RELATÓRIO DE PESQUISA 1994

ON THE PROBLEM OF CONFORMAL COUPLING IN FIELD THEORY IN CURVED SPACETIME

A.A. Grib

and

W.A. Rodrigues Jr.

Outubro RP 61/94

INSTITUTO DE MATEMÁTICA
ESTATÍSTICA E CIÊNCIA DA COMPUTAÇÃO

UNICAMP

UNIVERSIDADE ESTADUAL DE CAMPINAS

ABSTRACT – In this paper we study the properties of the solutions of the minimal and conformal coupled scalar fields in curved spacetime. We show that, contrary to claims published in the literature, anomalous R-forces between two "scalar charged" particles don't exist for the conformal coupled scalar fields.

Even more, it is the minimal coupled scalar field that has a pathological and unexpected behavior. The origin of the erroneous claim is investigated due to its great methodological meaning.

IMECC – UNICAMP
Universidade Estadual de Campinas
CP 6065
13081-970 Campinas SP
Brasil

Outubro – 1994

On the problem of conformal coupling in field theory in curved spacetime

A.A. Grib(*) and *W.A. Rodrigues Jr.*
Depart. de Matemática Aplicada - IMECC
Universidade Estadual de Campinas
CP 6065, 13081-970, Campinas, SP

Abstract

In this paper we study the properties of the solutions of the minimal and conformal coupled scalar fields in curved spacetime. We show that, contrary to claims published in the literature, anomalous R-forces between two "scalar charged" particles don't exist for the conformal coupled scalar fields.

Even more, it is the minimal coupled scalar field that has a pathological and unexpected behavior. The origin of the erroneous claim is investigated due to its great methodological meaning.

As it is well known in field theory, in curved space-time we have an ambiguity [1,2] concerning massless scalar particles. The Klein-Gordon equation in curved space-time for these particles can be written in two different forms. The first results from the principle of minimal coupling and is:

$$(\nabla_i \nabla^i)\varphi(x) = 0, \tag{1}$$

where ∇_i is the Levi-Civita connection of the metric g_{ik} which has the signature $(+,-,-,-)$ and $\varphi(x)$ is a scalar field. The second possible form results from conformal coupling and is:

$$(\nabla_i \nabla^i + \frac{R}{6})\varphi(x) = 0. \tag{2}$$

Here R is the scalar curvature of spacetime.

(*)on leave in absence: Friedmann Lab. for Theoretical Physics, St. Petersburg, Russia.

This ambiguity is preserved for massive scalar particles, for which one can write either

$$(\nabla_i\nabla^i + m^2)\varphi(x) = 0\ ,\ \text{minimal coupling} \tag{3}$$

or

$$(\nabla_i\nabla^i + \frac{R}{6} + m^2)\varphi(x) = 0\ ,\ \text{conformal coupling.} \tag{4}$$

a) One of the advantages of eq.(2) is that it is conformally invariant (invariant under Weyl scale transformations). This means that if one changes the metric g_{ik} into $\tilde{g}_{ik} = \exp[-2\sigma(x)]g_{ik}$, and the function $\varphi(x)$ into $\tilde{\varphi}(x) = \exp(\sigma(x))\varphi(x)$, then $\tilde{\varphi}(x)$ is the solution of eq.(2) in the metric $\tilde{g}_{ik}$.

Eq.(1) doesn't share the property of conformal invariance with equation (2).

It is interesting to notice here, that the Dirac equation for a massless particle (for example a neutrino) in curved space-time is conformally invariant. The same is true for Maxwell's equations in curved space-time. But there is no conformal invariance for the graviton, which is described by a massless tensor field $h_{ik}(x)$ with spin 2 [2]. It satisfies the linearized equation:

$$\nabla^j\nabla_j h_{ik}(x) + 2R_{jikl}h^{jl}(x) = 0 \tag{5}$$

with the gauge conditions

$$\nabla_k h_i^k(x) = 0, \qquad h(x) = h_i^i(x) = 0, \tag{6}$$

where R_{jikl} is the curvature tensor of the background space-time. Notations are the same as in [2].

b) The other advantage of the eq.(2), and also of the eq.(4) for the massive case, concerns the properties of quasiclassical solutions. As shown by Chernikov and Tagirov, [3] only for eq.(2) or eq.(4) we have in the quasiclassical limit particles moving along geodesics of the corresponding space-time. This occurs if R is large enough, so that it has the order $R \sim m^2$ in units $\hbar = c = 1$ and we can't neglect this term using the quasiclassical approximation.

