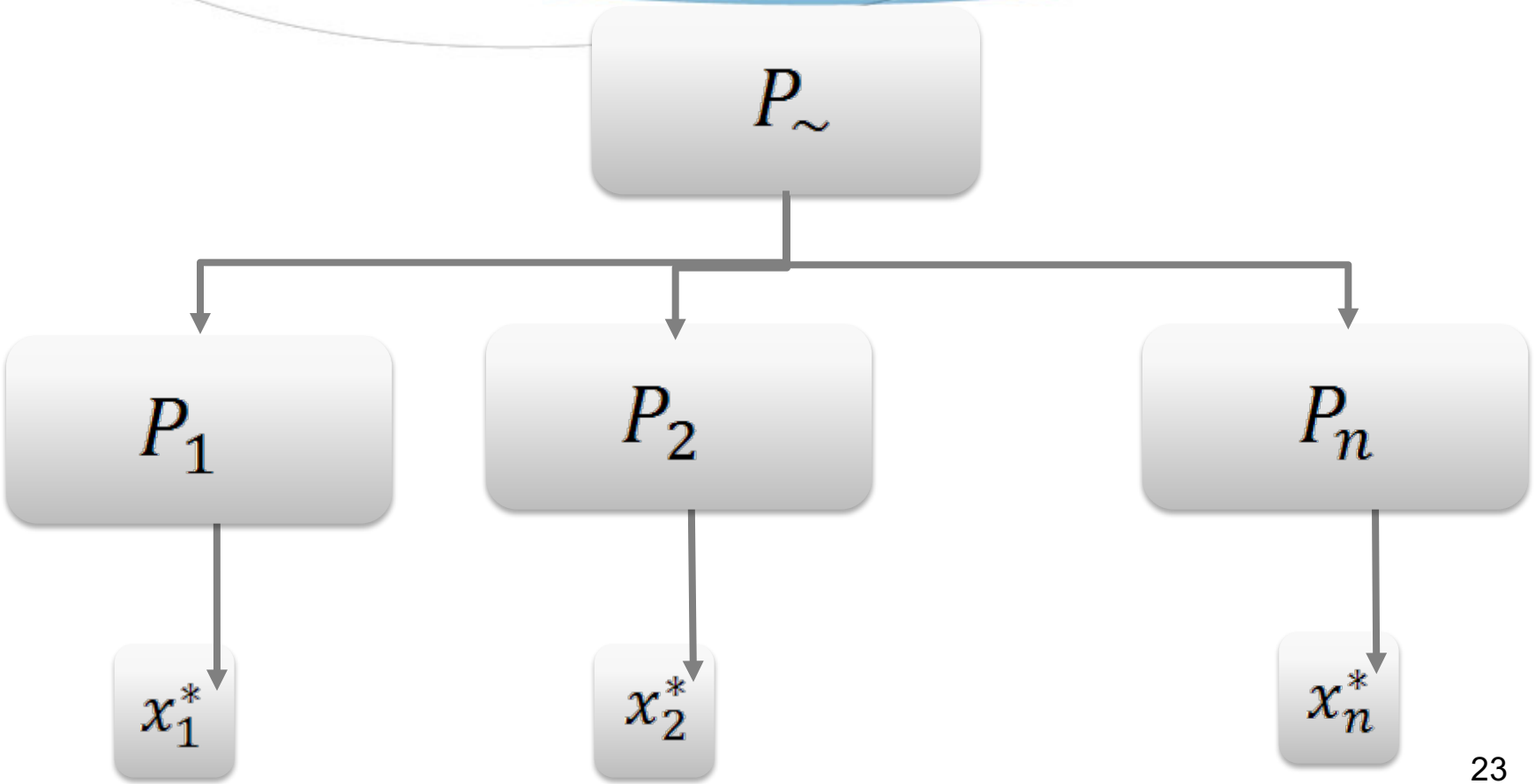
FUZZY PARAMETRIC APPROACH PHASE 1



FUZZY PARAMETRIC APPROACH PHASE 2



