Flag Codes with Gallery Distance

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- We have considered matricial codes from the point of view of flag variety and we study basic coding theoric problems for such codes.

$$\begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{bmatrix} \longrightarrow \langle v_1 \rangle \subset \langle v_1, v_2 \rangle \subset \cdots \subset \langle v_1, v_2, ..., v_n \rangle$$

where $v_i \in \mathbb{F}_q^n$.

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- **Motivations:**

Flag Variety is a natural generalization of Grassmann Variety (widely used in network coding)

With the gallery distance, Flag Variety, in some sense, generalizes permutation codes with Kendall metric(used in Rank Modulation Schemes)

Basics

- A **flag** is a maximal nested sequence of subspaces of a n-dimensional vector space over a field with q elements.
- Two flags are **adjacent** if the two sequences are equal, but one subspace.
- A **gallery** is a finite sequence of flags such that consecutive flags are adjacent.
- The **gallery distance** between two flags is the minimum size of a gallery joining them.

Some Results

$B_r :=$ Ball of radius r,
$A_q(n,d) :=$ Maximum cardinality of a flag code of size n and minimum distance d over the finite field with q-elements

- **Sphere's Size:** $\displaystyle\prod_{i=1}^{n} \frac{(qx)^i - 1}{qx - 1}$ is the generating function for the cardinality of the spheres.
- **Packing Radius:** $R = \left\lfloor \dfrac{d-1}{2} \right\rfloor$.
- **G-V and S-P bounds:**
$$\frac{\prod_{i=2}^{n+1} \binom{i}{i-1}_q}{|B_{(d-1)}|} \leq A_q(n,d) \leq \frac{\prod_{i=2}^{n+1} \binom{i}{i-1}_q}{|B_k|}.$$
- $\vdots$