For massive vector bosons, when we use the Proca equation in the Riemannian space-time we don't have the $R/6$ term. Instead we have

$$\nabla_i f^{ik}(x) + m^2\varphi^k(x) = 0 \tag{7}$$

where $f^{ik}(x) = \nabla_i\varphi_k - \nabla_k\varphi_i = \partial_i\varphi_k - \partial_k\varphi_i$, where φ_i is a vector field.

As shown in [4], the longitudinal component of a vector field behaves as some minimally coupled scalar massive field.

From all this it seems that in nature one can have both kinds of fields and we must investigate their different physical manifestations.

In this paper we shall discuss carefully the scalar case. The minimally coupled scalar field, as it is well known [5], plays an important role in inflation theory, being popular in cosmology. If we choose the conformal coupling we don't have the properties necessary for inflation.

Lack of conformal invariance of eq. (1) can be interpreted in the sense that this field is not really massless and has some scale, defined by the curved space-time. On the opposite, eq. (2) describes really massless particles.

This is confirmed by the structure of the Green functions $G(x, x')$ for the scalar fields for eqs. (1) and (2). Generally for an arbitrary coupling of the form ξR we have [2] for the Green function the following equation:

$$\sqrt{-g}(\nabla_i\nabla^i + m^2 + \xi R)G(x, x') = \delta(x - y'), \tag{8}$$

where $\xi = 0$ corresponds to minimal coupling and $\xi = \frac{1}{6}$ to conformal coupling. Here g is the determinant of the metrical tensor. The singularities of the Green function can be obtained by the Schwinger-DeWitt technique [6, 7] and has the following representation [1]:

$$G(x, x') \underset{\lim x \to x'}{=} \frac{\Delta^{1/2}m^2}{4\pi^2}\Big\{-\frac{1}{m^2\sigma} + L\Big(1 - \frac{m^2\sigma}{4}\Big)$$

$$-\frac{1}{2} + \frac{5}{16}m^2\sigma + \ldots - \frac{a_1(x, x')}{m^2}\Big[L\Big(1 - \frac{m^2\sigma}{2}\Big) + \frac{m^2\sigma}{2} + \ldots\Big]$$

$$+ \frac{a_2(x, x')}{m^4}\Big[\frac{1}{2} - \frac{Lm^2\sigma}{2} + \frac{m^2\sigma}{4} + \ldots\Big] + \ldots\Big\}, \tag{9}$$

where $\sigma(x,x') = \eta_{ik}(x^i - x^{i'})(x^k - x^{k'})/2]$ and $\eta_{ik} = diag(1,-1,-1,-1)$ is the metrical tensor of Minkowski spacetime:

$$\Delta(x,x') = -\det\left(\frac{\partial^2\sigma(x,x')}{\partial x'^k}\right)\cdot[g(x)g(x')]^{-1/2} \tag{10}$$
$$\Delta(x,x) = 1$$

and

$$L = \frac{1}{2}\ln[\left(\frac{1}{2}\right)m^2\sigma] + c; \tag{11}$$

where $c = 0,577,\ldots$ is the Euler constant, and

$$a_1(x,x) = a_1 = \left(\frac{1}{6}-\xi\right)R \tag{12}$$

$$a_2(x,x) = a_2 = \frac{1}{180}(R^{iklm}R_{iklm} - R^{ik}R_{ik}) - \frac{1}{6}\left(\frac{1}{5}-\xi\right)\nabla^k\nabla_k R$$
$$+\frac{1}{2}\left(\frac{1}{6}-\xi\right)^2 R^2. \tag{13}$$

Eq.(9) is written for terms up to $0(m^2\sigma)$ and $0((m\rho)^{-k})$ included, where $R_{ijkl}R^{ijkl} \sim \rho^{-k}$.

