MODELING, MIGRATION, AND DEMIGRATION

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Most recently, a new process has been introduced in seismic reflection imaging being called *seismic demigration.* It has been designed as the inverse process to seismic migration and is, thus, easily confused with *seismic forward modeling.* In this paper, we want to clarify in a simple way the similarities and differences of modeling and demigration as well as how both processes are related to *seismic migration.* All three processes cannot be completely understood without talking about "true amplitudes," i.e., the correct treatment of geometrical-spreading effects. These topics are without question of central importance to the understanding of the modern seismic reflection method and to correctly interpret, for instance, the many depth-migrated sections that are shown in this journal. Let us start by commenting on the often-heard statement that "seismic forward modeling is the inverse of seismic migration." We do not agree with this. In our opinion, seismic demigration deserves much more to be called the "inverse of seismic migration," and forward modeling is, in turn, the "inverse of migration/inversion." However, there exists a close relationship between seismic modeling and demigration, which needs to be understood and which we shortly want to elaborate. In fact this latter relationship provides the foundation for a new seismic modeling method that we call "seismic modeling by demigration." With this method, one obtains in our opinion for the first time a simple geometrical understanding of the role of a Fresnel zone in seismic forward modeling. It is recognized as the region on a reflector that contributes to an observed primary reflection. Before explaining seismic modeling by demigration, let us shortly review what is commonly understood by seismic modeling, migration, migration/inversion, and demigration as true-amplitude imaging processes.

Seismic Modeling

Let us consider the dome-like reflector in Figure 1a which separates two homogeneous, acoustic media in which the propagation velocities are $v_1 = 2500$ m/s above the interface and $v_2 = 3000$ m/s below it, and the densities are $\rho_1 = \rho_2 = 1$ g/cm^3. In Figure 1a, we also see a certain shot location $S = (x_S, 0)$ and the corresponding receiver location $G = (x_G, 0)$ on the *seismic line* which coincides with the x-axis. Both points are separated by the offset $2h = x_G - x_S = 500$ m. They have the midpoint P with coordinate $x_m = (x_G + x_S)/2$. The simulation of seismograms for an Earth model like in Figure 1 is called *seismic modeling.* We restrict this simulation here to the computation of primary reflections only, because these are the key events in seismic modeling, imaging, inversion, and interpretation.

One of the most popular seismic modeling techniques, providing the primary reflection from the dome-like reflector at G for a point source at S, is the *seismic ray method* described in many books and papers. It requires the construction of the ray that connects S and G being reflected at the interface. By dislocating the shot S and the receiver G such that the offset remains fixed but each new midpoint P coincides with an equidistant sequence of points on the seismic line, we can construct all reflected rays of Figure 1b as well as the *common-offset section* of Figure 2a, where the seismic traces, showing the reflections, are plotted at the midpoint P of each respective shot-receiver pair. This section is quite inaccurate in the encircled regions, where the socalled caustic events are not well described by zero-order ray theory. If we had applied the often-used *Kirchhoff integral technique* to perform the the seismic forward modeling, we would have obtained the common-offset section of Figure 2b. Here, the modeling of the caustic events yields more realistic results. However, we see some spurious events in the encircled regions of Figure 2b. Obviously, we have obtained by the two popular seismic forward modeling techniques, ray theory and the Kirchhoff integral, inaccurate synthetic seismograms in the enclosed regions of Figures 2a and 2b. These would certainly lead to seismic misinterpretations in cases where those synthetic seismogram sections would be compared with field data. Such severe inaccuracies are however not observed in Figure 2c, which was constructed by the new modeling by demigration method of Santos et al. (*Modeling by demigration, Extended Abstracts, SEG's 67th Annual International Meeting, 1997*) explained below. Rather than including in the comparison in Figure 2 results of other commonly used seismic forward modeling techniques, let us quickly review the seismic reflection imaging processes of migration and demigration to better understand their relationship to seismic forward modeling.

Observe that in Figure 1a we have also plotted three *isochrones* for the points N_1, N_2, and N_3 indicated in the common-offset section of Figure 2c. These isochrones, one of which touches the reflector (see Figure 1a), are of fundamental importance in the seismic imaging processes to be explained below.

Seismic Migration

Seismic migration, as recently well explained from a new perspective in various issues of this journal by Norman Neidell (*Acquisition/Processing – Perceptions in Seismic Imaging, Parts I-IV, TLE, 1997*), involves the construction of " reflector images" from a seismic record (e.g., a common-offset section as the ones depicted in Figure 2). For the construction of the reflector image of the dome in Figure 1a, one needs a macrovelocity model to be given. The migrated reflector image of the dome-like reflector is, however, not the "sharp interface" in Figure 1a along which the physical medium parameters change discontinuously. It is rather given by a "string of wavelets" or "strip of wavelets" attached to the sharp reflector interface at depth $z = \Sigma(x)$ using the given macrovelocity field. The reflector image of the dome-like reflector is shown in Figure 3. The strip of wavelets along the reflector is of varying thickness, i.e., each wavelet attached to the reflector is stretched differently.

