

Practical Design and Decoding of Polar Codes

Vera Miloslavskaya

Saint-Petersburg State Polytechnic University
veram@dcn.icc.spbstu.ru

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Polar Codes

- Polar codes can achieve the capacity of an arbitrary binary-input output-symmetric memoryless channel

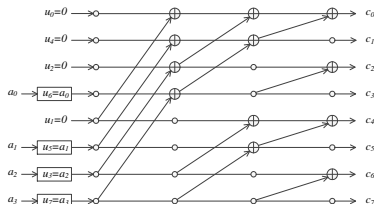
- An $(n = l^m, k)$ polar code is generated by k rows of matrix $G_n = A^{\otimes m}$, where

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

- Problems:

- The successive cancellation decoder is far from maximum likelihood
- Minimum distance of polar code $\sim \sqrt{n}$
- The original Arikan construction results in codes of length 2^m

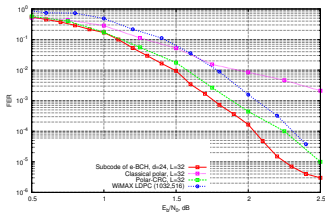
$$A^{\otimes 3} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$



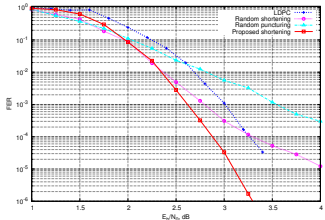
Design of Polar Codes

- A generalization of polar codes: dynamic frozen symbols
 - A dynamic frozen symbol is equal to some linear combination of information symbols
 - Polar subcodes of extended BCH codes
- An algorithm for construction of shortened polar codes
 - Error probability under the successive cancellation decoding is minimized
 - Optimal for the code length lower than 64, suboptimal for other lengths

Length=1024, Dimension=512



Length=768, Dimension=512



Decoding of Polar Codes

- A sequential decoding algorithm is proposed
 - An instance of the stack successive cancellation (SC) decoding method
 - A new metric for paths u_0^i is used
 - Significant complexity reduction compared to the list/stack SC decoding
- The algorithm can be applied to
 - Polar codes with the 2×2 kernel
 - Polar codes with an arbitrary binary kernel
 - Reed-Solomon codes represented as polar codes with dynamic frozen symbols

Average decoding complexity for (1024, 512) codes, $\times 10^3$ real operations

E_b/N_0 , dB	Polar code, proposed approach						LDPC, Belief prop.	
	Additions			Comparisons			Additions	$\log \tanh(\frac{x}{2})$
	$L = 32$	$L = 256$	$L = 2048$	$L = 32$	$L = 256$	$L = 2048$	≤ 200 iter.	≤ 200 iter.
0	141	833	5231	227	1332	8374	2617	1307
0.5	133	752	4265	218	1224	6968	2333	1112
1	73	286	1232	122	477	2065	1469	722
1.5	32	88	267	54	151	461	394	185
2	18	27	42	31	48	74	140	62