So from eq.(9) it can be seen that, when $x \to x'$ and $m \to 0$, the structure of the singularity for $\xi = \frac{1}{6}$ (when $a_1 = 0$) is the same as in flat space-time. But if $\xi = 0$ we have an additional singularity, as if we had some mass due to the curvature R of the space-time.

Nevertheless, in the literature as, e.g., in [8], we can find claims that eq.(2) leads to violation of the strong equivalence principle and to the appearence of anomalous R-term forces between two "scalar charged" particles! In this paper we shall show that the situation is quite the opposite! The argumentation of [8] is the following: write eq.(2) in a locally Lorentz coordinate system (with origin in a given point P_0) where a point source lives. We have

$$\Box\,\varphi + \frac{R}{6}\varphi = \mu_1\delta(\vec{r}). \tag{14}$$

Then we obtain a Yukawa's potential solution;

$$\varphi = -\frac{\mu_1}{r}\exp-\left[\left(\frac{r}{a\sqrt{6}}\right)\right], \tag{15}$$

where $a = R^{-\frac{1}{2}}$.

It is easy to see, nevertheless, that, if we find an exact solution of eq.(2) with the source term, we don't arrive at the Yukawa solution. To see this, take the most simple case of a conformally flat Friedmann quasieuclidean space-time with the metric:

$$ds^2 = a^2(\eta)(d\eta^2 - d\vec{l}^2) \tag{16}$$
$$d\vec{l}^2 = dx^2 + dy^2 + dz^2,$$

where η is the conformal time.

Write eq.(2) as:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}\left(\sqrt{-g}g^{ik}\frac{\partial\varphi}{\partial x^k}\right) + \frac{R}{6}\varphi = 0 \tag{17}$$

or as

$$\frac{1}{a^4}\frac{\partial}{\partial x^i}\left(a^4\frac{1}{a^2}\frac{\partial\varphi}{\partial x^i}\right) + \frac{R}{6}\varphi = 0 \tag{18}$$

Here

$$R = 6a^{-2}\left(\frac{a''}{a}\right), \qquad a'' = \frac{d^2a}{d\eta^2}. \tag{19}$$

From eq.(18) we have

$$\frac{1}{a^4}\frac{\partial}{\partial\eta}\left(a^2\frac{\partial\varphi}{\partial\eta}\right) - \frac{1}{a^2}\Delta\varphi + \frac{R}{6}\varphi = 0 \tag{20}$$

or

$$\frac{a^2}{a^4}\frac{\partial^2\varphi}{\partial\eta^2} + \frac{2}{a^3}a'\frac{\partial\varphi}{\partial\eta} - \frac{1}{a^2}\Delta\varphi + \frac{R}{6}\varphi = 0. \tag{21}$$

To find a solution of eq.(21) we make a conformal transformation of φ by

$$\varphi \longmapsto \tilde{\varphi} \quad ; \quad \varphi = \frac{1}{a}\tilde{\varphi}. \tag{22}$$

Then we get from eq.(21):

$$\frac{1}{a^2}[\frac{\partial^2\varphi}{\partial\eta^2} + \frac{2a'}{a}\frac{\partial\varphi}{\partial\eta} - \Delta\varphi + (\frac{R}{6})a^2\varphi] = 0 \tag{23}$$

or for $\tilde{\varphi}$

$$\frac{1}{a^2}[\frac{\tilde{\varphi}''}{a} - \tilde{\varphi}\frac{a''}{a^2} + \frac{R}{6}a\tilde{\varphi} - \frac{1}{a}\Delta\tilde{\varphi}] = 0 \tag{24}$$

Using R from eq.(19) we see that due to compensation of terms we get

$$\frac{1}{a^3}[\tilde{\varphi}'' - \Delta\tilde{\varphi}] = 0 \tag{25}$$

Now let us put the source on the right hand side of (2). Instead of eq.(2) we now have

$$(\nabla_i\nabla^i + \frac{R}{6})\varphi(x) = \mu\frac{\delta(\vec{x})}{\sqrt{g^{(3)}}} \tag{26}$$

Here $\delta(\vec{x})$ is the usual δ-function, but we must have $\sqrt{g^{(3)}}$ where $g^{(3)}$ is the determinant of the 3-metric, i.e., $\sqrt{g^{(3)}} = a^3$.

