Universidade Estadual de Campinas

Workshop in Stochastic Analysis and Applications

- August, 2018 -

Book of Abstracts

• Long time behavior of the Gross-Pitaevskii equation at positive temperature. Anne de Bouard (CMAP, Ecole Polytechnique - France)

Abstract

The modeling of cold atoms systems has known an increasing interest in the theoretical physics community, after the first experimental realizations of Bose Einstein condensates, some twenty years ago. In the recent years, mean field models taking account of thermal fluctuations, aiming to describe condensates close to critical condensation temperature have been introduced, as e.g. the so called Stochastic Projected Gross Pitaevskii equation. We will describe some mathematical results about the global existence of solutions and long time dynamics of a non-truncated version of the model which is a complex Ginzburg Landau equation with a confining harmonic potential and additive space-time white noise. We take advantage in particular of the fact that the Gibbs measure is invariant for both the reversible and non reversible dynamics. This is a joint work with A. Debussche and R. Fukuizumi.

• 2D Navier-Stokes equation with cylindrical fractional Brownian noise. Benedetta Ferrario (Universitá di Pavia - Italy)

Abstract

We consider the Navier-Stokes equation on the 2D torus, with a stochastic forcing term which is a cylindrical fractional Wiener noise of Hurst parameter H. When $H = \frac{1}{2}$, there are results on global existence of a unique strong solution. We consider the case of a fractional Brownian motion, proving a local existence and uniqueness result when $\frac{7}{16} < H < \frac{1}{2}$ and a global existence and uniqueness result when $\frac{1}{2} < H < 1$. This is based on a joint work with C. Olivera.

• Path-by-path regularization by noise for scalar conservation laws. Benjamin Guess (Max Planck Institute for Mathematics in the Sciences - Germany)

Abstract

In this talk we will revisit regularizing effects of noise for nonlinear SPDE. In this regard we are interested in phenomena where the inclusion of stochastic perturbations leads to increased regularity of solutions as compared to the unperturbed, deterministic case. Closely related, we study effects of production of uniqueness of solutions by noise, i.e. instances of SPDE having a unique solution, while non-uniqueness holds for the deterministic counterparts. The talk will concentrate on a path-by-path regularization by noise result in the case of nonlinear scalar conservation laws. In particular, this proves regularizing properties for scalar conservation laws driven by fractional Brownian motion and generalizes the respective results obtained in [G., Souganidis; Comm. Pure Appl. Math. (2017)]. We show that (ρ, γ)-irregularity is a sufficient path-by-path condition implying improved regularity. In addition, we introduce a new path-by-path scaling property which is also shown to be sufficient to imply regularizing effects.

• **Asymptotic expansion for random vectors**. Ciprian Tudor (SAMM, Université Paris 1 - France)

Abstract

We develop the asymptotic expansion theory for vector-valued sequences F_N of random variables. We find the second-order term in the expansion of the density of F_N, based on assumptions in terms of the convergence of the Stein-Malliavin matrix associated to the sequence F_N . Our approach combines the classical Fourier approach and the recent theory on Stein method and Malliavin calculus. We find the second order term of the asymptotic expansion of the density of F_N and we illustrate our results by several examples.

• **Non-equilibrium fluctuations for the simple symmetric exclusion process with a slow bond**. Dirk Erhard (UFBA - Brazil)

Abstract

The simple symmetric exclusion process with a slow bond in one dimension is an interacting particle system that can be described as follows: particles perform one-dimensional independent simple random walks subject to
(i) two particles never occupy the same site at the same time;
(ii) the jump rate over any fixed edge is 1, except over the edge connecting 0 to 1, here the jump rate is α/n, where α is a positive constant and n is a parameter that will be send to infinity. In this talk I consider the case in which the above process starts out of equilibrium and I will discuss its fluctuations around its mean. It turns out that at large scales they can be described by a generalised Ornstein-Uhlenbeck process, which formally is given by a linear SPDE. This is joint work with Tertuliano Franco, Patrícia Gonçalves, Adriana Neumann and Mariana Tavares.

