A Modified Sum-Product Algorithm over Graphs with Isolated Short Cycles

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January 6, 2015
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Motivation

• Low Density Parity Check code, Pioneering work by Gallager [1].


• Capacity approaching codes.

Practical challenges in achieving Shannon capacity

1. Ultra-large block lengths (Computational complexity).
2. Higher girth random codes (Code construction and encoding difficulty).

Causes for Error Floor

• Code construction: Presence of combinatorial structures like trapping sets in Tanner graph.
• Decoding algorithm: Message passing algorithm being sub-optimal over graphs with cycles.

Can cycles aid in decoding?

Heuristically, improve correlations → Better decoding.
**Message Passing Algorithm in Probability Domain**

### Check node update:

\[
c_{jk}(t)(0) = \frac{1 + \prod_{v_i \in N(c_j) \setminus v_k} (2v_{ij}^{(t-1)}(0) - 1)}{2}.
\]

- Ensures parity is satisfied at the check node.
- Denotes probability of even number of ones among the extrinsic messages (Gallager’s lemma).
- Gallager’s lemma assumes statistical independence among variable node messages.

### Variable node update:

\[
v_{ij}^{(t)}(0) = p_i \prod_{c_k \in N(v_a) \setminus c_j} c_{ki}^{(t)}(0).
\]

- Product of all extrinsic check node messages and the channel reliability \( p_i \).
- Assumes statistical independence among the check node messages.

*Independence assumption breaks down badly for message passing over short cycles.*
Presence of Short Cycles and Suboptimal Algorithm

**Remedy?** Avoid short cycles by constructing Tanner graphs of higher girth. Is it the ideal solution??

**Fundamental approach:**
Investigate the assumptions and approximations used in the message passing algorithm.

* Significant when short cycles play a role:
  - 2-D factor graphs modeling both ISI channel and decoder have short cycles.
  - Motivation towards error floor mechanism.
Research Work

- We analysed the dependency among messages passed within an isolated cycle of length 4 and modified the update equations.
- Modified message passing algorithm for LDPC codes shows 0.6 dB improvement over sum-product algorithm for a code with block length of 200 and rate 0.5 with 25 isolated cycles of length 4.
- More the number of cycles in the graph, the modified algorithm is able to use the dependency for effective decoding.

Conclusion

- The improvement at high SNR region suggests a possible solution to the error flooring effects in Tanner graphs.

Future works

- An investigation into the effect of nested 4 cycles on the algorithm.
- A unified theory for tractable statistical dependencies during belief propagation.