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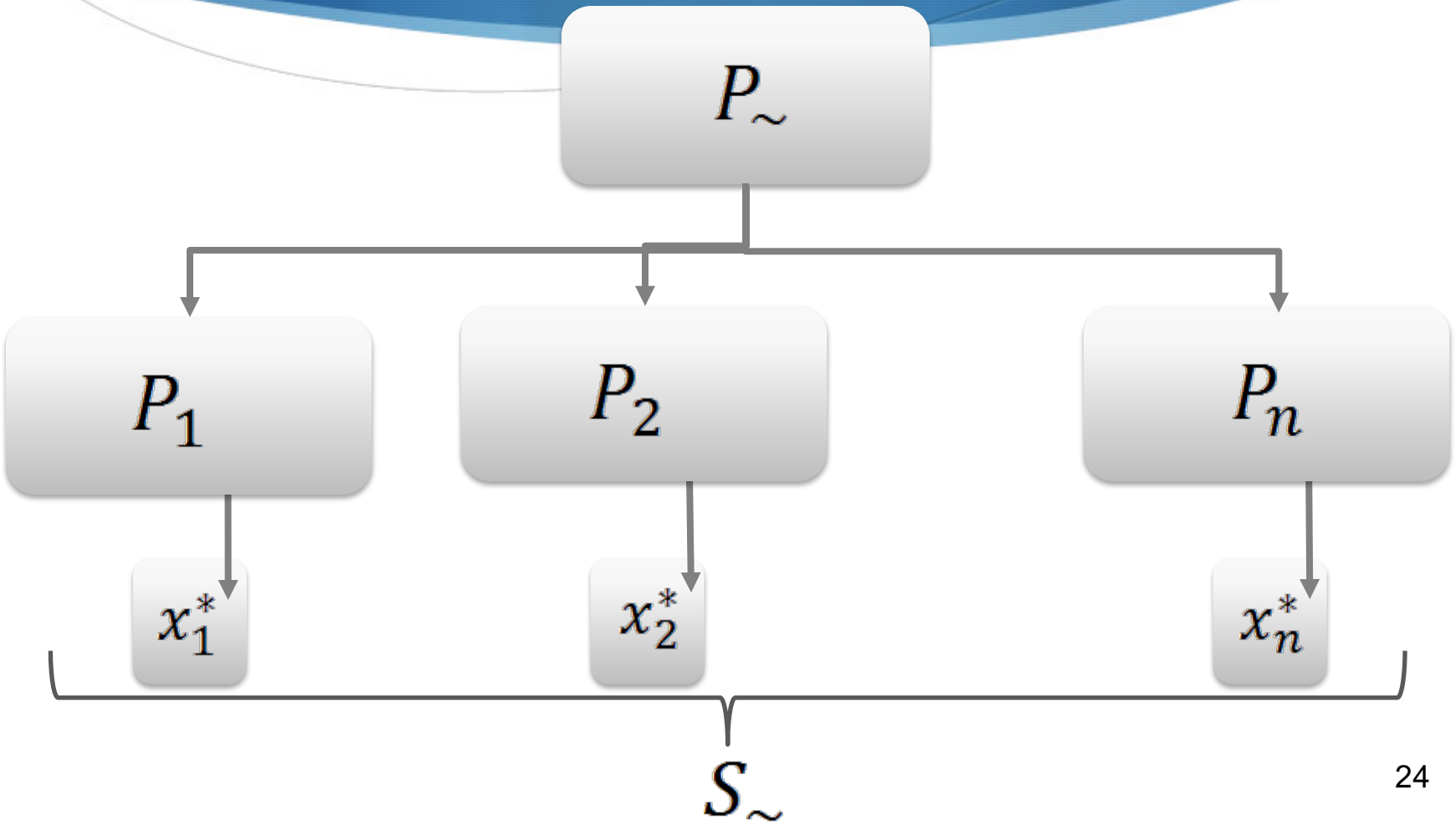