Let us now quickly review the Kirchhoff migration procedure, by which the reflector image in Figure 3 could, in principle, be obtained from any of the common-offset sections of Figure 2, e.g., from that of Figure 2c. The procedure can be conceived easily as follows. Assume a dense rectangular grid of points M in the (x, z)-space of Figure 3, in which we want to construct the depth-migrated reflector image from the given common-offset section in Figure 2c. Let us also assume that within the (x, z)-space of Figure 3 the macrovelocity model consists of a homogeneous medium velocity of $v_1 = 2500\ m/s$. Moreover, let all depth points M on the grid be treated like "diffraction points" in this macrovelocity model. Three of the many depth points, M_1, M_2, and M_3, are shown in Figure 3. Their diffraction-traveltime trajectories – which would pertain to actual diffraction responses if the points M would be actual diffraction points in the medium – are included in the simulated common-offset section of Figure 2c. Now, a Kirchhoff depth migration involves in principle nothing else but performing a summation (or stack) of all the seismic trace amplitudes encountered along each diffraction-traveltime trajectory (which is also called diffraction-stack curve) in the common-offset section, and placing the summation value into the corresponding point M. In this way one obtains what is commonly called the "common-offset depth-migrated section." This does not reveal the sharp discontinuity of the dome-like reflector in Figure 1 but only its reflector image (see Figure 3) in which the primary-reflection signals remain present in a spatial form.

The above Kirchhoff migration procedure could be refined if the reflector image (i.e., each wavelet in the strip) would be expected to also provide at any point along the sharp reflector some *quantitative information* about the reflector such as its correct location in depth, its angle-dependent reflection coefficient, or its velocity and density contrasts, etc. To know these quantities – which in reality would laterally vary above and below the reflector at any reflector point – is of utmost importance for an amplitude-versus-offset (AVO) or amplitude-versus-angle (AVA) analysis with which one aims at ultimately constructing or recovering the exact physical Earth model. The indicated refinement that provides reflector images that can be used for this purpose is commonly referred to as *true-amplitude migration*. It requires, e.g., sources and receivers with identical characteristics along the seismic line. Then the simple kinematic Kirchhoff migration described above can be replaced by a so-called weighted "true-amplitude Kirchhoff migration." In the latter, more sophisticated migration procedure, the seismic trace amplitudes encountered along each "diffraction-stack curve" have to be weighted in the stack with a specific *true-amplitude weight* that varies along the stacking curve.

The theory of true-amplitude Kirchhoff migration can be found in Schleicher et al. (*3-D true-amplitude finite-offset migration, Geophysics, 1993*). Figure 3 provides in fact the true-amplitude reflector image as if obtained by a true-amplitude Kirchhoff migration of the common-offset section of Figure 2c. The peak amplitudes of the spatial wavelets in the migrated reflector image are proportional to the local reflection coefficient. The fact that this coefficient depends on the varying reflection angle of the rays (see Figure 1b) connecting the sources to the receivers explains why the peak amplitudes change along the true-amplitude reflector image. Note as well that the spatial primary-reflection pulses in the reflector image of Figure 3 do not have the same vertical length at all points on the reflector. They are in fact vertically stretched by a certain factor described by Tygel et al. (*Pulse distortion in depth migration, Geophysics, 1994*). This factor does not depend on the true-amplitude weights, but it depends on the reflector dip, the specular reflection angle, and the velocity at the reflector point.

We observe that the end product of seismic true-amplitude migration, as given by the reflector image in Figure 3, is not identical to the physical earth model depicted in Figure 1a. To determine the physical parameters of the Earth model from a true-amplitude migrated reflector image, one needs an additional process, usually called inversion. It involves and AVO or AVA analysis and is commonly applied in a chain with true-amplitude migration. The complete chain is then referred to as migration/inversion. We observe that only this two-step chain is justified to be called "the inverse to seismic forward modeling."

With this observation, we are left with the obvious question: What is, then, the actual inverse of seismic migration? Well, the answer is that this is the recently introduced process of seismic demigration. We will comment on this process in more detail in the next section.

