

Optimum modular check digit systems

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Systems based in Modular Arithmetic

Identifier $a_1a_2a_3 \cdots a_{m+1}$, where a_{m+1} is the check digit calculated by:

$$\sum_{i=1}^m \sigma_i(a_i) \equiv a_{m+1} \pmod{n}.$$

The most common errors and their relative frequencies:

Error type	Symbol	Form	Frequency
Single error	SGL	..a.. → ..b..	79.1 %
Adjacent transposition	ATR	..ab.. → ..ba..	10.2 %
Alternate transposition	LTR	..abc.. → ..cba..	0.8 %
Adjacent twin	ATW	..aa.. → ..bb..	0.5 %
Alternate twin	LTW	..aca.. → ..bcb..	0.3 %
Fonetic	FON	..1a.. → ..0a..	0.5 %
Others			8.6 %

A modular check digit system is able to detect:

- 1 the individual error, if and only if the image of σ_i is a permutation.
- 2 the digit transposition error between the positions i and j , $a_i \neq a_j$, if and only if

$$(\sigma_i - \sigma_j)(a_i) \not\equiv (\sigma_i - \sigma_j)(a_j) \pmod{n}.$$

- 3 the twin error between the positions i and j , $a \neq b$, if and only if

$$(\sigma_i + \sigma_j)(a) \not\equiv (\sigma_i + \sigma_j)(b) \pmod{n}.$$

- 4 the fonetic error, if and only if

$$\sigma_i(1) + \sigma_{i+1}(a) \not\equiv \sigma_i(a) + \sigma_{i+1}(0) \pmod{n}.$$

Optimum systems for prime base

Systems used in Brazil

System	SGL	ATR	LTR	ATW	LTW	WPE %
Mod 11 complete	0	0	0	14.29	0	0.07
Mod 11 restrict	1.82	1.82	1.82	15.85	18.18	1.77
Mod 11 2 check digits	0.04	0.14	0.14	0.83	0.03	0.05
Alphanumeric Mod 10	7.81	18.02	18.02	48.76	41.57	8.53

Improvements proposed

System	SGL	ATR	LTR	ATW	LTW	FON	WPE
Mod 11 complete	0	0	0	0	0	0	0
Mod 11 restrict	1.82	1.82	1.82	1.82	1.82	0	1.65
Mod 11 2 CDs	0	0	0	0	0	0	0
Alphanum Mod 37	0	0	0	0	0	0	0

Optimum modulus 10 systems

We have developed the Mod 10 3P that has the lowest rate of non-detection of errors to identifiers with at least 7 digits. And proved that the Verhoeff system is optimum for identifiers with up to 6 digits. Systems using modulus 10 on literature

System	SGL	ATR	LTR	ATW	LTW	FON	WPE
2 permutations	0	11.11	100.0	11.11	11.11	0	2.02
3 permutations	0	11.11	11.11	49.23	40.73	0	1.59
IBM	0	2.22	100.0	6.67	11.11	12.50	1.16
General. IBM	0	2.22	17.78	6.67	4.44	11.67	0.47
PTT	0	3.49	3.70	5.40	5.19	3.33	0.45
Modif. Gen. IBM	0	2.54	4.44	6.67	4.44	11.67	0.40
Colenbrander	0	2.22	4.44	6.67	4.44	16.67	0.39
Vehroeff (≤ 6)	0	2.22	4.44	2.22	4.44	0	0.28
Mod 10 3P	0	2.22	2.22	5.93	5.93	0	0.29
Vehroeff (> 6)	0	2.22	4.44	2.22	4.44	3.57	0.30