PSEUDO PERIODIC ORBITS OF THE PLANAR COLLISION RESTRICTED 3–BODY PROBLEM IN ROTATING COORDINATES

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Abstract. By using the continuation method of Poincaré, we characterize the periodic circular orbits and the symmetric periodic elliptic orbits that can be prolonged from the planar Kepler problem in rotating coordinates to pseudo periodic orbits of the planar restricted 3–body problem in rotating coordinates with the two primaries moving in an elliptic collision orbit.

1. Introduction and statement of the main results

We consider a restricted 3–body problem where the two primaries with different mass describe an elliptic collision orbit of the 2–body problem, and the infinitesimal mass is moving in a plane that contain the line of motion of the primaries. In what follows this restricted 3–body problem will be called the *planar collision restricted* 3*–body problem*.

The main tool of this paper is the Poincaré's continuation method for prolonging a known periodic orbit of a system of differential equations depending on a parameter. This is one of the most frequently used methods for proving the existence of periodic orbits but of course not the unique see for instance [7]. Poincaré started the study of the periodic orbits of the n–body problem studying the periodic orbits of the planar circular restricted 3–body problem, see [11, 14]. There is an extensive literature on the existence of the periodic solutions of the n–body problem, especially for the restricted 3–body problems, see the books [6, 10, 13] and the papers [1]–[5], [8], etc.

The results of this paper are philosophically close to the ones of [8], but three main differences must be mentioned. The first one is that in our case the parameter used in the prolongation of the periodic orbits is related with the mass of the primaries and in [8] this parameter is related with the distance between the primaries. The second is that in our case the primaries are moving in a plane following a rotating elliptic collision orbit and in [8] the primaries are moving in a straight line perpendicular to the motion of the infinitesimal body. Moreover in [8] the primaries always have equal masses and in our case the masses of the primaries are μ and $1-\mu$ with $\mu \in [0, 1]$.

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Finally the tool for studying the prolongation of the elliptic orbits is the same than in [8], but the tool for studying the circular periodic orbits is completely different in both papers.

Another difference with many of the restricted 3–body problems studied until now is that when we prolonged from $\mu = 0$ to $\mu > 0$ small the symmetric elliptic orbits of the Kepler problem, we distinguished between 32 different families of this class of orbits, and in general this distinction, except in [8], is not done.

As usual a *periodic orbit* of a restricted 3–body problem is an orbit of this problem such that after a time T (the period of the orbit) the three bodies are at the same position with the same velocity. Here we shall study only periodicity with respect to the motion of the infinitesimal body. Such kind of orbits will be called *pseudo periodic orbits* of the planar collision restricted 3– body problem.

We begin by describing the equations of the motion for our problem, see Section 2. In Section 3 we present the symmetry of our problem, and in Section 4 we analyze the symmetric pseudo periodic orbits that can be prolonged from the symmetric periodic elliptic orbits of the Kepler problem in rotating coordinates to the planar restricted 3–body problem with the primaries moving in an elliptic collision orbit also in rotating coordinates. The main result of that section is the following one. This theorem is stated using the Delaunay variables which are introduced later on.

Theorem 1. *Let p and q be relatively prime integers and $T = 2\pi p/q$. Then the elliptic T–periodic orbit of the Kepler problem in rotating coordinates which satisfies*

$$l(0) = n_1\pi, \quad g(0) = n_2\pi, \quad L^3(0) = p/q, \quad G(0) = \text{ constant}, \tag{1}$$

can be continued for small values of $\mu = \mu(T) > 0$ into a S_1–symmetric pseudo periodic orbit of period near to $2\pi p$ of the planar collision restricted 3–body problem in rotating coordinates.

In Section 5 we characterize the periodic circular orbits that can be prolonged from the Kepler problem in rotating coordinates to pseudo periodic orbits of the planar restricted 3–body problem with the primaries moving in an elliptic collision orbit also in rotating coordinates. The main result of that section is the following.

Theorem 2. *Let T_0 be the period of a circular periodic orbit of the Kepler problem in rotating coordinates. If $2\pi/T_0 \in (1 - 1/\sqrt{8}, 1 + 1/\sqrt{8}) \setminus \{1/k : k = 1, 2, 3, \dots\}$, then for $\mu = \mu(T_0)$ small enough there exists a pseudo periodic solution of the planar collision restricted 3–body problem in rotating coordinates with period $T(\mu)$ such that tends to the circular periodic orbit of the Kepler problem in rotating coordinates with period T_0 when $\mu \to 0$.*