Seismic Demigration

Let us now address the subject of seismic demigration, which is given increasing attention in seismic reflection imaging and which is contributing to quite some confusion. Corresponding to a seismic true-amplitude migration, one can define its inverse, *true-amplitude demigration.* It involves nothing more than the formulation of an reflection-imaging process by which one can return from a true-amplitude depth-migrated section (e.g., the true-amplitude reflector image in Figure 3) to the original common-offset section (e.g., that of Figure 2c). In other words, (true-amplitude) demigration is designed to be the inverse of (true-amplitude) migration.

A true-amplitude Kirchhoff demigration can be performed in a completely analogous way to true-amplitude Kirchhoff migration as described above. Rather than defining a grid of points M in the (x, z)-space, we now define a dense grid of points N in the (x, t)-space in which we desire to construct (or recover) the common-offset section from the depth-migrated section in Figure 3. Each grid point N together with the offset $2h$ and the macrovelocity model defines an isochrone in the (x, z)-space. Figure 2c shows three of the grid points N_1, N_2, and N_3, the isochrones of which are shown in Figure 1a and Figure 3. In the same way as a diffraction-traveltime stacking trajectory can be viewed as the "time response in the common-offset section of a diffraction point M in depth, i.e., in the migrated section," an isochrone can be viewed as the "depth response in the migrated section of a point N in time, i.e., in the common-offset section." In other words, the isochrone determines all subsurface points for which a common-offset reflection from a possible true reflector would be recorded at N, after traveling along a primary reflected ray from S to G. This definition of the isochrone implies the following observation. If the isochrone was considered as a reflector, the spherical wave originating at the source S at the time zero would focus at the receiver G at the time defined by point N. In our example, the above isochrones are ellipses because of the constant macrovelocity field.

We recall that one can realize a Kirchhoff migration by distributing (smearing) the amplitude value found at N in the seismic time section along the corresponding isochrone. For the inverse process, demigration, we now have to perform a weighted true-amplitude *stack* along each isochrone on the depth-migrated section amplitudes of Figure 3. Then, we place the resulting stack value into the corresponding point N. As in Kirchhoff migration, the isochrone-stack demigration will only result in significant values if the point N is in the near vicinity of an actual reflection-traveltime curve. Elsewhere, it will yield negligible results. Thus, this process provides again the original (or true-amplitude) common-offset section of Figure 2c. For a more mathematical treatment of true-amplitude Kirchhoff demigration, we refer the reader to Tygel et al. (*A unified approach to 3-D seismic reflection imaging. – Part II: Theory, Geophysics, 1996*).

We hope that with the present discussion we can help to avoid any confusion between seismic modeling and demigration, recognizing that there is a similar difference as between migration and migration/inversion. From what has been said above we obviously have to clearly distinguish between seismic modeling and true-amplitude demigration, though the results of both procedures (in our example the synthetic common-offset sections of Figure 2) are the same. In seismic modeling the point of departure for the construction of the common-offset sections is the physical Earth model with the sharp reflectors (in our example represented by Figure 1a), while in true-amplitude demigration it is the migrated section with the reflector images (in our example represented by Figure 3). So, the true inverse to seismic (true-amplitude) migration is (true-amplitude) demigration, whereas seismic forward modeling is the inverse to migration/inversion.

From what has been said above, there remains one final question that needs to be answered. As true-amplitude migration plays an important role in migration/inversion (i.e., the inverse to seismic modeling), could it be that true-amplitude migration can also play part in seismic modeling (i.e., the inverse to seismic migration/inversion)? The answer is, Yes, it can, i.e., true-amplitude demigration can in deed be made part of a two-step seismic modeling procedure. This new method we will call seismic modeling by demigration. It provides an important new inside in the relationship between reflectors and observed reflections.

Seismic modeling by demigration

After having addressed the differences between seismic forward modeling and seismic demigration, let us now show that we can directly, i.e., from the given sharp reflector of the Earth model in Figure 1a together with a chosen source pulse, construct its true-amplitude reflector image in the depth-migrated section of Figure 3. After this has been done, we can in a second step perform the true-amplitude demigration, thus offering to the geophysical community a new seismic modeling method which we call modeling by demigration. This procedure reveals the close relationship between seismic forward modeling and seismic reflection imaging. It was applied, as indicated, for the construction of the common-offset section of Figure 2c directly from the Earth model of Figure 1a.

