On a Generalized Fibonacci and Lucas numbers

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UFMS

- For positive integers a, b,

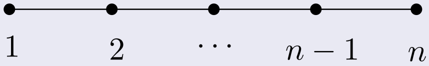
$$F_{(a,b)}(k,n) = a^n(n+1), \quad \text{para } n = 0, \dots, k-1$$
$$F_{(a,b)}(k,n) = aF_{(a,b)}(k,n-1) + bF_{(a,b)}(k,n-k), \quad \text{for } n \geq k$$

$$L_{(a,b)}(k,n) = a^n(n+1), \quad \text{for } n = 0, 1, \dots, k-1$$
$$L_{(a,b)}(k,n) = ka^n + (n-k+1)ba^{n-k}, \quad \text{for } n = k, \dots, 2k-1$$
$$L_{(a,b)}(k,n) = a^{k-1}\left\{b(k-1)F_{(a,b)}(k,n-(2k-1)) + \right.$$
$$\left. + F_{(a,b)}(k,n-(k-1))\right\}, \quad \text{for } n > 2k-1.$$

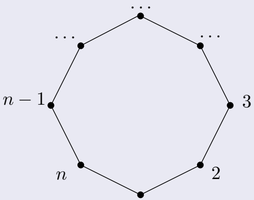
- We have studied these sequences to find new identities for well-known sequences.

Motivation

- Let a simple graph $X = (E, V)$ comprising a set V of vertices together with a set E of edges.
- A subset $S \subseteq V$ is called $k-$independent of X if for any two vertices x, y in S, $d_X(x, y) \geq k$, where $d_X(x, y)$ is the minimum path joining x to y.
- $F(k, n)$ is the number of subsets $k-$independent of P_n



- $L(k, n)$ is the number of subsets $k-$independent of C_n



Theorem

Let positive integers n and $k \geq 2$ with condition $n \geq 2k+1$. Then

$$F_{(a,b)}(k,n) = a^{k+2}F_{(a,b)}(k,n-k-2) + \sum_{i=0}^{k+1} ba^{i-1}F_{(a,b)}(k,n-k-i).$$

Theorem

Let n, k positive integers with conditions $k \geq 2$ and $n \geq 2k$. Then

$$L_{(a,b)}(k,nk+1) = a\sum_{i=0}^{n} b^{n-i}L_{(a,b)}(k,ik).$$

We studied a possibility to introduce a weight function on these sequences to find new identities involving generating functions.

$$P_{(a,b)}(k,n) = (n+1)\prod_{i=1}^{n}\frac{1-q^{i(a+1)}}{1-q^{i}}, \text{ for } n = 0,1,\dots k-1;$$

$$P_{(a,b)}(k,n) = \frac{1-q^{n(a+1)}}{1-q^{n}}P_{(a,b)}(k,n-1)$$

$$+\frac{1-q^{(n-k+1)(b+1)}}{1-q^{n-k+1}}P_{(a,b)}(k,n-k), \text{ for } n \geq k;$$