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FIXED POINTS THEOREM AND STABILITY RESULTS FOR FUZZY-MULTIVALUED MAPPING

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ABSTRACT. – In this paper we extend the fixed point theorem of Banach to the fuzzy context and we prove a stability result for the fixed points set.

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FIXED POINTS THEOREM AND STABILITY RESULTS FOR FUZZY-MULTIVALUED MAPPING

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Abstract

In this paper we extend the fixed point theorem of Banach to the fuzzy context and we prove a stability result for the fixed points set.

1. Introduction

The classic fixed point theorem of Banach establish that if (X, d) is a complete metric space and $f : X \to X$ is a contractive function, then f has a unique fixed point.

In this work we present a "Banach Theorem" type for a function $\Gamma : X \to \mathbb{F}(X)$, where $\mathbb{F}(X)$ denote the metric space of fuzzy sets. Also, we prove a result of stability of fixed points set.

We remark that the Theorem 3.5, in this paper, extend the principal result of Heilpern[1] where he supposed that X has a linear structure and the fuzzy-multivalued mapping have compact-convexes levels.

Also, we would like to say that the Theorem 3.5 was presented in[6], and that we gave the proof for completeness, since this result is fundamental to study the stability of fixed points set.

The results given in this paper wild be interesting for the study of the differential inclusions with right-hand side being fuzzy sets, and the stability of the solutions as well as study control problems where the control is a fuzzy set. These questions are being presently under investigation.

Finally, this paper is organized as follows: in Section 2 we state some preliminary results that will be useful for the rest of the paper. In Section 3 we give the fixed point theorem and in Section 4 we give a result of stability for the fixed points set.

2. Preliminaries

Let (X,d) be a metric space, we consider

$$\mathbb{C}(X) = \{A \subseteq X \mid A \quad \text{is closed, bounded an nonempty}\},$$
$$\mathbb{K}(X) = \{A \subseteq X \mid A \quad \text{is compact and nonempty}\},$$
$$N(A,r) = \{x \in X \mid d(x,a) < r \quad \text{for some} \quad a \in A\}.$$

The Hausdorff metric on $\mathbb{C}(X)$ is defined by

$$H(A,B) = \inf\{r > 0 \mid B \subseteq N(A,r) \quad \text{and} \quad A \subseteq N(B,r)\}$$

with $A, B \in \mathbb{C}(X)$.

As we know $\mathbb{C}(X)$ form a metric space with the Hausdorff metric and $\mathbb{K}(X)$ is a metric subspace of $\mathbb{C}(X)$. Now, following Nadler[3], we define the notion of set-valued contraction:

Definition. Let (X,d) and (Y,d') be two metric spaces and $f : X \to \mathbb{C}(Y)$ be a set-valued mapping (or multivalued function). The set-valued mapping f is said to be set-valued contraction if for all $x, y \in X$ there exist a constant $\lambda < 1$ such that

$$H(f(x), f(y)) \leq \lambda d(x,y).$$

Remark 2.1. A set-valued contraction f is H-continuous, i.e., if $d(x_n, x) \to 0$ as $n \to \infty$, then $H(f(x_n), f(x)) \to 0$ as $n \to \infty$.

Definition. Let $f : X \to \mathbb{C}(X)$ be a set-valued mapping. We say that $x \in X$ is a fixed point of f if $x \in f(x)$.

To prove the principal result of next section we need the following remark due to Nadler[3], p. 450.

Remark 2.2. Let $A, B \in \mathbb{C}(X)$, $a \in A$ and $\eta > 0$. Then, there exist $b \in B$ such that

$$d(a,b) \leq H(A,B) + \eta.$$

If $A, B \in \mathbb{K}(X)$, then we can take $\eta = 0$.

Now, we recall the following concepts of fuzzy theory.

Let $u : X \to [0,1]$ a fuzzy set, we denote by $L_\alpha u$ the α-level of u $(\alpha \in (0,1])$, defined by

$$L_\alpha u = \{x \in X \mid u(x) \geq \alpha\}.$$

The closure of the set $\{x \in X \mid u(x) > 0\}$ is called the support of the fuzzy set u and it's denoted by $L_0 u$.