The last question to be answered is obviously on how to perform the transformation of the earth model with a sharp reflector in Figure 1a into its migrated image with the reflector image of Figure 3? The obvious answer is that we just have to place the correctly stretched and scaled source pulses at the locations of the reflector. Both the pulse stretch and its amplitude depend on the given macrovelocity model. A detailed description of how the construction can be technically realized is out of scope of this paper. It will be thoroughly described in a follow-up paper in GEOPHYSICS. It was in fact this construction technique by which we obtained the artificial depth-migrated section of the common-offset section of Figure 2c. In other words, the latter section shows the synthetic common-offset reflections from the sharp reflector in Figure 1a as obtained by what we call "modeling by demigration". It is to be remarked that the construction of the artificial migrated section (Figure 3) can be done implicitly during the demigration process. In this way, modeling by demigration becomes a one-step modeling scheme that determines the seismic time section (Figure 2c) directly from the Earth model (Figure 1a) like any other modeling scheme.

This new seismic modeling method provides an extremely useful aid to seismic interpreters, who are always keen to know what region on a reflector contributes to an observed seismic primary reflection. The new modeling technique gives an easy answer to this question. For instance, all contributions to the primary reflection at point N_2 in Figure 2c stem clearly from that particular segment of the dome-like reflector in Figure 1a, for which its corresponding reflector image in Figure 3 is transversed by the isochrone for point N_2. This isochrone is tangent to the reflector at M_2 (see also Figure 1a). The region of intersection between an isochrone and a reflector strip we can call a "time-domain Fresnel zone" on the reflector. It increases with the length of the seismic source pulse and in regions, where the reflector curvature is very similar to the isochrone curvature. The time-domain Fresnel zone just defined is not to be confused with the classical frequency-dependent Fresnel zone, with which it is, however, closely related, but which is a much more difficult concept to understand.

Conclusions

In this paper, we have carefully studied the seismic processes of forward modeling, migration, demigration, and migration/inversion. We have seen that demigration is related to seismic forward modeling in the same way as migration is related to migration/inversion. We have established two pairs of inverse seismic imaging processes, being migration and demigration on the one hand, and forward modeling and migration/inversion on the other hand. In this way, we have also shown that seismic forward modeling is not an isolated process but can be conceived as a part of the great set of seismic imaging methods.

As a result of this investigation, we have introduced a new seismic modeling technique. This we call "seismic modeling by demigration." Though we have only discussed it for a simple dome-like reflector with an homogeneous velocity overburden, we want to emphasize that the new method can handle arbitrary reflectors in 3D laterally inhomogeneous velocity models. Provided the velocity-model is smooth (as commonly assumed in seismic migration and demigration), we no longer have to insist on smooth representations of the reflecting interfaces, because we require no longer a two-point ray tracing as in the standard ray method. Moreover with the new seismic modeling technique we get seismic events in regions where, e.g., the standard ray theory would fail.

Hence, we can consider modeling by demigration as a novel, interesting and more accurate approach to improved seismic modeling. Considering seismic migration to be a highly developed imaging tool we can look upon modeling by demigration as the key to make this world available to all geophysicists, who may have seen seismic modeling as either being unrelated to seismic migration and demigration or who have simply considered it to be the inverse to seismic migration without any foundation.

