Searching MDS Burst-Correcting Codes

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Linear Block Codes

$$f : GF(2)^{k} \longrightarrow GF(2)^{n}$$

k - information bits	**n - k check bits**

$$\underbrace{\hspace{6cm}}_{\mathbf{n}}$$

The Reiger Bound

- The Reiger bound states:

  For a given **[n, k]** single-burst-correcting code: $2b \leq n - k$

- That is to say, there is a relationship between the single-burst-correcting capability of a code and its redundancy.

Search Algorithm

Given $\boldsymbol{n, k,\ b \le \frac{(n-k)}{2}}$ and $\boldsymbol{1 \le l \le b}$, the algorithm finds out if there is a cyclic or shortened cyclic $[\boldsymbol{n, k\langle b,l\rangle}]$ code $\boldsymbol{C}$ with generator polynomial

$$\boldsymbol{g(x) = g_0 + g_1 x + \cdots + g_{n-k} x^{n-k}, g_0 = g_{n-k} = 1.}$$

1. Declare an initial pair $(\boldsymbol{b, g})$
2. Obtain $\boldsymbol{n}$ (length of the code) → $(\boldsymbol{n = g + l}$ for $\boldsymbol{1 \le l \le b})$
3. Obtain $\boldsymbol{k}$ for optimal codes ($\boldsymbol{2b = n - k}$)
4. Construct all possible generator polynomials with degree $\boldsymbol{n-k}$. Excluding inverse, not initiating with 0s etc.
5. Start checking each possible generator polynomial ,if it is burst correcting code for the pair $(\boldsymbol{b, g})$
6. Create the Generator matrix $\boldsymbol{G}$ for the actual polynomial
7. Find the parity check matrix $\boldsymbol{H}$.
8. Create all possible error patterns for the defined $\boldsymbol{b}$.
9. Find the set $\boldsymbol{S}$ for the NAA Syndromes.
10. Check for uniqueness in the set $\boldsymbol{S}$.
11. Create all possible the AA error patterns for the defined $\boldsymbol{l > 1}$ or $\mathbf{l < b}$.
12. Find the AA Syndromes and check for uniqueness in S. if it is not unique decrease $\boldsymbol{k}$ and go to (4).
13. Declare that code $\boldsymbol{C}$ generated by code $\boldsymbol{g(x)}$ is an $[\boldsymbol{n, k\langle b,l\rangle}]$ code.

Table for b = 8

g	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	$g(x)$	Cyclic?
20	[21,5]	[22,6]	[23,7]	[24,8]	[25,9]	[26,10]	$\boxed{[28,12]}$	[27,11]	10111	Yes
21	[22,6]	[23,7]	[24,8]	[25,9]	[26,10]	[27,11]	[29,12]	$\boxed{[28,12]}$	10111	Yes
22	[23,7]	[24,8]	[25,9]	[26,10]	[27,11]	[28,12]	$\boxed{[30,14]}$	[29,13]	10115	Yes
23	[24,8]	[25,9]	[26,10]	[27,11]	[28,12]	[29,13]	$\boxed{[31,15]}$	[30,14]	10117	Yes
24	[25,9]	[26,10]	[27,11]	[28,12]	[29,13]	[30,14]	[32,15]	$\boxed{[31,15]}$	10117	Yes
25	[26,10]	[27,11]	[28,12]	[29,13]	[30,14]	[31,15]	[33,16]	$\boxed{[32,16]}$	19F17	No
26	[27,11]	[28,12]	[29,13]	[30,14]	[31,15]	[32,16]	[34,17]	$\boxed{[33,17]}$	15B2D	No
27	[28,12]	[29,13]	[30,14]	[31,15]	[32,16]	[33,17]	$\boxed{[35,19]}$	[34,18]	15533	No
28	[29,13]	[30,14]	[31,15]	[32,16]	[33,17]	[34,18]	[36,19]	$\boxed{[35,19]}$	15533	No
29	[30,14]	[31,15]	[32,16]	[33,17]	[34,18]	$\boxed{[35,19]}$	[37,19]	[36,19]	11109	No
30	[31,15]	[32,16]	[33,17]	[34,18]	[35,19]	[36,20]	$\boxed{[38,22]}$	[37,21]	11105	No
31	[32,16]	[33,17]	[34,18]	[35,19]	[36,20]	[37,21]	[39,22]	$\boxed{[38,22]}$	11105	No
32	[33,17]	[34,18]	[35,19]	[36,20]	[37,21]	$\boxed{[38,22]}$	[40,23]	[39,22]	11105	No
33	[34,18]	[35,19]	[36,20]	[37,21]	[38,22]	$\boxed{[39,23]}$	[41,24]	[40,23]	1B1B7	No
34	[35,19]	[36,20]	[37,21]	[38,22]	$\boxed{[39,23]}$	[40,23]	[42,24]	[41,24]	1B1B7	No
35	[36,20]	[37,21]	[38,22]	[39,23]	$\boxed{[40,24]}$	[41,24]	[43,25]	[42,25]	1B1B7	No
…	…	…	…	…	…	…	…	…	…	…
198	[199,181]	[200,182]	$\boxed{[201,183]}$	[202,183]	[203,184]	[204,185]	[206,186]	[205,185]	574B3	No
199	[200,182]	[201,183]	[202,184]	$\boxed{[203,185]}$	[204,185]	[205,186]	[207,187]	[206,187]	5795B	No
200	[201,183]	[202,184]	$\boxed{[203,185]}$	[204,185]	[205,186]	[206,187]	[208,188]	[207,187]	574B3	No

Conclusions and Future Work

- We have presented an efficient algorithm finding the best cyclic or shortened cyclic single burst-correcting codes for different parameters, in the sense that if a found $[n, k]$ code can correct any burst of length up to b, k is the largest possible number among (shortened) cyclic codes. The algorithm minimizes the number of syndrome checks by using Gray codes. It can be adapted to take into account both non-all-around and all-around bursts.

Future Work

- **Multiple burst-correcting codes:** it is interest to find efficient multiple burst-correcting codes that are optimal in terms of redundancy.