So, from (25) we have that eq.(26) can be written,

$$\frac{1}{a^3}[\tilde{\varphi}'' - \Delta\tilde{\varphi}] = \frac{1}{a^3}\mu\delta(\vec{x}), \tag{27}$$

and the static solution of this equation is the usual Coulomb potential, but with a conformal factor:

$$\tilde{\varphi} = -\frac{\mu}{r} \quad \text{and} \quad \varphi = -\frac{1}{a}\frac{\mu}{r}. \tag{28}$$

On the contrary, for minimal coupling instead of eq.(25) we get

$$\frac{1}{a^2}\left(\frac{\tilde{\varphi}''}{a} - \tilde{\varphi}\frac{a''}{a^2} - \frac{1}{a}\Delta\tilde{\varphi}\right) = 0 \tag{29}$$

or

$$\frac{1}{a^3}\left(\tilde{\varphi}'' - \tilde{\varphi}\frac{a''}{a} - \Delta\tilde{\varphi}\right) = 0, \tag{30}$$

so that, if there is a source, we have

$$\frac{1}{a^3}\left(\tilde{\varphi}'' - \tilde{\varphi}''\frac{a''}{a} - \Delta\tilde{\varphi}\right) = \mu\delta(\vec{x})/\sqrt{g^{(3)}} \tag{31}$$

The situation is the same as if in flat space-time we had some mass term due to $\frac{a''}{a} \neq 0$. This is the reason why, contrary to [8], it is here that we don't have the usual massless behaviour.

For a dust-like Universe we have $R \neq 0$ and $a(\eta) = a_0\eta^2$, so that the term $\frac{a''}{a} > 0$ and for $\eta \approx$ const it has properties of $m^2a^2 < 0$, i.e., of "tachyonic mass".

Now let us discuss for this case the quasiclassical behaviour. We write

$$\varphi = \frac{1}{a}\rho e^{i\frac{S}{\hbar}} \tag{32}$$

Then it is easy to see that for eq.(2) and eq.(25) we have the following equations (33) and (34), respectively, valid until the order $\hbar^{-2}$:

$$\frac{\partial S}{\partial\eta}\frac{\partial S}{\partial\eta} - \sum_{\alpha=1}^{3}\frac{\partial S}{\partial x_\alpha}\frac{\partial S}{\partial x_\alpha} = 0, \tag{33}$$

$$\frac{1}{a^2}\left\{\frac{\partial S}{\partial\eta}\frac{\partial S}{\partial\eta} - \sum_{\alpha=1}^{3}\frac{\partial S}{\partial x_\alpha}\frac{\partial S}{\partial x_\alpha}\right\} = m^2 \tag{34}$$

which is just $g^{00}\frac{\partial \delta}{\partial\eta}\frac{\partial S}{\partial\eta} - g^{\alpha\beta}\frac{\partial S}{\partial x^\alpha}\frac{\partial S}{\partial x^\beta} = m^2$.

For the minimal coupling case we obtain from eq.(30)

$$\frac{1}{a^2}\left\{\frac{\partial S}{\partial\eta}\frac{\partial S}{\partial\eta} - \sum_{\alpha=1}^{3}\frac{\partial S}{\partial x_\alpha}\frac{\partial S}{\partial x_\alpha}\right\} = -\frac{a''}{a^3} = -\frac{R}{6} \tag{35}$$

So, it is easy to see that geodesics for these particles can't live on light-cones. Even worse, they can be space like (the meaning of this for gravitons and vector mesons for large η is still to be understood!).

And now in the end of our presentation we return to the question: what is the error, when we naively (as it was done in [8]) write eq.(2) in a locally Lorentz coodinate system as

$$\Box\varphi + \frac{R}{6}\varphi = \mu_1\delta(\vec{r}), \tag{36}$$

or better

$$\Box\varphi + \frac{R}{6}\varphi = \frac{\mu_1}{\sqrt{g^{(3)}}}\delta(\vec{r}) \tag{37}$$

The answer to this question has a great metodological meaning.