• **Forward-backward SDEs with jumps and classical solutions to nonlocal quasilinear parabolic PDEs**. Evelina Shamarova (UFPB - Brazil)

Abstract

We obtain an existence and uniqueness theorem for fully coupled forward-backward SDEs (FBSDEs) with jumps via the classical solution to the associated quasilinear parabolic partial integro-differential equation (PIDE), and provide the explicit form of the FBSDE solution. Moreover, we embed the associated PIDE into a suitable class of non-local quasilinear parabolic PDEs which allows us to extend the methodology of Ladyzhenskaya et al (O. Ladyzenskaja, V. Solonnikov, N.N. Uralceva. Linear and Quasi-Linear Equations of Parabolic Type, 1968) to non-local PDEs of this class. Namely, we obtain the existence and uniqueness of a classical solution to both the Cauchy problem and the initial-boundary value problem for non-local quasilinear parabolic second-order PDEs.

• Recent developments in stochastic calculus via regularizations with jumps and applications to BSDEs.. Francesco RUSSO (ENSTA ParisTech - France)

Abstract

The aim of this talk consists in mentioning recent developments about stochastic calculus via regularizations for jump processes. We recall that a $weak\ Dirichlet\ process\ X$ with respect to a given underlying filtration is the sum of a local martingale and a process A such that $[A, N] = 0$ for every continuous local martingale. We introduce the notion of special weak Dirichlet process; whenever such a process is a semimartingale, then it is a special semimartingale. We will provide conditions on a function $u\colon [0, T] \times \mathbb{R}^d \to \mathbb{R}$ and on an adapted cadlag process S such that $u(t, S_t)$ is a special weak Dirichlet process. Two applications will be discussed.
1. The existence of a solution to a (strong) solution of a BSDEs with distributional driver, with underlyng Brownian filtration (with Elena Issoglio, Leeds).
2. Consider the case a BSDE driven by a random measure: a solution is a triplet (Y, Z, K) where K is a rendom field. The function $u(s, x) \colon = Y_s^{s,x}$ is deterministic. If u has some minimal regulartiy, the calculus will allow to link $Z,\ K$ to u (with Elena Bandini, Milano Bicocca).

• Asymptotic stability of stochastic differential equations driven by fractional Brownian motions. Hoang Duc Luu (Vietnam Academy of Science and Technology - Vietnam)

Abstract

In this talk I am going to present some criteria for the exponential stability of the trivial solution of stochastic differential equations driven by fractional Brownian motions.

• Nonlinear SPDEs and irregular nonlinearities. Jean François Colombeau (Institut Fourier - France/ UNICAMP - Brazil)

Abstract

In nonlinear SPDEs one encounters irregular "'functions"' or distributions that need to be manipulated nonlinearly. In the early distribution theory L. Schwartz concluded in the impossibility of such manipulations *L. Schwartz, Impossibility of the multiplication of distributions, Comptes Rendus 239,1954, 847-848.* Later he presented the note *JF Colombeau, A general multiplication of distributions, Comptes Rendus 236, 1983, 357-360* from some originality not perceived in 1954. This context is suited for explicit calculations and theoretical proofs. Usually nonlinear PDEs without distributional solutions have solutions in this context. In some cases in which there are different possible solutions it has been possible to select a physically correct one giving new formulas in agreement with experimental observation, see the book *JF. Colombeau, Multiplication of distributions, Springer Lecture Notes in Math 1532, 1992.* In the talk we will introduce the above very simply without any prerequisite, hoping this method could be reproduced not only for existence (it works) but also for uniqueness from a

choice motivated by physics.

• Regularity properties and simulations of Gaussian random fields on the sphere cross time. Jorge Clarke (Université de Lille - France)

Abstract

We study the regularity properties of Gaussian fields defined over spheres cross time. In particular, we consider two alternative spectral decompositions for a Gaussian field on $S^d \times \mathbb{R}$. For each decomposition, we establish regularity properties through Sobolev and interpolation spaces. We then propose a simulation method and study its level of accuracy in the L^2 sense. The method turns out to be both fast and efficient.

• Renewal Contact Processes. Maria Eulalia Vares (UFRJ - Brazil)

Abstract

This talk is based on joint works in collaboration with L. R. Fontes, D. Marchetti, and T. Mountford. We investigate a non-Markovian analogue of the Harris contact process on Z^d. An individual is attached to each site and it can be infected or healthy; the infection propagates to healthy neighbors as in the usual contact process, according to independent exponential times with a fixed rate. Nevertheless, the possible recovery times for an individual are given by the points of a renewal process with heavy tail; the renewal processes are assumed to be independent for different sites. In [1], we show that if the interarrival distribution has a tail bounded from below by t^{-a} for some $a < 1$ (plus some regularity conditions), then the process survives for any positive value of the infection rate. In [2], a robust argument shows that the critical infection rate is positive in any dimension whenever the interarrival distribution has finite second moment. We also show that in one dimension the same holds when the interarrival distribution has decreasing hazard rate and tail bounded by t^{-a} with $a > 1$.