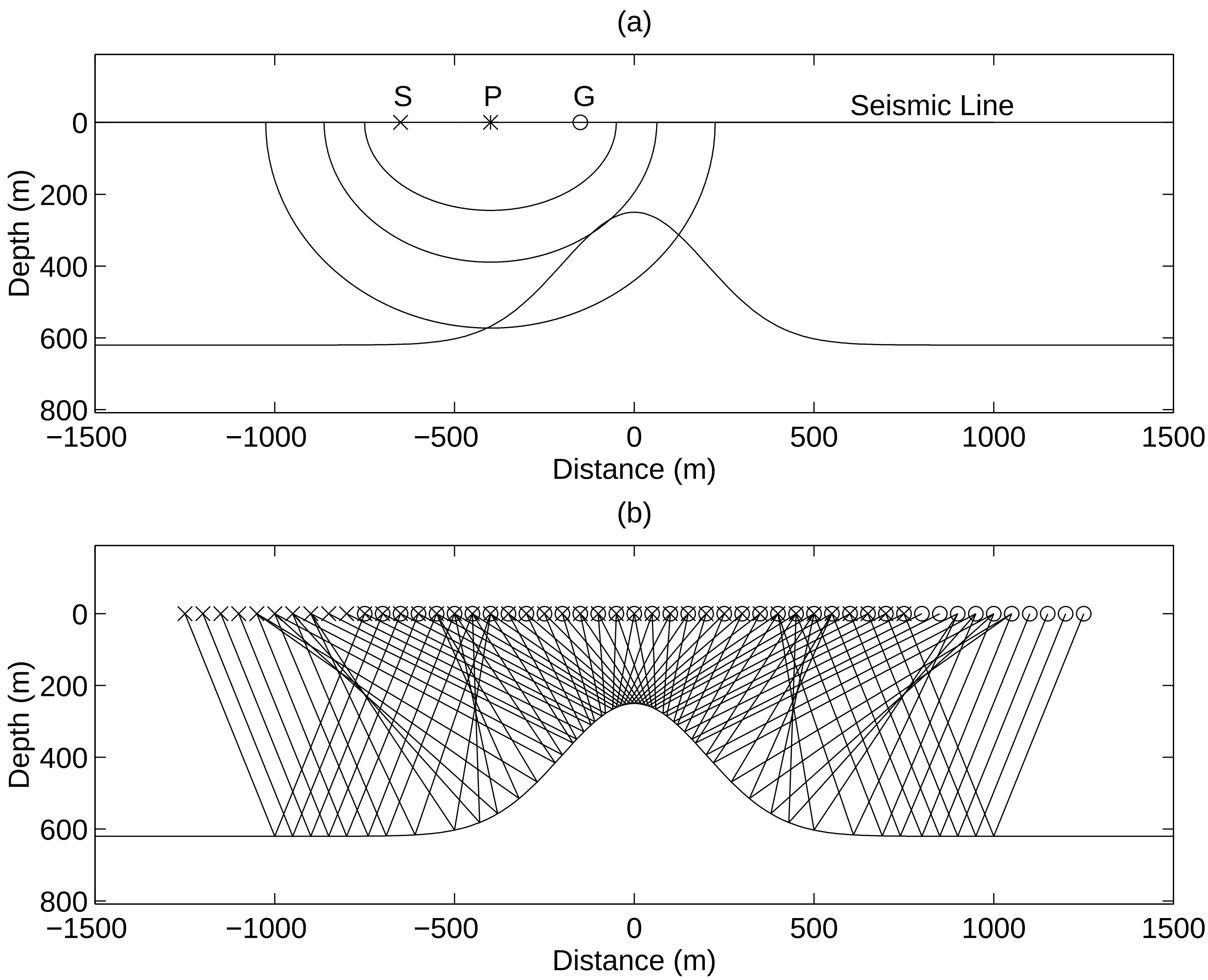

Fig. 1 (a) Dome-like reflector denoting the discontinuity between two homogeneous acoustic half spaces. Also shown is the source-receiver pair S, G and three corresponding isochrones. (b) Ray family for the primary reflections of a common-offset experiment with a half-offset of $h = 250$ m over the dom-like reflector of Figure 1a.