We observe that the family $\{L_\alpha u \mid \alpha \in [0,1]\}$ satisfies the following properties

$$L_0 u \supseteq L_\alpha u \supseteq L_\beta u \qquad \text{for all} \quad 0 \leq \alpha \leq \beta$$

and

$$u = v \Leftrightarrow L_\alpha u = L_\alpha v \qquad \text{for all} \quad \alpha \in [0,1].$$

We denote by $\mathbb{F}(X)$ the following set

$$\mathbb{F}(X) = \{u : X \to [0,1] \mid L_\alpha u \in \mathbb{C}(X) \qquad \text{for all} \quad \alpha \in [0,1]\}.$$

We observe that $L_\alpha u \neq \phi$ for all $\alpha \in [0,1]$ is equivalent to $u(x) = 1$ for some $x \in X$.

Definition. Let $u, v \in \mathbb{F}(X)$ and $\alpha \in [0,1]$. We define

$$h_\alpha(u,v) = \inf\{d(x,y) \mid x \in L_\alpha u, \quad y \in L_\alpha v\};$$
$$H_\alpha(u,v) = H(L_\alpha u, L_\alpha v);$$
$$D(u,v) = \sup_{\alpha \in [0,1]} H_\alpha(u,v).$$

We observe that h_α is an increasing monotonically function in α and that D is a metric on $\mathbb{F}(X)$. Also, if X is a complete metric space, then the metric space $(\mathbb{F}(X), D)$ is also complete[5].

The space $(\mathbb{F}(X), D)$ is called a fuzzy metric space.

Moreover, we can define a partial order, $\subseteq_F$ on $\mathbb{F}(X)$ by setting

$$u \subseteq_F v \quad \Leftrightarrow \quad u(x) \leq v(x)\,, \quad \text{for all } \ x \in X$$
$$\Leftrightarrow \quad L_\alpha u \subseteq L_\alpha v\,, \quad \text{for all } \ \alpha \in [0,1].$$

Remark 2.3. It is easy to see that

$$(X, d) \hookrightarrow (\mathbb{C}(X), H) \hookrightarrow (\mathbb{F}(X), D)$$

are isometrics embeddings.

In fact, we observe that for every $A \in \mathbb{C}(X)$ we can associate the characteristic function $\chi_A : X \to \{0,1\}$, defined by $\chi_A(x) = 0$ if $x \notin A$ and $\chi_A(x) = 1$ if $x \in A$; then we have

$$D(\chi_A, \chi_B) = H(A, B).$$

In particular, if $A = \{x\}$ and $B = \{y\}$,

$$D(\chi_{\{x\}}, \chi_{\{y\}}) = H(\{x\}, \{y\}) = d(x,y).$$

To easy the notation, we will denote $\chi_{\{x\}}(x)$ by χ_x.

Definition. Let X, Y two metric spaces. A fuzzy set-valued mapping or fuzzy-multivalued mapping is an application $\Gamma : X \to \mathbb{F}(Y)$.

We say that Γ is a fuzzy set-valued contraction (D-contraction, for short) if for all $x, y \in X$ there exists a constant $\lambda < 1$ such that

$$D(\Gamma(x), \Gamma(y)) \leq \lambda d(x,y).$$

3. Fixed Point Theorem

To establish the principal result of this section, we recall the following Lemma's due to Heilpern; We observe that the compact-convex levels is not requered to prove the following results.

Lemma 3.1. Let $x \in X$ and $u \in \mathbb{F}(X)$. Then $\chi_x \subseteq_F u \Leftrightarrow h_\alpha(\chi_x, u) = 0$, $\forall\, \alpha \in [0,1]$.

Lemma 3.2. $h_\alpha(\chi_x, u) \leq d(x,y) + h_\alpha(\chi_y, u)$ for any $x, y \in X$ and $u \in \mathbb{F}(X)$.

