Stochastic continuity equation with non-smooth velocity

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Abstract

In this work we study the stochastic continuity equation:

$$\begin{cases} \partial_t u(t,x) + Div\big((b(t,x) + \dfrac{dB_t}{dt}) \cdot u(t,x)\big) = 0\,, \\ u|_{t=0} = u_0\,, \end{cases} \tag{1}$$

The main issue is to prove uniqueness of L^2-weak solutions for one-dimensional stochastic continuity equation (1) with unbounded measurable field vector (drift) without assumptions on the divergence. More precisely, we assume that b (drift) satisfies

$$|b(x)| \leq k(1 + |x|).$$

The proof is based in the fact that one primitive V is regular and it verifies the transport equation

$$\partial_t V(t,x) + (b(t,x) + \frac{dB_t}{dt}) \cdot \nabla V(t,x) = 0\,. \tag{2}$$

Then using a modified version of the "commutator Lemma" and the characteristic systems associated to the stochastic partial differential equation (2) we shall show that $V = 0$ with initial condition equal to zero, which implies that $u = 0$. For more details to see [1]. Joint work with Christian Olivera (Universidade Estadual de Campinas).

Referências

[1] D. A. C. Mollinedo and C. Olivera , *Stochastic continuity equation with non-smooth velocity* , Annali di Matematica Pura ed Applicata (1923 -), page 1-16, 2017 .