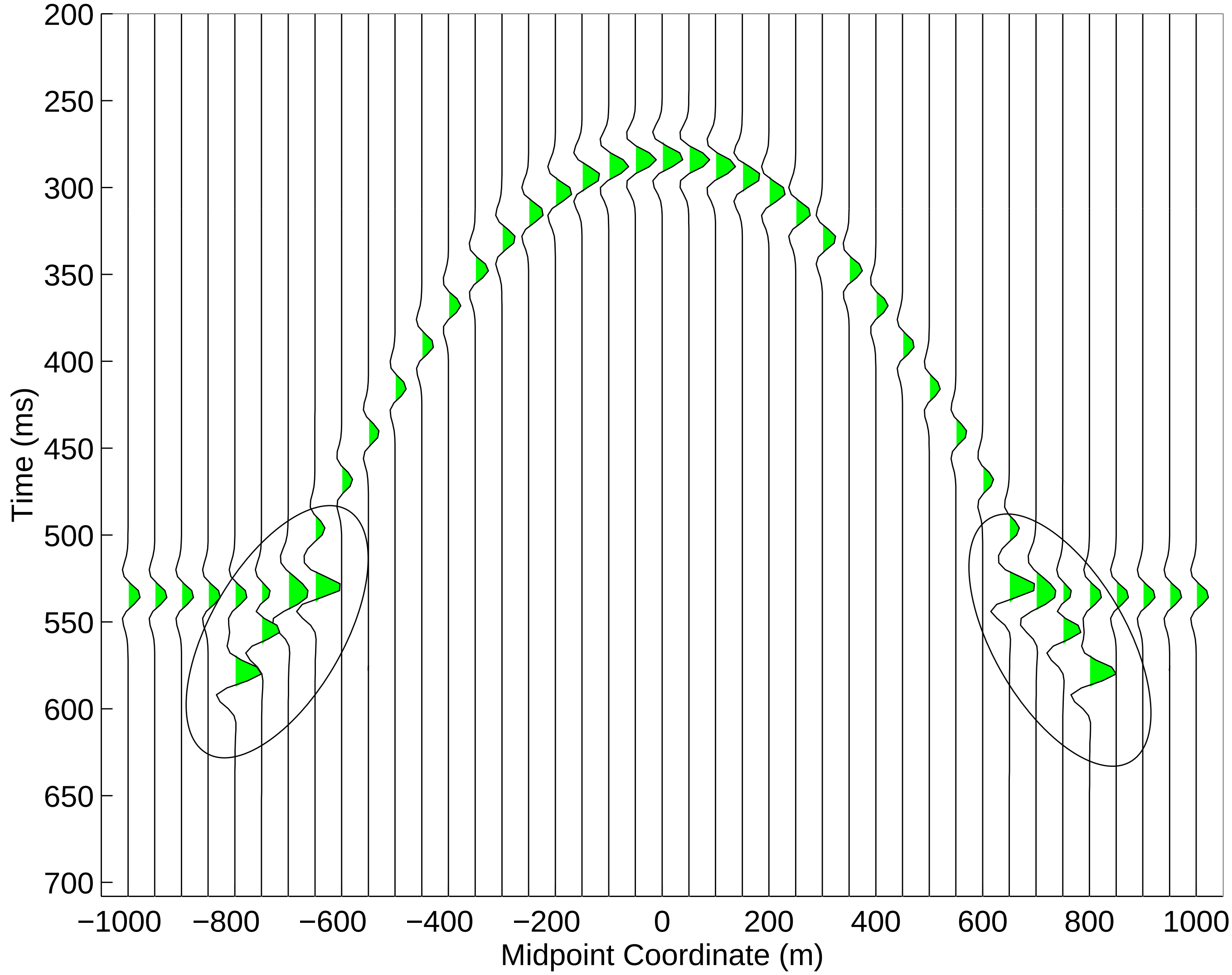

FIG. 2A: Synthetic common-offset sections showing the modeled reflections from the dome-like reflector as obtained from the standard seismic ray method.