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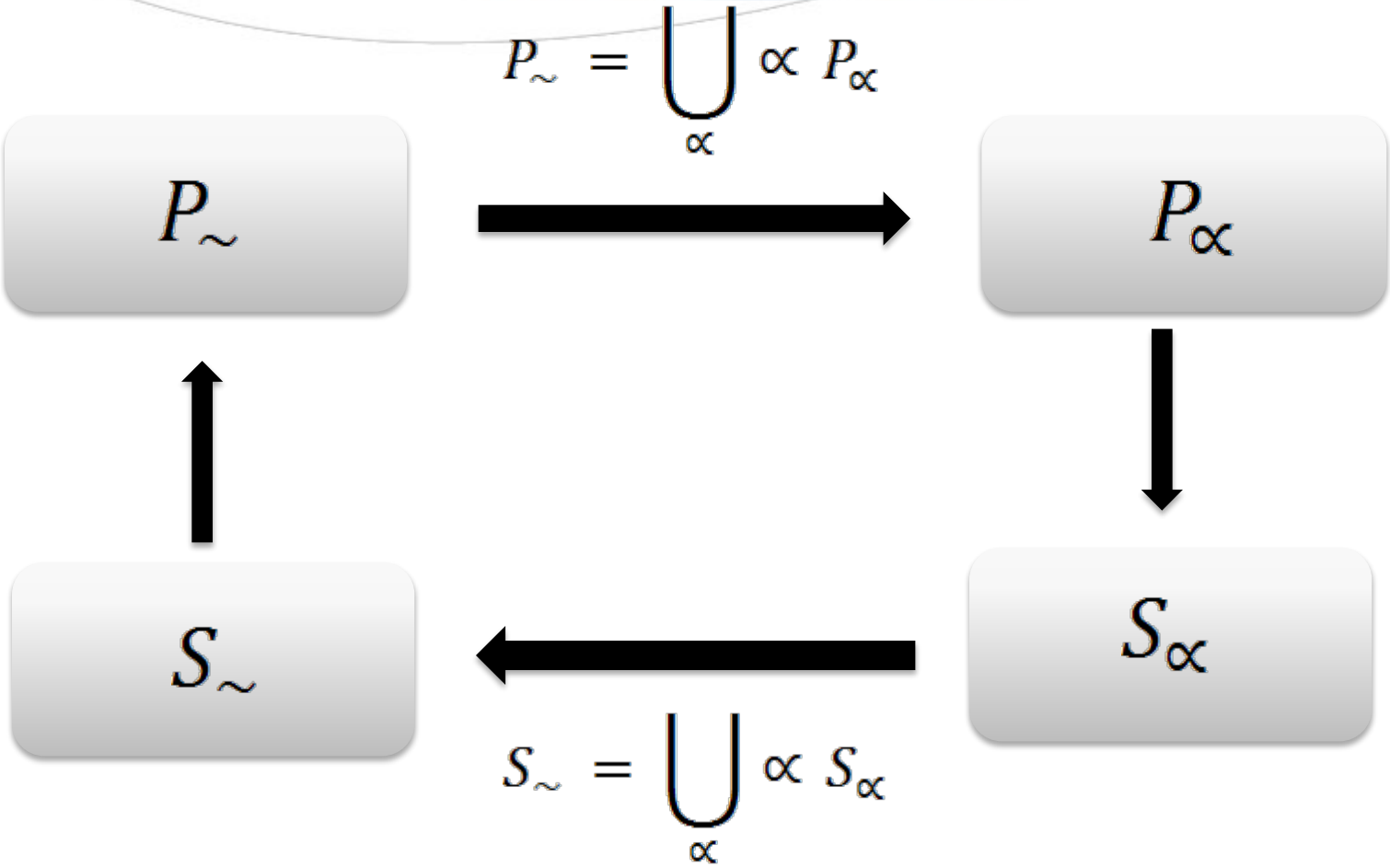

GOAL TO OPTIMIZATION METHODS WITH PARAMETRIC APPROACH

FUZZY PROBLEM

FUZZY SOLUTION (set of solutions)

CHRONOLOGY OF METHODS WITH PARAMETRIC APPROACH

Fuzzy Information and Decision Processes
M.M. Gupta and E. Sanchez (eds.)
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231

FUZZY MATHEMATICAL PROGRAMMING

J. L. Verdegay

Departamento de Estadistica Matematica
Facultad de Ciencias. Universidad de Granada
Granada. Spain

This paper presents a new method to solve the Fuzzy Mathematical Programming (FMP) problem. The α-cuts of the constraints fuzzy set and the α - cuts of the objective are considered for a FMP problem. One can obtain a mathematical programming problem for each α-cut, these particular problems are then solved and thus the fuzzy family of solutions defines the fuzzy solution to the original FMP problem. It is proved that the solutions calculated by other methods are obtained as particular values of this fuzzy solution. In this way, a new formulation to the sensitivity analysis in the FMP problem is given. Finally, a new approach to the fuzzy objective is made.