The operator $\nabla_i\nabla^i$ in eq.(2) acting on the scalar field has only one Christoffel symbol, so it seems that at "the point" (i.e., in P_0, the origin of the locally Lorentz coordinate system) we can write $\Box\varphi$ as in Minkowski spacetime. But, as discussed in [9] in order to find the solution of a given differential equation at a given point of the manifold, one must look for properties of this solution in the neighborhood of this point taking the boundary conditions properly into account. Indeed, if (ξ^μ) are locally Lorentz coordinates and (x^μ) are arbitrary coordinates and if (X^μ) are the coordinates of P_0 (the origin of the locally Lorentz coordinate systen), then as is well known

$$\xi^\alpha(x) = a^\alpha + b^\alpha_\mu(x^\mu - X^\mu) + \frac{1}{2}b^\alpha_\lambda\Gamma^\lambda_{\mu\nu}(X)(x^\mu - X^\mu)(x^\nu - X^\nu),$$

with $a^\alpha = \xi^\alpha(x)|_{x=X}$; $b^\alpha_\lambda = \left.\frac{\partial\xi^\alpha}{\partial x^\lambda}\right|_{x=X}$.

Also:

$$\left.\Gamma^\alpha_{\beta\gamma,\mu}(\xi^\mu)\right|_{\xi^\mu_0} = -\frac{1}{3}\left.(R^\alpha_{\beta\gamma\mu}(\xi^\mu) + R^\alpha_{\gamma\beta\mu}(\xi^\mu))\right|_{\xi^\mu_0}.$$

Then, for $\xi^\mu \neq \xi^\mu(P_0) = \xi^\mu_0$, quantities $\Gamma^\alpha_{\beta\mu}(\xi^\mu)$ are not null. This means that, in order to get a solution for eq.(2) with a source term in the (ξ^μ) coordinates valid for all spacetime and taking into account the boundary conditions, we can't simply solve eq.(36). What is necessary is to solve a very complicated equation in these coordinates. If this is done carefully we must obtain the right result founded above.

Before ending we find useful to discuss how to solve eq.(2) in a coordinate system used by astronomers. To do this, besides the sinchronous reference frame

$$ds^2 = c^2dt^2 - a^2(t)d\vec{l}^2 = c^2dt^2 - a^2(t)[dr^2 + r^2(\sin^2\theta' d\varphi^2 + d\theta'^2)] \tag{38}$$

connected with the conformal one [(16)] by $cdt = ad\eta$, we introduce another coordinate system where the space distance is given by $D = a(t)r$, so, that

$$dD = adr + rda \tag{39}$$

and

$$ds^2 = (1 - D^2\frac{\dot{a}^2}{c^2a^2})c^2dt^2 + 2D\frac{\dot{a}}{ca}dDcdt - dD^2 - D^2(\sin^2\theta d\varphi^2 + d\theta^2). \tag{40}$$

The advantage of the reference frame associated to these coordinates is that for an observer on the Earth, when $\varepsilon = D\frac{\dot{a}}{ca}$ is small, one has a good approximation to the Minkowski metric, and only for large enough D one has curved spacetime. Really, we can use the parameter ε as a small parameter. Let us write equation ((7) in these coordinates. From:

$$\begin{cases} g_{00} = 1 - \left(\frac{D\dot{a}}{ca}\right)^2 \\ g_{01} = 2D\left(\frac{\dot{a}}{ca}\right); g_{ii} = -1; g_{\theta\theta} = -D^2, \\ g_{\varphi\varphi} = -D^2\sin^2\theta' \end{cases} \tag{41}$$

and using the relations [9]

$$\begin{cases} g^{0r}g_{r0} + g^{00}g_{00} = 1 \\ g^{r0}g_{00} + g^{rr}g_{r0} = 0 \\ 2g^{r0}g_{0r} + g^{rr}g_{rr} = 1 \end{cases}, \tag{42}$$

we obtain

$$g^{0r} = \frac{2D\frac{\dot{a}}{ca}}{1 + 7\left(D\frac{\dot{a}}{ca}\right)^2}. \tag{43}$$

