

**Workshop in Stochastic Analysis and
Applications**

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**$W^{1,p}$ -solutions of the transport
equation by stochastic perturbation**

Abstract

In this work we study the stochastic transport equation given by:

$$\partial_t u(t, x) + \left(b(t, x) + \frac{dB_t}{dt} \right) \nabla u(t, x) = 0, \quad u|_{t=0} = u_0 \quad (1)$$

where $(t, x) \in [0, T] \times \mathbb{R}^d$, $\omega \in \Omega$, $b: \mathbb{R}_+ \times \mathbb{R}^d \rightarrow \mathbb{R}^d$ is a vector field (drift) and $B_t = (B_t^1, \dots, B_t^d)$ is a standard Brownian motion in \mathbb{R}^d . Indeed, considering a Holder continuous, possibly unbounded, divergence-free drift we will prove well-posedness of the Cauchy problem (1), namely, we show existence, uniqueness and strong stability of $W^{1,p}$ -weak solutions. In particular, this result implies the *persistence of regularity* for initial conditions $u_0 \in W^{1,p}(\mathbb{R}^d)$, with $1 < p < \infty$.