On the existence of self-dual codes invariant under permutation groups

Tendai . M. Mudziiri Shumba

School of Maths, Stats and Computer Science, UKZN

January 2015

- Given a group G and a finite field F of characteristic p, p prime, how do we determine the self-dual codes invariant under G of a given length n over F?
- A result by Harada et al shows that a special construction $C(G, \Omega) = \langle \mathsf{Fix}(\sigma) | \sigma \in \mathsf{Inv}(G) \rangle^{\perp}$ is such that every self-orthogonal code $\mathcal{C}$ of length $|\Omega|$ is a subcode of $C(G, \Omega)$.
- Further every G-invariant self-dual code $\mathcal{C}$of length $|\Omega|$ is such that $C(G, \Omega)^{\perp} \subseteq \mathcal{C} \subseteq C(G, \Omega)$.

- Regarding a code as an FG-module, we find all FG modules satisfying the last inclusion using GAP and MAGMA and then filter all such whose dimensions are exactly $|\Omega|/2$.
- These are the G-invariant self-dual codes of length Ω
- *a priori* one can determine whether there are self-dual codes invariant under a given group using representation and character theory of groups.
- However this does not mean the enumeration is easy.

- We use this scheme to determine self-dual codes of various lengths invariant under some sporadic simple and almost simple groups as well as symmetric and alternating groups.
- We found that there are no self-dual codes of length n invariant under the symmetric group S_n.
- Tatenda!