A Modified Sum-Product Algorithm over Graphs with Isolated Short Cycles

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January 6, 2015

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Motivation

- Low Density Parity Check code, Pioneering work by Gallager [1].

[1] R. G. Gallager, Low-density parity-check codes, 1963.

- Capacity approaching codes.

Practical challenges in achieving Shannon capacity

1. Ultra-large block lengths (Computational complexity).
2. Higher girth random codes (Code construction and encoding difficulty).
3. Optimal decoding algorithm (ML decoder: NP hard).

Causes for Error Floor

- Code construction: Presence of combinatorial structures like trapping sets in Tanner graph.
- Decoding algorithm: Message passing algorithm being sub-optimal over graphs with cycles.

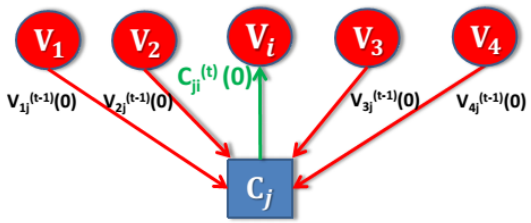
Can cycles aid in decoding?

Heuristically, improve correlations $\rightarrow$ Better decoding.

Message Passing Algorithm in Probability Domain

Check node update:

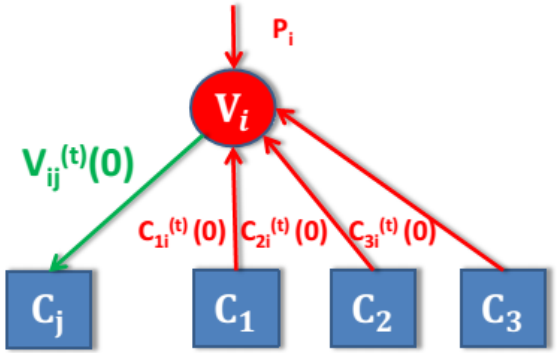
$$c_{jk}^{(t)}(0) = \frac{1 + \prod_{v_i \in N(c_j) \setminus v_k} (2v_{ij}^{(t-1)}(0) - 1)}{2}.$$



- Ensures parity is satisfied at the check node.
- Denotes probability of even number of ones among the extrinsic messages (Gallager's lemma).
- Gallager's lemma assumes statistical independence among variable node messages.

Variable node update:

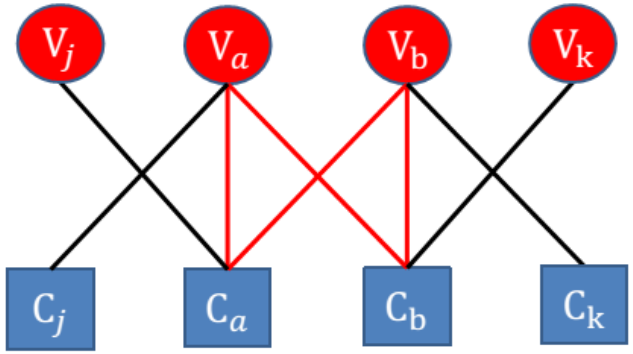
$$v_{ij}^{(t)}(0) = p_i \prod_{c_k \in N(v_a) \setminus c_j} c_{ki}^{(t)}(0).$$



- Product of all extrinsic check node messages and the channel reliability p_i.
- Assumes statistical independence among the check node messages.

Independence assumption breaks down badly for message passing over short cycles.

Presence of Short Cycles and Suboptimal Algorithm



Remedy? Avoid short cycles by constructing Tanner graphs of higher girth. Is it the ideal solution??

Fundamental approach:
Investigate the assumptions and approximations used in the message passing algorithm.

⋆ Significant when short cycles play a role:

- 2-D factor graphs modeling both ISI channel and decoder have short cycles.
- Motivation towards error floor mechanism.

Research Work

- We analysed the dependency among messages passed within an isolated cycle of length 4 and modified the update equations.
- Modified message passing algorithm for LDPC codes shows 0.6 dB improvement over sum-product algorithm for a code with block length of 200 and rate 0.5 with 25 isolated cycles of length 4.
- More the number of cycles in the graph, the modified algorithm is able to use the dependency for effective decoding.

Conclusion

- The improvement at high SNR region suggests a possible solution to the error flooring effects in Tanner graphs.

Future works

- An investigation into the effect of nested 4 cycles on the algorithm.
- A unified theory for tractable statistical dependencies during belief propagation.