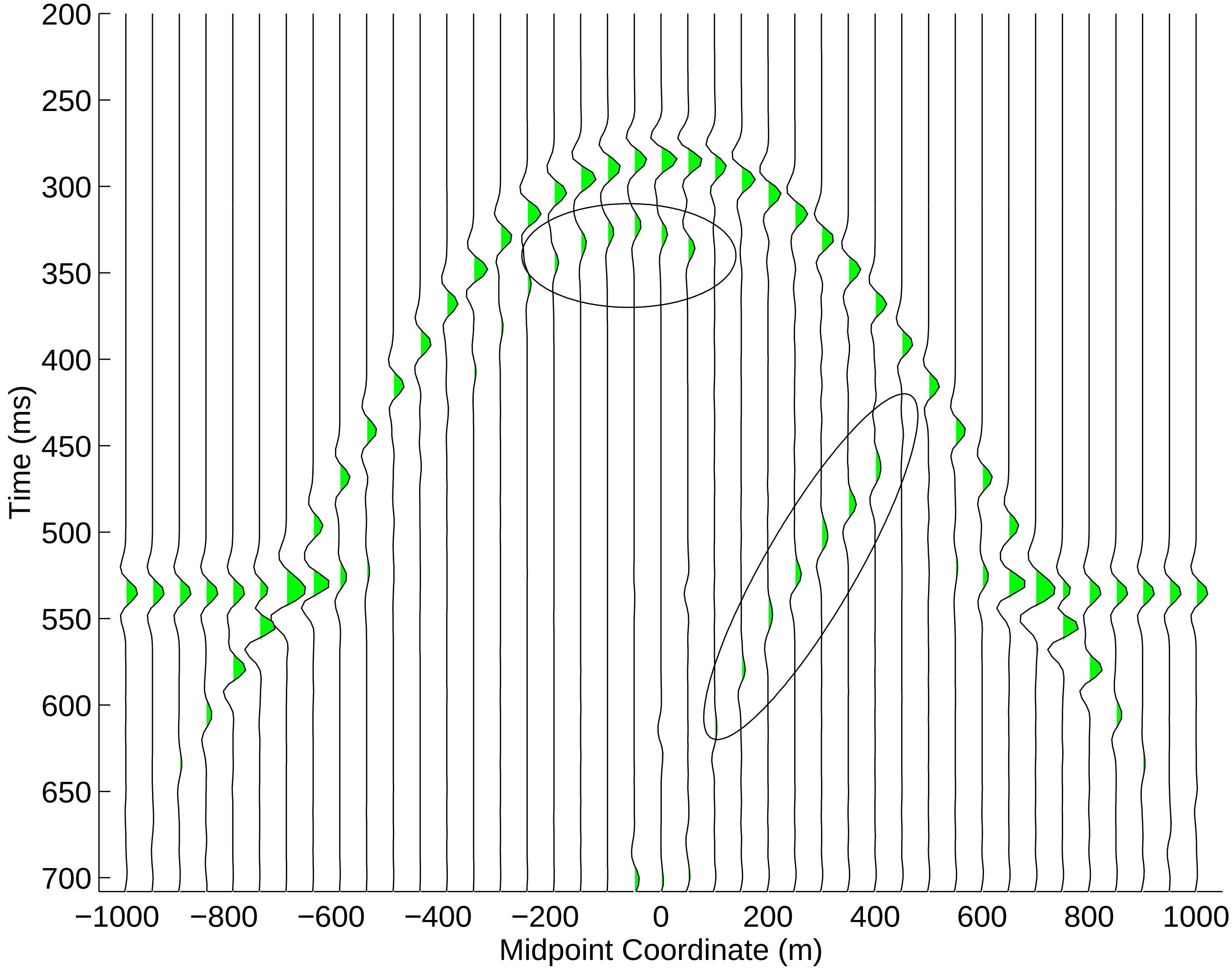

FIG. 2B: Synthetic common-offset sections showing the modeled reflections from the dome-like reflector as obtained from the Kirchhoff modeling technique.

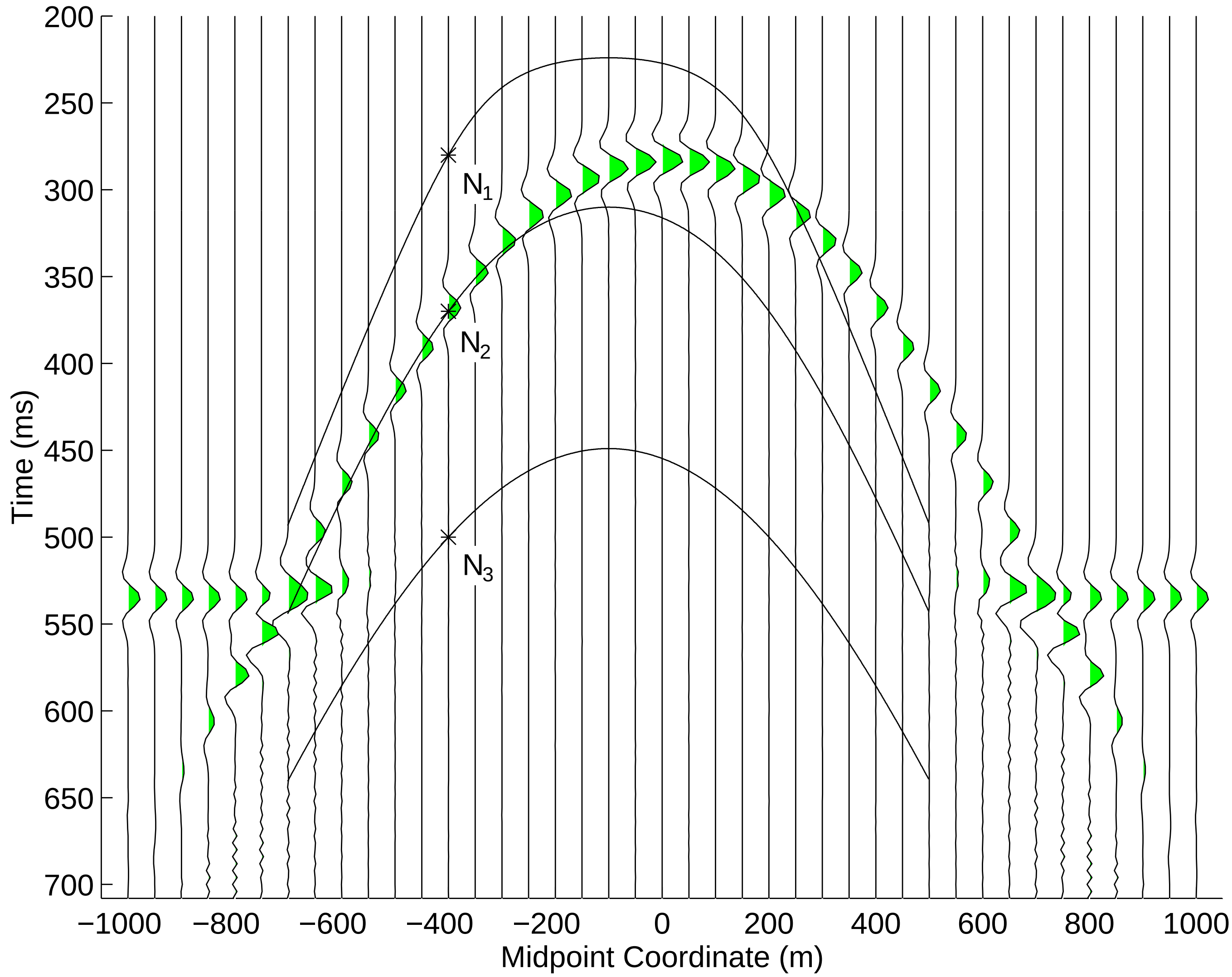

Fig. 2c: Synthetic common-offset sections showing the modeled reflections from the dome-like reflector as obtained from modeling by demigration. Also shown are three diffraction stack curves.

