A COMMENT ON THE TWIN PARADOX AND THE HAFELE–KEATING EXPERIMENT

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ABSTRACT. We show that a theoretical and experimental analysis of the twin paradox and the Hafele–Keating experiment recently presented in this journal is non sequitur from the epistemological point of view and wrong according to the mathematical structure of Relativity Theory.

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From time to time, like a phoenix the twin paradox reborn in some physical journal with someone trying to show that Relativity Theory do not implies an unequal age for two twins after one of then take a tripstarting and ending at the location of his brother, or that the canonical calculated ages are wrong . It always happens that a paper of this kind generates a controversy which many fellows presenting their arguments for the unequal age solution whereas others insists on the equal age solution and /or non "canonical" calculations. Here hystory repeats itself since we are going to show that the theoretical and experimental analysis of the thin paradox recently put forth by Cornille [1] is non sequitur from the epistemological point of view, being moreover wrong within the the mathematical structure of Relativity Theory.

To begin , let us remember that the most important feature of Relativity Theory is the hypothesis that the collection of all possible happenings, i. e., all possible events constituting space-time, i. e., ST = (M, g, D) is a connected 4- dimensional oriented and time oriented Lorentzian manifold (M, g) together with the Levi- Civita connection D of g on M. The events in U M in a paricular chart of a given atlas have coordinates (x^0,x^1,x^2,x^3), x^0 is called the time-like ccordinate and the x^i , i = 1,2,3 are called the space-like coordinates. These labels according to Einstein [2] do not necessaraly have a metrical meaning, i. e., are not measured by the standard clocks and the standard rulers of the theory . The metrical of the manifold (in a coordinate basis) is

$$g = g_{\mu\nu} dx^\mu \otimes dx^\nu \qquad (1)$$

with $g_{\mu\nu} = g(\partial/\partial x^\mu, \partial/\partial x^\nu)$ being calculated, of course, for each $x \in M$ in $T_x M$, the tangent space to M at x.[The properties of the vector space $T_x M$, = $R^{1,3}$ (Minkowski vector space), and in particular the so called anti- Minkowski inequality for time- like vectors in the same class are fundamental for the understanding of clocks problem of Relativity Theory. We discussed these properties at length in [3] and the reader is adressed to this reference for details of notation and the proofs of the results we are going to use]. Here we quote the

Anti - Minkowski inequality (proposition 9 in [3]) : Let v, w $\in \tau^+ R^{1,3}$ (where τ^+ is the class of future pointing time - like vectors). Then it holds,

$$[g(v+w,v+w)]^{1/2} \geq [g(v,v)]^{1/2} + [g(w,w)]^{1/2} \qquad (2)$$

Now, tangent space magnitudes defined by the metric are related to magnitudes on the manifold in the following way :

Let I R be an open interval on the real line and $\Gamma: I \to M$ a map. We suppose that Γ is a C^0, piecewise C^1 curve in M. We denote the inclusion function $I \to R$ by u, and the

Definition 1. An observer in ST is a future - pointing time -like curve [3] $\Gamma: R \supseteq I \rightarrow M$, by $I \ni u \rightarrow \Gamma(I)$ M, and such that $g(\Gamma_* u, \Gamma_* u) = 1$.

We now introduce**:

Definition 2: (Standard Clock Postulate) - Let Γ be an observer . Then there exists **standard clocks** that "can be carried by Γ" and such that they register (in Γ) **proper** time, i.e., the inclusion parameter u of the definition of observer. Standard clocks "tic - tac" with a constant period, which means that in Γ there are a sequence of events separeted byequal intervals of proper time.

The question regarding the physical objects that realize the standard clocks of Relativity Theory is of course central to the present issue and will be discussed bellow. We shall need,

Definition 3: A reference frame in $U \subseteq M$ is a time- like vector field $Q \in TU$ such that each one of its integral lines is an observer.

Definition 4: A chart in $U \subseteq M$ of the maximal oriented atlas of M is said to be a naturally adapted coordinate system to a a reference frame Q (nacs/Q) if in the natural ccordinate basis of TU associated with the chart the space- like components of Q are null.

Old treatments of the clocks problem involve at least two reference frames $Q \in TU$ and $Q' \in TV$ each one containing a standard clock at (coordinate) rest at the origins of $\langle x^\mu \rangle$ and $\langle x'^\mu \rangle$, respectively the (nacs/Q) and (nacs/Q') .For $U \cap V \subseteq M$ where both Q and Q' are defined we have the coordinate transformations $\langle x^\mu \rangle \rightarrow \langle x'^\mu \rangle$. In particular we have $x'^0 = f(x^0, x^1, x^2, x^3)$ relating the time- like coordinate of an event $e \in U \cap V$ in Q' with the time- like and the space- like coordinates of the same event in Q. In what follows we are not using the ccordinate transformation laws to solve the clocks problem.

