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ON THE GENERALIZED D'ALEMBERT WAVE EQUATION

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ABSTRACT - In this paper we study a particular case of the generalized d'Alembert wave equation with two independent variables, a spatial dimension x and a temporal dimension t, at a fixed point $(x = a, t = b)$ of the Castelnuovo spacetime, a projective representation of the de Sitter universe that generalizes Minkowski spacetime [1]. We find a d'Alembert wave equation associated to this point, and give the complete solution for the special case of an observer at the origin of spacetime coordinates.

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Abstract

In this paper we study a particular case of the generalized d'Alembert wave equation with two independent variables, a spatial dimension x and a temporal dimension t, at a fixed point $(x = a, t = b)$ of the Castelnuovo spacetime, a projective representation of the de Sitter universe that generalizes Minkowski spacetime [1]. We find a d'Alembert wave equation associated to this point, and give the complete solution for the special case of an observer at the origin of spacetime coordinates.

1 Introduction

The Fantappié-de Sitter group, a ten-parameter rotation group, contains as limiting cases the Galilean and the Poincaré groups. To this group one may associate a wave equation called generalized d'Alembert wave equation. This association gives rise to the variable aperture light cone structure of projective relativity, which has as limiting case the fixed aperture light cone structure of special relativity.

We consider the generalized bidimensional d'Alembert wave equation [2,3] for a function of degree of homogeneity zero in a de Sitter universe of radius one, and use units for which the velocity of light parameter $c = 1$. The equation is

$$(1+x^2)\frac{\partial^2 u}{\partial x^2} + 2xt\frac{\partial^2 u}{\partial x \partial t} - (1-t^2)\frac{\partial^2 u}{\partial t^2} + 2(x\frac{\partial u}{\partial x} + t\frac{\partial u}{\partial t}) = 0. \quad (1)$$

Consider a neighbourhood of the point $(x = a, t = b)$ with radius ε, with $\varepsilon << 1$, and take the limit $\varepsilon \to 0$. Then this equation becomes an equation with constant coefficients,

$$(1+a^2)\frac{\partial^2 u}{\partial x^2} + 2ab\frac{\partial^2 u}{\partial x \partial t} - (1-b^2)\frac{\partial^2 u}{\partial t^2} + 2(a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial t}) = 0, \quad (2)$$

for $u = u(x,t)$. This is an equation of telegraph type, which is associated to the propagation of a wave with a source at the point (a, b).

2 Fixed point: $P(a,b)$

Let's analyse a general Cauchy problem for equation (2), using Riemann's method [4] to obtain the solution. We begin with the generalized d'Alembert wave equation

$$(1+a^2)\frac{\partial^2 u}{\partial X^2} + 2ab\frac{\partial^2 u}{\partial X \partial T} - (1-b^2)\frac{\partial^2 u}{\partial T^2} + 2a\frac{\partial u}{\partial X} + 2b\frac{\partial u}{\partial T} = 0 \quad (3)$$

where $u = u(X,T)$, with the initial conditions

$$\begin{aligned} u(X,0) &= f(X), \\ u_T(X,0) &= g(X), \end{aligned} \quad (4)$$

where $f(X)$ and $g(X)$ are well-behaved functions.

Now, introduce in equation (3) the change of independent variables

$$\begin{aligned} X &= x + \frac{c_2 + c_1}{c_2 - c_1}t, \\ T &= \frac{2t}{c_1 - c_2}. \end{aligned} \quad (5)$$

where $c_1 = \dfrac{ab + A}{1-b^2}$, $c_2 = \dfrac{ab - A}{1-b^2}$ and $A^2 = 1 + a^2 - b^2$. The inverse transformation[1] is

$$\begin{aligned} x &= X + \frac{ab}{1-b^2}T, \\ t &= \frac{A}{1-b^2}T. \end{aligned} \quad (6)$$

We then obtain the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} + \frac{2a}{A^2}\frac{\partial u}{\partial x} + \frac{2b}{A}\frac{\partial u}{\partial t} = 0 \quad (7)$$

for $u = u(x,t)$. The initial conditions are transformed in the expressions

$$\begin{aligned} u(x,0) &= f(x), \\ u_t(x,0) &= \frac{1-b^2}{A}\left[g(x) - \frac{ab}{1-b^2}f'(x)\right], \end{aligned} \quad (8)$$

where $'$ denotes the derivative with respect to x.

Introducing in equation (7) the function

$$\Phi(x,t) = u(x,t)e^{\lambda x + \mu t}, \quad (9)$$

where $\lambda = a/A^2$ and $\mu = -b/A$, we get the equation

$$\frac{\partial^2 \Phi}{\partial x^2} - \frac{\partial^2 \Phi}{\partial t^2} + \alpha^2 \Phi = 0, \quad (10)$$

where $\alpha^2 = -\left(\dfrac{a^2}{A^4} - \dfrac{b^2}{A^2}\right)$. The initial conditions assume the form

$$\begin{aligned} \Phi(x,0) &= f(x)e^{(a/A^2)x} \equiv f_1(x), \\ \Phi_t(x,0) &= e^{(a/A^2)x}\left[\Psi(x) - \frac{b}{A}f(x)\right] \equiv g_1(x). \end{aligned} \quad (11)$$

[1] Notice that the jacobian is not null.