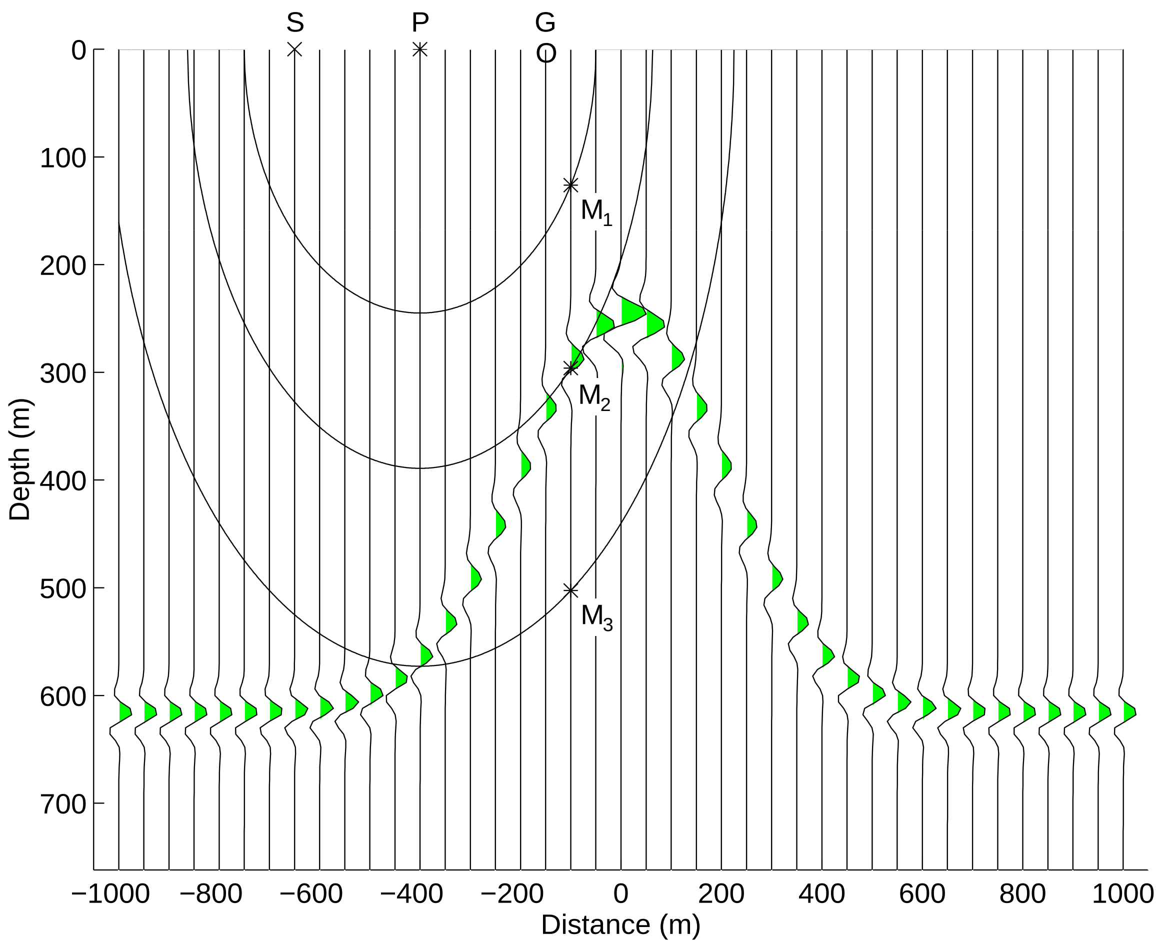

FIG. 3: Artificial true-amplitude reflector image constructed from the model in Figure 1. Also shown are three isochrones for the three points N_1, N_2, and N_3 in Figure 2c, i.e., pertaining to the indicated shot receiver pair and the traveltimes $t_1 = 0.28\ s$, $t_2 = 0.37\ s$, and $t_3 = 0.50\ s$. The three points M_1, M_2, and M_3 pertain to the diffraction hyperbolas in Figure 2c.