Then, on the first approximation in ε we have:

$$g^{0r} = 2D\frac{\dot{a}}{ca} = g_{0r} \tag{44}$$

Putting this into eq (17), noting that terms depending on ε^2 after differentiation still will contain ε, we can put them away when $\varepsilon \to 0$. The only term which can't be put away is the one containing derivatives in D of g^{0r}. So, taking for $\sqrt{-g}$ the Minkowski value, we obtain the equation:

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right]\varphi + 2\frac{\dot{a}}{ca}\frac{\partial}{\partial t}\varphi + \frac{R}{6}\varphi = 0 \tag{45}$$

So, the nondiagonal components g_{0r} lead to an extra term in the approximation $\varepsilon \to 0$.

But in order to solve this equation we must take

$$\varphi = \frac{1}{a}\tilde{\varphi} \tag{46}$$

which, as in the previous case of conformal, time, immediately leads to a cancellation of the $\frac{R}{6}$ term, so that we end with:

$$\frac{1}{c^2 a}\ddot{\tilde{\varphi}} - \frac{1}{a}\Delta\tilde{\varphi} = 0. \tag{47}$$

Can we put away the term $2\frac{\dot{a}}{c^2 a}\frac{\partial}{\partial t}\varphi$ but retain the $\frac{R}{6}$ term? From Einstein's equations, we can see that for the usual cosmological models we must put away the term $\frac{R}{6}\varphi$ if we take $\frac{\dot{a}}{a}$, the Hubble's constant, equal to zero. At the modern epoch of evolution of the universe, we can do this and use the Minkowski metric as a very good approximation near the Earth.

But for R large enough in the early epochs one can't do this. But, then, the nondiagonal terms $g^{0r} \neq 0$ lead to the impossibility of having a unique time and to define space distance unambiguously. It follows that we can't write the usual $\delta(\vec{r})$-function for the charge distribution and eq (36) doesn't have unambiguous sense.

Acknowledgements: The authors are grateful to CNPq for research grants.

References

[1] N.O. Birrell, P.C.W. Davies. Quantum Fields in Curved space, Cambridge, Cambridge Univ. Press, 1982.

[2] A.A. Grib, S.G. Mamayev and V.M. Mostepanenko", Vacuum Quantum Effects in Strong Fields", Friedmann Laboratory Publishing, St. Petersburg, Russia (1994)

[3] N.A. Chernikov, E.A. Tagirov, Ann. Inst. H. Poincaré **9A**, 109 (1968)

[4] A.A. Grib, A. V. Nesteruk, Gen. Relat. Grav. 15, 31, (1983)

[5] A.D. Linde, Physics of Elementary Particles and Inflationary Cosmology, Moscow, Nauka (1990)

[6] J. Schwinger Phys. Rev. **93**, pp. 615 (1951)

[7] B.S. DeWitt Phys. Rep. C. **19**, pp. 296 (1975)

[8] A.P. Lightman, W.H. Press, R.H. Price, S.A. Teukolsky "Problem Book in Relativity and Gravitation, "problem 14.14, pp. 85, Princeton Univ. Press., Princeton, N. Jersey, (1975)

[9] W.A. Rodrigues Jr. "On locally inertial reference frames in General Relativity" subm. for publication (1994).

[10] L.D. Landau E.M. Lifshits, The Classical Theory of Fields, (fourth revised english edition), Pergamon Press, Oxford, (1975).

RELATÓRIOS DE PESQUISA — 1994

01/94 Stability of the Sucker Rod's Periodic Solution — *Aloisio Freiria Neves.*

02/94 A New Strategy for Solving Variational Inequalities in Bounded Polytopes — *Ana Friedlander, José Mario Martínez* and *Sandra Augusta Santos.*

03/94 Prime and Maximal Ideals in Polynomial Rings — *Miguel Ferrero.*

04/94 A New Globalization Strategy for the Resolution of Nonlinear Systems of Equations — *Ana Friedlander, Márcia A. Gomes-Ruggiero, José Mario Martínez* and *Sandra Augusta Santos.*

