

Seminário de sistemas dinâmicos e estocásticos

IMECC - UNICAMP

Título: Path Cohomology of Locally Finite Digraphs, Hodge's Theorem and the p-Lazy Random Walk.

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Resumo:

The study of Markov chains on discrete spaces, such as digraphs, has captivated mathematicians in recent decades due to its interconnectedness with topology, geometry, dynamics, spectral theory, and differential equations. Furthermore, extensive exploration of these multifaceted relationships has been pursued for their practical utility in diverse fields, including machine learning and image segmentation. In recent times, these interrelations have been generalized to higher dimensions within the framework of finite-dimensional simplicial complexes. In this paper, we embark on a further extension of these concepts. Initially, we introduce a cohomology of infinite (though locally finite) digraphs in arbitrary dimensions. Subsequently, in the latter portion of this manuscript, we define a fresh family of Laplace operators and conduct an examination of their spectrum, culminating in the proof of the Hodge Decomposition Theorem within this framework. Finally, we conclude by presenting a Markov chain, the p-Lazy Random Walk, whose asymptotic behavior is intrinsically linked to these cohomologies, while its mixing time is related to the spectrum of our Laplace operators. This development opens doors to numerous unexplored questions, particularly regarding potential generalizations of the Ollivier-Ricci curvature to this topology and these Laplacians. It is known for expanding dynamical systems and finite state Markov chains that the asymptotic behaviour of the minimal distance between two orbits up to time n is given by its correlation dimension.

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