Lemma 3.3. If $\chi_x \subseteq_F u$, then

$$h_\alpha(\chi_x, v) \leq H_\alpha(u,v) \quad \forall\, \alpha \in [0,1], \quad \forall\, v \in \mathbb{F}(X).$$

Definition. Let $\Gamma : X \to \mathbb{F}(X)$ a fuzzy-multivalued mapping, then $x^* \in X$ is a fixed point of Γ if $\chi_{x^*} \subseteq_F \Gamma(x^*)$.

Remark 3.4. The above definiton generalizes the correspondent definition of multivalued mapping.

Now, we can give the fixed point theorem for the fuzzy multivalued case.

Theorem 3.5. Let (X,d) a complete metric space and $\Gamma : X \to \mathbb{F}(X)$ a D-contraction. Then, Γ has fixed points.

Proof. Let $x_0 \in X$ and we choose $x_1 \in L_1\Gamma(x_0)$. This implies that $\mathcal{X}_{x_1} \subseteq_F \Gamma(x_0)$. By using the Remark 2.2, we can to find $x_2 \in L_1\Gamma(x_1)$ such that

$$\begin{aligned} d(x_1,x_2) &\leq H_1(\Gamma(x_0),\Gamma(x_1)) + \lambda \\ &\leq D(\Gamma(x_0),\Gamma(x_1)) + \lambda \end{aligned}$$

therefore $\mathcal{X}_{x_2} \subseteq_F \Gamma(x_1)$ and $d(x_1,x_2) \leq \lambda d(x_0,x_1) + \lambda$.

Analogously, we can find $x_3 \in X$ such that $\mathcal{X}_{x_3} \subseteq_F \Gamma(x_2)$ and

$$\begin{aligned} d(x_2,x_3) &\leq H_1(\Gamma(x_2),\Gamma(x_1)) + \lambda^2 \\ &\leq \lambda d(x_2,x_1) + \lambda^2 \\ &\leq \lambda^2 d(x_0,x_1) + 2\lambda^2. \end{aligned}$$

We can continue this process to obtain a sequence $\{x_k\}$ of points in X such that

$$\mathcal{X}_{x_k} \subseteq_F \Gamma(x_{k+1})$$

and

$$d(x_k,x_{k+1}) \leq \lambda^k d(x_0,x_1) + k\lambda^k.$$

Now, exactly as in the proof of the Nadler[1] Theorem for contractive set-valued function, we prove that $\{x_k\}$ is a Cauchy-sequence in X and, since X is a complete metric space, there exists $x^* \in X$ such that $d(x_k,x^*) \to 0$ as $k \to \infty$.

Let us now show that x^* is a fixed point of Γ.

Indeed, we have

$$\begin{aligned} h_\alpha(\mathcal{X}_{x^*},\Gamma(x^*)) &\leq d(x^*,x_k) + h_\alpha(\mathcal{X}_{x_k},\Gamma(x^*)) \quad \text{(by Lemma 3.2)} \\ &\leq d(x^*,x_k) + H_\alpha(\Gamma(x_{k-1}),\Gamma(x^*)) \quad \text{(by Lemma 3.3)} \\ &\leq d(x^*,x_k) + D(\Gamma(x_{k-1}),\Gamma(x^*)) \\ &\leq d(x^*,x_k) + \lambda d(x_{k-1},x^*) \to 0 \quad \text{as} \quad k \to \infty. \end{aligned}$$

Thus, $h_\alpha(\mathcal{X}_{x^*},\Gamma(x^*)) = 0, \quad \forall\alpha \in [0,1]$, and the Lemma 3.1 implies that $\mathcal{X}_{x^*} \subseteq_F \Gamma(x^*)$.