05/94 An Ambrosetti-Prodi Type Result for a System of Elliptic Equations via Leray-Schauder Degree — *Daniel Cordeiro de Morais Filho.*

06/94 On Attractivity of Discontinuous Systems — *Marco Antonio Teixeira.*

07/94 Weakly Elliptic Systems of Variational Inequalities: a 2×2 Model Problem with Obstacles in both Components — *D. R. Adams* and *H. J. Nussenzveig Lopes.*

08/94 A New Method for Large-Scale Box Constrained Convex Quadratic Minimization Problems — *Ana Friedlander, José Mario Martínez* and *Marcos Raydan.*

09/94 Weak - Strong Continuity of Multilinear Mappings Pelczyǹski - Pitt Theorem — *Raymundo Alencar* and *Klaus Floret.*

10/94 Removable Singularities Theorems for the Yang-Mills Functional — *Antonella Marini.*

11/94 Magneto-Micropolar Fluid Motion: Existence and Uniqueness of Strong Solution — *Marko A. Rojas-Medar.*

12/94 Absolutely Summing Analytic Operators and the Generalized Khintchine Inequality — *Mário C. Matos.*

13/94 Minimal Immersions of Surfaces Into n-Dimensional Space Forms — *Irwen Valle Guadalupe.*

14/94 Inexact - Newton Methods and the Computation of Singular Points — *Daniel N. Kozakevich, José Mario Martínez* and *Sandra Augusta Santos.*

15/94 Existence and non-existence of radial solutions for elliptic equations with critical growth in R^2 — *Djairo G. de Figueiredo* and *B. Ruf.*

16/94 Numerical Solution of the Leptonic Model — *M. F. Tome, J. M. Martínez* and *Waldyr A. Rodrigues Jr.*

17/94 On Acyclic Knots — *Ricardo N. Cruz.*

18/94 Optimal Stopping Time for a Poisson Point Process — *Nancy Lopes Garcia.*

19/94 Field Theory of the Spinning Electron: I - Internal Motions — *Giovanni Salesi* and *Erasmo Recami.*

20/94 Field Theory of the Spinning Electron: II - The New, Non-Linear Field Equations — *Erasmo Recami* and *Giovanni Salesi.*

21/94 The Weak Solutions and Reproductive Property for a System of Evolution Equations of Magnetohydrodynamic Type — *Marko A. Rojas-Medar* and *José Luiz Boldrini.*

22/94 A Fixed Point Theorem of Banach in the Fuzzy Context — *Marko A. Rojas-Medar, Heriberto Román-Flores* and *Rodney C. Bassanezi.*

23/94 Spectral Galerkin Approximations for the Equations of Magneto Hydrodynamic Type: Local in Time Error Estimates — *Marko Rojas-Medar* and *José Luiz Boldrini.*

24/94 Solving Nonsmooth Equations by Means of Quasi-Newton Methods with Globalization — *Marcia A. Gomes-Ruggiero, José Mario Martínez* and *Sandra Augusta Santos.*

25/94 Os Trabalhos de Leopoldo Nachbin (1922-1993) — *Jorge Mujica.*

26/94 On a New Class of Polynomials — *D. Gomes* and *E. Capelas de Oliveira.*

27/94 Level-Convergence and Fuzzy Integral — *H. Román-Flores, A. Flores-Franulic* and *Rodney C. Bassanezi, Marko A. Rojas-Medar.*

28/94 Estimating Leaf Area Index for Canopies using Coverage Processes — *Nancy Lopes Garcia.*

29/94 The Isoperimetric Problem in Cylindrical Spaces I - Symmetrization and General Results — *Renato H.L. Pedrosa.*

30/94 The Isoperimetric Problem in Cylindrical Spaces II - Classification Results — *Renato H.L. Pedrosa.*

31/94 Chain Control Sets for Semigroup Actions on Homogeneous Spaces — *Carlos José Braga Barros* and *Luiz A.B. San Martin.*

32/94 Linear Semigroups Acting on Stiefel Manifolds — *Luiz A.B. San Martin.*

33/94 Wave Series Expansions for Stratified Acoustic Media — *Lúcio Tunes dos Santos, Bjørn Ursin* and *Martin Tygel.*

