On the existence of self-dual codes invariant under permutation groups

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Given a group $G$ and a finite field $F$ of characteristic $p$, $p$ prime, how do we determine the self-dual codes invariant under $G$ of a given length $n$ over $F$?

A result by Harada et al shows that a special construction $C(G, \Omega) = \langle \text{Fix}(\sigma) | \sigma \in \text{Inv}(G) \rangle^\perp$ is such that every self-orthogonal code $C$ of length $|\Omega|$ is a subcode of $C(G, \Omega)$.

Further every $G$-invariant self-dual code $C$ of length $|\Omega|$ is such that $C(G, \Omega)^\perp \subseteq C \subseteq C(G, \Omega)$. 
Regarding a code as an $FG$-module, we find all $FG$ modules satisfying the last inclusion using GAP and MAGMA and then filter all such whose dimensions are exactly $|Ω|/2$.

These are the $G$-invariant self-dual codes of length $Ω$.

$a priori$ one can determine whether there are self-dual codes invariant under a given group using representation and character theory of groups.

However this does not mean the enumeration is easy.
We use this scheme to determine self-dual codes of various lengths invariant under some sporadic simple and almost simple groups as well as symmetric and alternating groups.

We found that there are no self-dual codes of length $n$ invariant under the symmetric group $S_n$.

Tatenda!