Example 1. Let $X = [0,1], 0 < \lambda < 1$ and we consider $\Gamma : X \to \mathbb{F}(X)$ given by

$$\begin{aligned} \Gamma(t)(x) &= \frac{x}{\lambda t} \text{ if } 0 \leq x \leq \lambda t,\ t \neq 0 \\ &= 1 \text{ if } \lambda t < x \leq 1,\ t \neq 0 \end{aligned}$$

and $\Gamma(0) = \mathcal{X}_{[0,1]}$

Then, $L_\alpha\Gamma(t) = [\alpha\lambda t, 1]$ for all α, t. Consequentely,

$$\begin{aligned} D(\Gamma(t_1),\Gamma(t_2)) &= \sup_\alpha H(L_\alpha\Gamma(t_1), L_\alpha\Gamma(t_2)) \\ &= \sup_\alpha H([\alpha\lambda t_1, 1], [\alpha\lambda t_2, 1]) \\ &= \sup_\alpha \alpha\lambda|t_1 - t_2| \\ &= \lambda|t_1 - t_2|. \end{aligned}$$

Therefore, Γ is D-contraction with constant λ. We observe that all t is a fixed point of Γ, since

$$\lambda t < t \leq 1 \Rightarrow \Gamma(t)(t) = 1 \quad \text{for all} \quad t.$$

Exemple 2. Let $X = [0,1], 0 < \lambda < 1$ and we consider $\Gamma : X \to \mathbb{F}(X)$ given by

$$\begin{aligned} \Gamma(t)(x) &= \frac{x}{\lambda t} \quad \text{if} \quad 0 \leq x \leq \lambda t, t \neq 0 \\ &= 0 \quad \text{if} \quad \lambda t < x \leq 1, t \neq 0 \\ \textit{and} \quad & \Gamma(0) = \mathcal{X}_{\{0\}} \end{aligned}$$

Then,

$$L_\alpha\Gamma(t) = [\alpha\lambda t, \lambda t] \quad \text{for all} \quad \alpha, t.$$

Consequentely,

$$\begin{aligned} D(\Gamma(t_1),\Gamma(t_2)) &= \sup_\alpha H(L_\alpha\Gamma(t_1), L_\alpha\Gamma(t_2)) \\ &= H([\alpha\lambda t_1, \lambda t_1], [\alpha\lambda t_2, \lambda t_2]) \\ &= \lambda|t_1 - t_2|. \end{aligned}$$

So, Γ is a D-contraction with constant λ. We observe that if $t > 0$ and $x = t$ then

$$\lambda t < t \Rightarrow \Gamma(t)(t) = 0 \quad \text{for all} \quad t > 0.$$

On the other hand $\Gamma(0)(0) = \mathcal{X}_{\{0\}}(0) = 1$. Therefore, $t = 0$ is a unique fixed point of Γ.

4. The Stability of Fixed Points Set

In this Section we define the fixed points fuzzy set associated with a fuzzy-multivalued mapping. We study some of their properties and we will prove a theorem of stability.

Proposition 4.1 - Let $\Gamma : X \to \mathbb{F}(X)$ a D-contraction with constant λ. Then, for each $\alpha \in [0,1]$, the multifunction $\Gamma_\alpha : X \to \mathcal{C}(X)$ given by $\Gamma_\alpha(x) = L_\alpha\Gamma(x)$ is also a H-contraction with constant λ.

Proof. We have

$$\begin{aligned} H(\Gamma_\alpha(x_1),\ \Gamma_\alpha(x_2)) &= H(L_\alpha\Gamma(x_1), L_\alpha\Gamma(x_2)) \\ &= \sup_\alpha H(L_\alpha\Gamma(x_2), L_\alpha\Gamma(x_1)) \\ &= D(\Gamma(x_1), \Gamma(x_2)) \\ &\le \lambda d(x_1, x_2)\ . \end{aligned}$$

If we denote by $S_\alpha = \{$ fixed points of $\Gamma_\alpha\}$ then the Nadler[3] Theorem 5, p. 479 implies $S_\alpha \neq \phi$ for all $\alpha \in [0,1]$. Being more explicit we have the following result:

Proposition 4.2 - Let $\Gamma : X \to \mathbb{F}(X)$ a D-contraction with constant λ. Then $S_\alpha \in \mathcal{C}(X)$ for all $\alpha \in [0,1]$.