Fuzzy Sets and Systems 29 (1989) 21–29
North-Holland

21

A GENERAL MODEL FOR FUZZY LINEAR PROGRAMMING

M. DELGADO, J.L. VERDEGAY, M.A. VILA
Departamento de Ciencias de la Computacion e Inteligencia Artificial, Facultad de Ciencias, Universidad de Granada, 18071 Granada, Spain

Received November 1986
Revised May 1987

Abstract: In the current literature there are several models of fuzzy linear programming problems. The aim of this paper is, first, to describe the more important problems in fuzzy linear programming and, second, to give a general model of fuzzy linear programming problems involving all of the above ones. Finally, a resolution method for that general model is proposed.

Keywords: Fuzzy linear programming problems; fuzzy constraint; fuzzy numbers.

1982 → … → 1986

A GENERAL METHOD

Fuzzy Sets and Systems 29 (1989) 21–29
North-Holland

A GENERAL MODEL FOR FUZZY LINEAR PROGRAMMING

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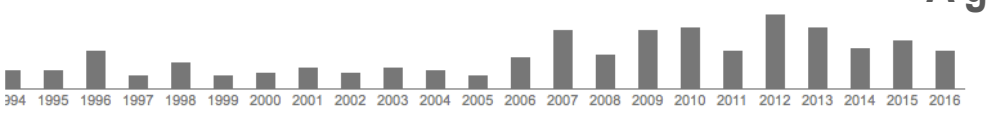
- Miguel Delgado
- J.L. Verdegay
- Amparo Vila



"A general model for fuzzy linear programming"
Fuzzy Sets and Systems,
29-1, 10 Janeiro 1989, Páginas 21–29

A general model for fuzzy linear programming
M Delgado, JL Verdegay, MA Vila - Fuzzy Sets and systems, 1989
Citado por 335 - Artigos relacionados - Todas as 5 versões

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Keywords: Fuzzy linear programming problems; fuzzy constraint; fuzzy numbers.

Solving Real-World Fuzzy Quadratic Programming Problems by a Parametric Method

Fuzzy Optim Decis Making (2011) 10:193–210
DOI 10.1007/s10700-011-9104-7

Extending and relating different approaches for solving fuzzy quadratic problems

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Fuzzy Optim Decis Making (2012) 11:337–349
DOI 10.1007/s10700-012-9131-z

Fuzzy optimization for distribution of frozen food with imprecise times

J. Brito · F. J. Martínez · J. A. Moreno · J. L. Verdegay

Published online: 29 April 2012

Abstract Problems concerning the distribution routes for frozen products need to incorporate constraints that avoid breaks in the cold chain. The decision making process under uncertain environments is a common one in real logistics problems. The purpose of this study is to apply a fuzzy approach which will provide an optimal solution to the distribution of frozen food with uncertainty in its time values. A soft computing approach is used where fuzzy constraints are included in the modeling and the solution of the problem.

Fuzzy Optim Decis Making
DOI 10.1007/s10700-013-9153-1

Fuzzy costs in quadratic programming problems

Ricardo C. Silva · Carlos Cruz · José L. Verdegay

IEEE TRANSACTIONS ON FUZZY SYSTEMS, VOL. 19, NO. 5, OCTOBER 2011

795

Using Fuzzy Numbers in Network Design Optimization Problems

Aplicación de la Soft Computing al Problema de Ruteo de Camiones y Remolques

Isis Torres Pérez[1], Carlos Alberto Cruz Corona[2], and José Luis Verdegay Galdeano[2] y Alejandro Rosete Suárez[1]