where $\Psi(x)$ is given by

$$\Psi(x) = \frac{1-b^2}{A} g(x) - \frac{ab}{A} f'(x). \tag{12}$$

We now define the Riemann function $v(x,t;\xi,\eta)$ for equation (10). Let Γ be a continuous smooth curve. Let ξ and η be characteristic coordinates and let's assume that the tangent to Γ isn't anywhere parallel to the ξ or η axis. Let M be a point that solves the Cauchy problem. The line MP, parallel to the x axis, intersects the curve Γ at P and the line MQ, parallel to the t axis, intersects Γ in Q. By definition, the Riemann function v must satisfy:

$$\begin{aligned} v_{\xi\xi} - v_{\eta\eta} + \alpha^2 v &= 0; \\ v &= 1 \text{ on MP}, \\ v &= 1 \text{ on MQ}. \end{aligned} \tag{13}$$

Then, the solution of equation (10) will be then given by

$$\Phi(M) = \frac{1}{2}[\Phi(P) + \Phi(Q)] + \frac{1}{2}\int_P^Q (v\Phi_\eta - \Phi v_\eta) d\xi. \tag{14}$$

Taking $v = v(s)$ where $s = \sqrt{(x-\xi)^2 - (t-\eta)^2}$ and substituting in equation (13) we have

$$\frac{\partial^2 v}{\partial s^2} + \frac{1}{s}\frac{\partial v}{\partial s} + \alpha^2 v = 0 \tag{15}$$

A regular solution at $s = 0$ for this equation is given by the Bessel function of order zero

$$v = J_0(\alpha s). \tag{16}$$

Then, writing v in terms of x, t, ξ and η we have

$$v(x,t;\xi,\eta) = J_0\left[\alpha\sqrt{(x-\xi)^2 - (t-\eta)^2}\right]. \tag{17}$$

Calculating the integral on the segment PQ, where $\eta = 0$ we have

$$\int_P^Q (v\Phi_\eta - \Phi v_\eta) d\xi = \int_{x-t}^{x+t} J_0(\alpha\Omega) g_1(\xi) d\xi - \int_{x-t}^{x+t} \frac{\alpha t J_0'(\alpha\Omega)}{\Omega} f_1(\xi) d\xi, \tag{18}$$

where $\Omega = \sqrt{(x-\xi)^2 - t^2}$. Therefore, using the initial conditions (11) we have

$$\begin{aligned} \Phi(x,t) &= \frac{1}{2}[f_1(x-t) + f_1(x+t)] + \\ &+ \frac{1}{2}\int_{x-t}^{x+t} J_0(\alpha\Omega) g_1(\xi) d\xi + \frac{\alpha t}{2}\int_{x-t}^{x+t} \frac{J_1(\alpha\Omega)}{\Omega} f_1(\xi) d\xi. \end{aligned} \tag{19}$$

Finally, using equations (9) and (6), the solution to the original problem is written as

$$\begin{aligned} u(X,T) &= \frac{1}{2}e^{(b/A)T}[f(X-T) + f(X+T)] - \\ &- \frac{1}{2}e^{(b/A)T}\int_{X-T}^{X+T}\left[\frac{b}{A}f(\xi) - \Psi(\xi)\right] J_0\left(\alpha\sqrt{(X-\xi)^2 - T^2}\right) e^{[-a(X-\xi)]/A^2} d\xi \\ &+ \frac{\alpha T}{2}e^{(b/A)T}\int_{X-T}^{X+T} \frac{J_1\left(\alpha\sqrt{(X-\xi)^2 - T^2}\right)}{\sqrt{(X-\xi)^2 - T^2}} e^{[-a(X-\xi)]/A^2} f(\xi) d\xi. \end{aligned} \tag{20}$$

This is the solution of the generalized d'Alembert wave equation in the bidimensional case for an arbitrary fixed point in the plane XT.

3 An application

In this section we reobtain the classical d'Alembert wave equation and its solution in terms of two arbitrary functions.

With this aim we take $a = b = 0$, i.e., we choose as fixed point the origin. Then $\alpha = 0$ and equation (3) takes the form

$$\frac{\partial^2 u}{\partial X^2} - \frac{\partial^2 u}{\partial T^2} = 0. \tag{21}$$

The solution is given by

$$u(X,T) = \frac{1}{2}[f(X-T) + f(X+T)] + \frac{1}{2}\int_{X-T}^{X+T} \Psi(\xi) d\xi \tag{22}$$

Equations (21) and (22) are, respectively, the classical d'Alembert equation and its solution. Moreover, we reobtain the fixed aperture light cone of special relativity.

Proceeding in this manner we may obtain the solutions to other particular cases, namely, for an observer moving on the t-axis, $P(0, b)$, an observer moving on the x-axis, $P(a, 0)$, and an observer moving on the light cone, $P(a, \pm a)$.

4 Conclusions

In studying the generalized d'Alembert wave equation associated with the Fantappié-de Sitter group with ten parameters, we verify that for a convenient limit, namely for an observer approaching the origin of spacetime, we obtain the classical d'Alembert equation, and that the same occurs with the respective solutions. We solve and discuss the d'Alembert equation for an observer on an arbitrary fixed point $P(a, b)$.

A natural sequence of this work is to study the conformal group, which has fifteen parameters and contains the Fantappié-de Sitter group as a limiting case. The conformal group is the largest group preserving a light cone structure and carries with it the accelerated motions [5].

Finally, for a convenient choice of the functions $f(x)$ and $g(x)$, the solutions given by equation (20) can be related to the so called X-wave solutions of the homogeneous wave equation, recently discovered [6].

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