Proof. Let $x \in \overline{S_\alpha}$, then there exists a sequence $(x_k) \subset S_\alpha$ such that $d(x_k, x) \to 0$ as $k \to \infty$.

On the other hand, $x_k \in S_\alpha$ imply $x_k \in \Gamma_\alpha(x_k)$, for all k. Since Γ_α is a H-contraction, by Proposition 4.1, we have

$$H(\Gamma_\alpha(x_k), \Gamma_\alpha(x)) \le \lambda d(x_k, x) \to 0 \text{ as } k \to \infty.$$

Consequently, $d(x_k, \Gamma_\alpha(x)) \to 0$ as $k \to \infty$.

We observe that

$$0 \le d(x, \Gamma_\alpha(x)) \le d(x, x_k) + d(x_k, \Gamma_\alpha(x)) \to 0$$

as $k \to \infty$. Since $\Gamma_\alpha(x) = L_\alpha\Gamma(x)$ is closed, we obtain $x \in \Gamma_\alpha(x)$. Thus, $x \in S_\alpha$.

Proposition 4.3 - Let $\Gamma : X \to \mathbb{F}(X)$ a D-contraction. Then there exists a unique $u \in \mathbb{F}(X)$ such that $L_\alpha u = S_\alpha$ for all $\alpha \in [0,1]$.

Proof. It is clear that

i) $\alpha \le \beta \Rightarrow S_\beta \le S_\alpha$.

Now, we prove that

ii) if $\alpha_n \uparrow \alpha$ then $S_\alpha = \bigcap_{n=1}^{\infty} S_{\alpha_n}$.

In fact, we suppose that $x_0 \in S_\alpha$ then, by using i) we have that $S_\alpha \subseteq S_{\alpha_n}$ for all n, that is $S_\alpha \subseteq \bigcap_{n=1}^{\infty} S_{\alpha_n}$.

Conversely, if $x_0 \in \bigcap_{n=1}^{\infty} S_{\alpha_n}$, then $x_0 \in \Gamma_{\alpha_n}(x_0)$ for all n. But, $\Gamma_\alpha(x_0) = \bigcap_{n=1}^{\infty} \Gamma_{\alpha_n}(x_0)$ since the level-application is left continuity. So, $x_0 \in \Gamma_\alpha(x_0)$, and, consequently, $x_0 \in S_\alpha$.

Moreover, the family (S_α) satisfies the hypothesis of Representation Theorem of Negoita-Ralescu[4] (with $S_0 = M$), therefore we conclude that there exists an unique $u \in \mathbb{F}(X)$ such that $L_\alpha u = S_\alpha$.

Definition - The fuzzy set u of the proposition 4.3 we will call the fuzzy set of fixed points associated with the fuzzy set-valued function Γ.

This definition is in agreement to the one given in the above section so as to show the following Proposition:

Proposition 4.4 - Let $\Gamma : X \to \mathbb{F}(X)$ and $u \in \mathbb{F}(x)$ such that $L_\alpha u = S_\alpha, \alpha \in [0,1]$. Then, $u(x) = (\Gamma(x))(x)$ for all $x \in X$.

Proof. Let $u(x_0) = \alpha_0$. Then $x_0 \in L_{\alpha_0} u = S_{\alpha_0}$. Hence $x_0 \in \Gamma_{\alpha_0}(x_0) = L_{\alpha_0}\Gamma(x_0)$, and consequently, $\Gamma(x_0)(x_0) \geq \alpha_0$.

If we suppose that $(\Gamma(x_0))(x_0) > \alpha_0$, then $(\Gamma(x_0))(x_0) = \alpha_0 + \varepsilon$ for some $\varepsilon > 0$. But, then $x_0 \in \Gamma_{\alpha_0+\varepsilon}(x_0)$ implies $x_0 \in S_{\alpha_0+\varepsilon} = L_{\alpha_0+\varepsilon} u$. Therefore $u(x_0) \geq \alpha_0 + \varepsilon$, which is a contradiction. So, $u(x_0) = (\Gamma(x_0))(x_0)$.