34/94 Estudo de Fractais gerados por Iterações de Ponto Fixo — *Ricardo Caetano Azevedo Biloti* and *Lúcio Tunes dos Santos.*

35/94 On Global Controllability of Discrete-Time Control Systems — *Luiz A. B. San Martin.*

36/94 Closed Formulas for the Exponentiation of Lie Algebra Elements and Applications to Kaluza Klein Theories — *Alex I. Shimabukuro* and *Márcio A. F. Rosa.*

37/94 **On Thirring's Approach to Mach's Principle: Criticisms and Speculations on Extensions of his Original Work** — *Fabio M. Peixoto* and *Márcio A. F. Rosa.*

38/94 **The Clifford Bundle and the Dynamics of the Superparticle** — *W.A. Rodrigues, Jr., J. Vaz, Jr.* and *M. Pavšič.*

39/94 **On the Level-continuity of Fuzzy Integrals** — *H. Román-Flores, A. Flores-Franulic, R.C. Bassanezi* and *M.Rojas-Medar.*

40/94 **On the Resolution of the External Penalization Problem** — *José Mario Martínez* and *Lúcio Tunes dos Santos.*

41/94 **On the Convergence of Quasi-Newton Methods for Nonsmooth Problems** — *Vera L. R. Lopes* and *José Mario Martínez.*

42/94 **On the Phenomenology of Tachyon Radiation** — *Ron Folman* and *Erasmo Recami.*

43/94 **Nonsingular Charged Particles in a Classical Field Theory** — *Werner M. Vieira.*

44/94 **Gradient-like Flows on High Dimensional Manifolds** — *R. N. Cruz* and *K. A. de Rezende.*

45/94 **Critical Point Theorems and Applications to a Semilinear Elliptic Problem** — *E. Alves de B. e Silva.*

46/94 **On the Convergence Rate of Spectral Galerkin Approximations for a Generalized Boussinesq Model** — *Sebastián A. Lorca* and *José Luiz Boldrini.*

47/94 **Um Modelo Para a Dengue Hemorrágica com Força de Infecção Periódica** — *R.C. Bassanezi, R. Zotin* e *M.B.F. Leite.*

48/94 **Approximations by Periodic Biorthogonal Wavelets: Numerical Algorithms** — *Sônia M. Gomes, Elsa Cortina* and *Irene Moroz.*

49/94 **Incompressible Tori Transverse to Anosov Flows in 3-Manifolds** — *Sérgio R. Fenley.*

50/94 **Non-Equilibrium Waterflood in Heterogeneous Porous Media** — *Cristina Cunha, Deise Ferreira* and *Antonio C. Corréa.*

51/94 **Fixed Points Theorem and Stability Results for Fuzzy-Multivalued Mapping** — *Marko A. Rojas-Medar* and *R. C. Bassanezi .*

52/94 **Density Estimation via Hybrid Splines** — *Ronaldo Dias.*

53/94 **The Equations of a Viscous Incompressible Chemical Active Fluid II: Regularity of Solutions** — *M.A. Rojas-Medar* and *S.A. Lorca.*

54/94 **Existence of Solutions for Quasilinear Elliptic Equations** — *João Marcos Bezerra do Ó.*

55/94 **Multiple Critical Points for Asymptotically Quadratic Functionals** — *E. Alves de B. e Silva.*

56/94 **On the Differentiability of Functions in Banach Spaces** — *Marko A. Rojas-Medar* and *Heriberto Román-Flores.*

57/94 **A Model for Distributions and Correlation Structure of Multivariate Polytomous Data From a Two-Stage Cluster Sample** — *Eliana H. de F. Marques* and *Gary G. Koch.*

58/94 **Best Subsets for Extentions of Dichotomous Logistic Regression** — *Tirza Aydar* and *Eliana H. de F. Marques.*

59/94 **Weak Continuity of Multilinear Mappings on Tsirelson's Space** — *Raymundo Alencar* and *Klaus Floret.*

60/94 **Magneto-Micropolar Fluid Motion: on the Convergence Rate of the Spectral Galerkin Approximations** — *Marko A. Rojas-Medar.*