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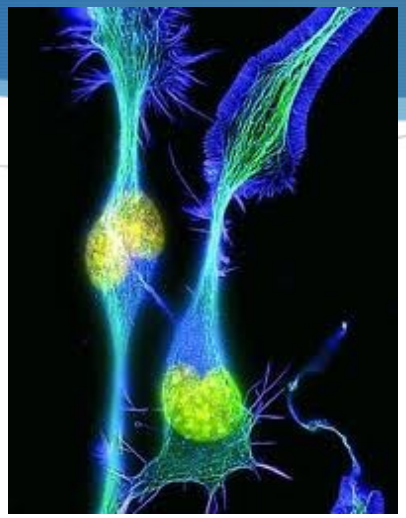
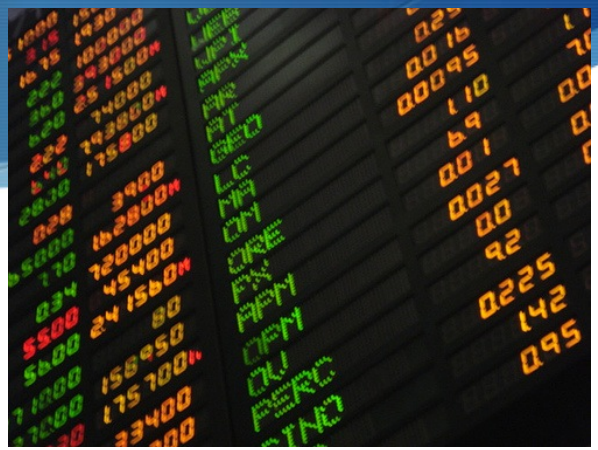
Resumen El Problema de Ruteo de Camiones y Remolques (TTRP) es una extensión del Problema de Ruteo de Vehículos (VRP), que tiene como rasgo distintivo el uso de remolques y restricciones de acceso a los clientes. Cada vez es mayor el número de situaciones reales donde se presenta este problema, siendo conveniente mejorar los modelos y métodos que se han utilizado en su resolución. En la literatura este problema se ha abordado sin considerar la subjetividad humana presente en los decisores cuando definen los elementos del modelo ; una suposición que no tiene en cuenta que en la mayoría de los casos el conocimiento disponible puede ser impreciso, incierto y vago. El presente trabajo se enfoca en como modelar y resolver el TTRP con información incierta haciendo uso de las metodologías de la Soft Computing. Además de presentar las principales estrategias de solución y las variantes más estudiadas del problema.

1982 → 1989 → … → 2012 → 2013

FUZZY MATHEMATICAL PROGRAMMING





FUZZY QUADRATIC PROGRAMMING

Solving Real-World Fuzzy Quadratic Programming Problems by a Parametric Method

Carlos Cruz[1], Ricardo C. Silva[2], and José Luis Verdegay[1]

[1] Department of Computer Science and Artificial Intelligence, CITIC–University of Granada, E-18071, Granada, Spain
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[2] Institute of Science and Technology, Federal University of São Paulo, rua Talim, 330, Vila Nair, 12231-280, São José dos Campos, SP, Brazil
ricardo.coelho@unifesp.br

Abstract. Although fuzzy quadratic programming problems are of the utmost importance in an increasing variety of practical fields, there are remaining technological areas in which has not been tested their applicability or, if tried, have been little studied possibilities. This may be the case of Renewable Energy Assessment, Service Quality, Technology Foresight, Logistics, Systems Biology, etc. With this in mind, the goal of this paper is to apply a parametric approach previously developed by authors to solve some of these problems, specifically the portfolio selection problem by using BM&FBOVESPA data of some Brazilian securities and the economic dispatch problem, which schedules a power generation in an appropriate manner in order to satisfy the load demand.

α	Decision Variables						FunObj
0.0	0.0501	0.2501	0.3751	0.9167	0.3751	0.3001	477.6953
0.1	0.0530	0.2525	0.3826	0.9217	0.3826	0.3030	484.2223
0.2	0.0560	0.2550	0.3901	0.9267	0.3901	0.3060	490.7662
0.3	0.0590	0.2575	0.3975	0.9317	0.3975	0.3090	497.3270
0.4	0.0620	0.2600	0.4050	0.9366	0.4050	0.3120	503.9047
0.5	0.0650	0.2625	0.4124	0.9416	0.4124	0.3150	510.4993
0.6	0.0680	0.2650	0.4199	0.9466	0.4199	0.3180	517.1108
0.7	0.0709	0.2675	0.4273	0.9516	0.4273	0.3209	523.7392
0.8	0.0739	0.2699	0.4348	0.9565	0.4348	0.3239	530.3846
0.9	0.0769	0.2724	0.4423	0.9615	0.4423	0.3269	537.0468
1.0	0.0799	0.2749	0.4497	0.9665	0.4497	0.3299	543.7259