Example 3 - If we consider Γ as in the example 1, we have that the fuzzy set of fixed points associated with Γ is such that

$u(t) = \Gamma(t)(t) = 1, \ \forall t \in (0,1]$, since $\lambda t < t \leq 1$, for $t > 0$, and

$u(0) = \Gamma(0)(0) = \chi_{[0,1]}(0) = 1$, for $t = 0$.

Thus, $u = \chi_{[0,1]}$.

We can interpret this result, saying that all the elements $t \in [0,1]$ are fixed points of maximum degree and equal to 1.

Example 4 If we consider Γ as is example 2, then the fuzzy set of fixed points associated with Γ is such that

$u(t) = \Gamma(t)(t) = 0, \ \forall t \in (0,1]$, since $\lambda t < t \leq 1$, for $t > 0$, and

$u(0) = \Gamma(0)(0) = \chi_{\{0\}}(0) = 1$, for $t = 0$.

Thus, $u = \chi_{\{0\}}$.

We can interpret this fact saying that $t = 0$ is a fixed point of maximum degree.

Lemma 4.5 - Let X be a complete metric space and we consider $\Gamma_1, \Gamma_2 : X \to \mathbb{F}(X)$ two D-contractions with constant λ and u_1, u_2 the fuzzy sets of fixed points associated with Γ_1 and Γ_2, respectively. Then

$$D(u_1, u_2) \leq \frac{1}{1-\lambda} \sup_{x \in X} D(\Gamma_1(x), \Gamma_2(x))$$

Proof. By using the Lemma 1 in Lim[2], p. 436, we have that for each $\alpha \in [0,1]$,

$$\begin{aligned} H(L_\alpha u_1, L_\alpha u_2) &= H(S_{1\alpha}, S_{2\alpha}) \\ &\leq \frac{1}{1-\lambda} \sup_{x \in X} H(\Gamma_{1\alpha}(x), \Gamma_{2\alpha}(x)) \\ &= \frac{1}{1-\lambda} \sup_{x \in X} H(L_\alpha \Gamma_1(x), L_\alpha \Gamma_2(x)) \\ &\leq \frac{1}{1-\lambda} \sup_{x \in X} \sup_{\alpha} H(L_\alpha \Gamma_1(x), L_\alpha \Gamma_2(x)) \\ &= \frac{1}{1-\lambda} \sup_{x \in X} D(\Gamma_1(x), \Gamma_2(x)). \end{aligned}$$

Finally, taking the supremum on α, we obtain

$$\sup_{\alpha} H(L_\alpha u_1, L_\alpha u_2) = D(u_1, u_2)$$
$$\leq \frac{1}{1-\lambda} \sup_{x \in X} D(\Gamma_1(x), \Gamma_2(x)).$$

Now, we can establish the folowing result for the stability of the fixed points in the fuzzy context.

Theorem 4.6 - Let X a complete metric space and $\Gamma_i : X \to \mathbb{F}(X)$ a sequence of D-contractions with constant λ, for all $i \in \mathbb{N}$. If $D(\Gamma_i(x), \Gamma_0(x)) \to 0$ as $i \to \infty$, uniformly in $x \in X$, then

$$D(u_i, u_0) \to 0 \quad \text{as} \quad i \to \infty.$$

Proof. Let $\varepsilon > 0$ and choose $n \in N$ such that

$$\sup D(\Gamma_i(x), \Gamma_0(x)) < (1-\lambda)\varepsilon x \text{ for all } i \geq n.$$

Then, by the Lemma 4.5 we have that for all $i \geq n$,

$$D(u_i, u_0) \leq \frac{1}{1-\lambda} \sup_{x \in X} D(\Gamma_i(x), \Gamma_0(x)) < \varepsilon.$$

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