[1] L. R. Fontes, T. S. Mountford, D. H. U. Marchetti, M. E. Vares. Contact process under renewals I. arXiv: 1803.01458 [math.PR]
[2] L. R. Fontes, T. S. Mountford, M. E. Vares. Contact process under renewals II. arXiv: 1803.01460 [math.PR]

• Entropy methods in large stochastic systems.. Milton Jara (IMPA - Brazil)

Abstract

We derive a new variational formula relating exponential moments of observables of Markov processes with the relative entropy of the law of the process with respect to arbitrary reference measures. As an application, we derive the scaling limit of fluctuations around its

hydrodynamic limit of non-equilibrium interacting particle systems.

• Rescaling noise near extremum points in stochastic Heat equation. Mikhail Neklyudov (UFAM - Brazil)

Abstract

We will show that the solution of 1D stochastic parabolic equation with colored additive noise can be controlled globally by rescaling noise locally.

• A strong averaging principle for Lévy diffusions in foliated spaces with unbounded leaves. Paulo H. P. da Costa (UNB - Brazil)

Abstract

This work extends a strong averaging principle for Lévy diffusions which live on the leaves of a foliated manifold subject to small transversal Lévy type perturbation to the case of non-compact leaves. The main result states that the existence of p-th moments of the foliated Lévy diffusion for $p \geq 2$ and an ergodic convergence of its coefficients in Lp implies the strong Lp convergence of the fast perturbed motion on the time scale t/ϵ to the system driven by the averaged coefficients. In order to compensate the non-compactness of the leaves we use an estimate of the dynamical system for each of the increments of the canonical Marcus equation derived in da Costa and Hoegele (2017), the boundedness of the coefficients in Lp and a nonlinear Gronwall-Bihari type estimate. The price for the non-compactness are slower rates of convergence, given as p-dependent powers of ϵ strictly smaller than $1/4$.

• Quantitative estimates of propagation of chaos for stochastic systems with $W^{-1,\infty}$ kernels.. Pierre-Emmanuel Jabin (CSCAMM - United States)

Abstract

We derive quantitative estimates proving the propagation of chaos for large stochastic systems of interacting particles. We obtain explicit bounds on the relative entropy between the joint law of the particles and the tensorized law at the limit. We have to develop for this new laws of large numbers at the exponential scale. But our result only requires very weak regularity on the interaction kernel in the negative Sobolev space $\dot{W}^{-1,\infty}$, thus including the Biot-Savart law and the point vortices dynamics for the 2d incompressible Navier-Stokes.

• Geodesic jumps in non-continuous SDE: applications to an averaging principle on foliated space. Paulo R. C. Ruffino (UNICAMP - Brazil)

Abstract

Semimartingales with jumps have been treated among others, by Kurtz, Pardoux and Protter, 1995, using the so called Marcus approach for jumps. Marcus interpretation describes the jumps as following an artificial deterministic flow of a vector field along a hidden time. Here we propose jumps of cadlag trajectories along geodesics, hence depending only on the point where the jumps start at. We get a generalized Itô-Kunita decomposition of the corresponding flow of local diffeomorphism and apply this approach to decomposition of flows and averaging along foliated manifolds.

• Path-dependent equations driven by Holder processes.

Rafael A. Castrequini (ENSTA ParisTech - France)

Abstract

This talk investigates existence results for path-dependent differential equations (pdDE) driven by a Hölder function where the integrals are understood in the Young sense. Typically in a pdDE $dY(t) = F(t, Y)dX(t)$, the second argument in the vector field $F = F(t, Y)$ depends on the trajectory of the path Y until time t (and not only the position of Y at time t, namely, $Y(t)$). In this talk, X is an Holder contiunous process suitable such that Young integral exists.