FUZZY QUADRATIC PROGRAMMING

Fuzzy Optim Decis Making (2011) 10:193–210
DOI 10.1007/s10700-011-9104-7

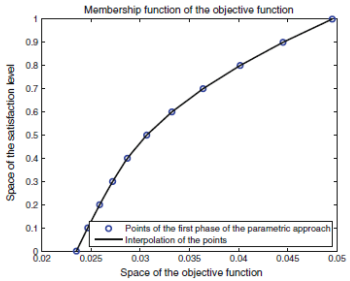
Extending and relating different approaches for solving fuzzy quadratic problems

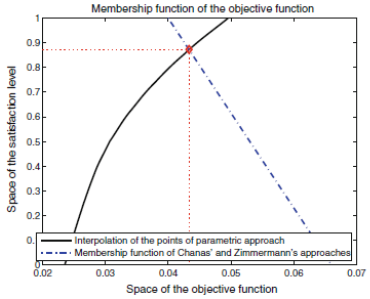
Carlos Cruz · Ricardo C. Silva · José L. Verdegay

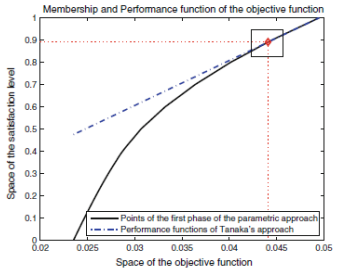
Published online: 22 June 2011

Abstract Quadratic programming problems are applied in an increasing variety of practical fields. As ambiguity and vagueness are natural and ever-present in real-life situations requiring solutions, it makes perfect sense to attempt to address them using fuzzy quadratic programming problems. This work presents two methods used to solve linear problems with uncertainties in the set of constraints, which are extended in order to solve fuzzy quadratic programming problems. Also, a new quadratic parametric method is proposed and it is shown that this proposal contains all optimal solutions obtained by the extended approaches with their satisfaction levels. A few numerical examples are presented to illustrate the proposed method.

α	x	Solution	Time
0.0	[0.0000; 0.1248; 0.1375; −0.0000; 0.0906; 0.0626; 0.5845; −0.0000; −0.0000]	0.0235	2.8140
0.1	[0.0000; 0.0694; 0.1531; 0.0000; 0.0903; 0.0600; 0.6272; 0.0000; −0.0000]	0.0247	0.2804
0.2	[−0.0000; 0.0139; 0.1687; 0.0000; 0.0901; 0.0574; 0.6699; 0.0000; −0.0000]	0.0259	0.3104
0.3	[0.0000; −0.0000; 0.1755; 0.0035; 0.1002; 0.0269; 0.6939; 0.0000; 0.0000]	0.0272	0.2504
0.4	[0.0000; 0.0000; 0.1626; 0.0306; 0.1169; −0.0000; 0.6900; −0.0000; 0.0000]	0.0287	0.2704
0.5	[0.0000; 0.0000; 0.1312; 0.0730; 0.1560; −0.0000; 0.6398; 0.0000; −0.0000]	0.0307	0.2303
0.6	[−0.0000; 0.0000; 0.0998; 0.1154; 0.1951; −0.0000; 0.5896; 0.0000; −0.0000]	0.0332	0.2203
0.7	[−0.0000; 0.0000; 0.0684; 0.1579; 0.2343; 0.0000; 0.5395; −0.0000; 0.0000]	0.0364	0.1502
0.8	[0.0000; −0.0000; 0.0370; 0.2003; 0.2734; 0.0000; 0.4893; 0.0000; −0.0000]	0.0402	0.1602
0.9	[−0.0000; 0.0000; 0.0056; 0.2428; 0.3125; 0.0000; 0.4391; 0.0000; −0.0000]	0.0445	0.3004
1.0	[0.0000; 0.0000; 0.0000; 0.2717; 0.3537; 0.0000; 0.3746; −0.0000; 0.0000]	0.0495	0.1102







In memorian to the professors
Bellman, Tanaka, Asai, and Chanas

Thanks for your attention!

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