• A strong invariance principle for the elephant random walk

. Renato Gava (UFSCar - Brazil)

Abstract

We consider a discrete- ime random walk on Z with unbounded memory called the elephant random walk (ERW). That is an interesting random walk since, depending on the value of its parameter $p \in (0, 1)$, it presents both normal and anomalous diffusion. We prove a strong invariance principle for the ERW. More specifically, we prove that, under a suitable scaling and in the diffusive regime as well as at the critical value $p_c = 3/4$ where the model is marginally superdiffusive, the ERW is almost surely well approximated by a Brownian motion. As a by-product of our result we get the law of iterated logarithm and the central limit theorem for the ERW. Joint work with Cristian Colleti (UFABC) and Gunter Schutz (Institute of Complex Systems).

• Stuck in Traffic

. Richard B. Sowers (University of Illinois at Urbana - United States)

Abstract

We discuss some recent problems and models related to urban mobility problems and related data. We discuss some big data problems and some models which are relevant for understanding intelligent transportation problems.

• **Uncertainty, Control and Filtering**. Samuel Cohen (University of Oxford - England)

Abstract

Combining learning and decision making, or more formally filtering and optimal control, is a classical problem. In this talk, we will look at problems related to doing this in the presence of uncertainty, in a coherent manner.

• **On the winding number of planar Brownian motion**. Stella Brassesco (IVIC - Venezuela)

Abstract

We consider a Brownian motion $\{B(t)\colon\ t \geq 0\}$ in $\mathbb{R}^2$ (identified with $\mathbb{C}$), starting at $z_0 \neq 0$. Let us denote by $R(t)$ its radial part, and by $\Theta(t) \in (-\infty, +\infty)$ its continuous argument. A formula for the density of $\big(\Theta(t)|R(0) = \rho\big)$ is obtained by standard methods, and several consequences are derived. In particular, the asymptotic behaviour of the argument both as $t \to \infty$ and as $t \to 0$ follow, as well as an expression for the density of the Brownian motion on a wedge of any angle.

• **Homogenization from Stochastic Deformations.**. Wladimir Neves (UFRJ - Brazil)

Abstract

We study in this talk the homogenization of the Liouville and Schrodinger equations. We are interested to model displacement of particles in non-cristaline materials, which can be represent from Stochastic Deformations. We also address the interesting question, which is related to semi-classical theory, and the probabilistic space structure required for the homogenization of the cited equations.

Posters

• $W^{1,p}$-solutions of the transport equation by stochastic perturbation. David Alexander Chipana Mollinedo (UTFPR - Brazil)

Abstract

In this work we study the stochastic transport equation given by:

$$\partial_t u(t,x) + \left(b(t,x) + \frac{dB_t}{dt} \right) \nabla u(t,x) = 0, \qquad u|_{t=0} = u_0 \qquad (1)$$

where $(t,x) \in [0,T] \times \mathbb{R}^d$, $\omega \in \Omega$, $b\colon \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}^d$ is a vector field (drift) and $B_t = (B_t^1, ..., B_t^d)$ is a standard Brownian motion in $\mathbb{R}^d$. Indeed, considering a Holder continuous, possibly unbounded, divergence-free drift we will prove well-posedness of the Cauchy problem (1), namely, we show existence, uniqueness and strong stability of $W^{1,p}$-weak solutions. In particular, this result implies the $persistence\ of\ regularity$ for initial conditions $u_0 \in W^{1,p}(\mathbb{R}^d)$, with $1 < p < \infty$.

• Method to Find a ϵ-Optimal Control Non-Markovian Systems. Francys Andrews de Souza (Unicamp - Brazil)

Abstract

We present a general solution for finding the epsilon-optimal controls for non-Markovian stochastic systems as stochastic differential equations driven by Brownian motion. Our theory provides a concrete description of a rather general class, among the principals, we can highlight financial problems such as portfolio control, hedging, super-hedging, pairs- rading and others. The pathwise analysis was made through a discretization structure proposed by Leão e Ohashi[1] jointly with measurable selection arguments, has provided us with a structure to transform an infinite dimensional problem into a finite dimensional. The theory is applied to stochastic control problems based on path-dependent SDEs where both drift and diffusion components are controlled. We are able to explicitly show optimal control with our method [2].

[1] Leão, D. and Ohashi, A. (2013). Weak approximations for Wiener functionals. Ann. Appl. Probab, 23, 4, 1660−1691
[2] Leão, D. Ohashi, A. and Souza, F. (2017). STOCHASTIC NEAR-OPTIMAL CONTROLS FOR PATHDEPENDENT SYSTEMS. arXiv: 1707.04976.