With the above definitions and given the Einstein synchronization procedure we can discover when a given reference frame is synchronizable,i.e., when the time- like coordinate function x^0 of the (nacs/Q) has the meaning of proper time registred by the standard clocks at (coordinate) rest in Q. All these points are discussed at length in [3] and here we quote that the condition for Q to be proper time synchronizable only if there exists $x^0: M \rightarrow R$ such that $\alpha = dx^0$, where

** in the mathematical presentation of a physical theory (viewed as the theory of a species of structure in the sense of Bourbaki, together with a physical interpretation) the non proper axioms are presented as definitions [4] distinguished vector field on I by d/du . For each $u \in I$, $\Gamma_* u$ denotes the tangent vectors at $\Gamma u \in M$; thus

$$\Gamma_* u = [\Gamma_*(d/du)](u) \in M_{\Gamma u}$$

Finally, the path- length between points $x_1 = \Gamma(a)$, $x_2 = \Gamma(b)$, $a, b \in I$, $x_1, x_2 \in M$ along the curve* $\Gamma: I \to M$ such that $g(\Gamma_* u, \Gamma_* u)$ has the same sign in all points along Γu, is the quantity

$$\int_a^b du[\,|g(\Gamma_* u, \Gamma_* u)|]^{1/2} \tag{3}$$

Observe now that taking the point $\Gamma(a)$ as a reference point we can use eq.(3) to define the function

$$s: \Gamma(I) \to R \ \text{ by } \ s(u) = \int_a^u du'[\,|g(\Gamma_* u', \Gamma_* u')|]^{1/2} \tag{4}$$

With eq.(3) we can calculate the derivative ds/du. We have

$$\frac{ds}{du} = [\,|g(\Gamma_* u, \Gamma_* u)|]^{1/2} = [\,|\,g_{\mu\nu} \frac{dx^\mu}{du}\frac{dx^\nu}{du}|]^{1/2} \tag{5}$$

From eq.(4) old textboobs on differential geometry and general relativity infers the equation,

$$(ds)^2 = g_{\mu\nu}\, dx^\mu dx^\nu \tag{6}$$

supposed to represent the square of the length of the " infinitesimal " arc determined by the coordinate displacement $x^\mu(a) \to x^\mu(a) + \frac{dx^\mu}{du}(a)\varepsilon$, where ε is an " infinitesimal " and $a \in I$.

The abusive and non carefull use of eq.(6) has produced many incorrect interpretations in Relativity Theory as illustrated ,e.g., in the odd paper [5] quoted by Cornille in support of his wrong view. For a critical reply to [5] and also [6] (also examples of the phoenix like nature of the twin paradox) see [3].

Now, given a time- like curve $\rho: R \supseteq I \to M$, any event $e \in \rho(I)$ separetes all other events in two disjoint classes , the past and the future [3]. The theory models an observer as

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* curves are classified as time - like, light - like and space - like when (for all $u \in I$) $g(\Gamma_* u, \Gamma_* u) > 0$, $g(\Gamma_* u, \Gamma_* u) = 0$, $g(\Gamma_* u, \Gamma_* u) < 0$ respectively.

$$\alpha = g(Q,) \tag{7}$$

is the 1 - form field physically equivalent to Q.

We are now theoretically prepared to analyse the clocks (or twins) problem (no paradox, of course), the Hafele - Keating experiment and some others claims done by Cornille.

Let Γ_1, $\overrightarrow{\Gamma_2}$, $\overleftarrow{\Gamma_2}$ be three future pointing time - like and staigth lines in $\mathcal{M}$ (an affine vector space) as in Fig. 1. Γ_1 and $\overrightarrow{\Gamma_2}$ has x_i as common point and Γ_1 and $\overleftarrow{\Gamma_2}$ has x_f as common point and $\overrightarrow{\Gamma_2}$ and $\overleftarrow{\Gamma_2}$ has x_m as common point .Γ_1 represents the path of a standard clock called 1 and $\Gamma_2 = \overrightarrow{\Gamma_2} + \overleftarrow{\Gamma_2}$ represents the path of a standard clock called 2 ; Now, according to Definition 2 and eq(4) the proper time registred by clock 1 between the events x_i and x_f is given $T_1 = [g(x_f - x_i, x_f - x_i)]^{1/2}$, i.e., the norm of the vector $x_f - x_i \in R^{1,3}$ The proper time registred by clock 2 is given by $T_2 = [g(x_m - x_i, x_m - x_i)]^{1/2} + [g(x_f - x_m, x_f - x_m)]^{1/2}$. According to the anti - Minkowski inequality we have

$$[g(x_f - x_i, x_f - x_i)]^{1/2} \geq [g(x_m - x_i, x_m - x_i)]^{1/2} + [g(x_f - x_m, x_f - x_m)]^{1/2} \tag{8}$$

and thus $T_1 \geq T_2$.

This result is an intrinsic consequence of the mathematical model of Relativity Theory. All observers in all reference frames in $\mathcal{M}$ must agree with the validity of the result $T_1 \geq T_2$.

We observe that path Γ_1 is a geodesic path between x_f and x_i as can be trivially proved, and then it follows that $T_1 > T_2$. We can also prove the following theorem [3] which is valid in a general ST (i. e., D does not need to be flat) :

Theorem: Among all future pointing time like cuves in ST = (M, g, D) passing through the points $x_i = \Gamma(a)$ and $x_f = \Gamma(b)$ the integral in eq.(4) is a maximum when Γ is a time - like geodesic.

To find the explicit relation betwe T_1 and T_2 we must introduce one reference frame in $\mathcal{M}$ and then give the parametric equations of Γ_1, $\overrightarrow{\Gamma_2}$, $\overleftarrow{\Gamma_2}$ in this frame.In $\mathcal{M}$ there exists infinite inertial reference frames { i } , i.e., frames such that $D\alpha_i = 0$, $\alpha_i = g(i ,)$. These frames are proper time synchronizable . Let i be an inertial frames and $\langle x^\mu \rangle$ the (nacs| i) , with x^0 , having the meaning of the proper time registred by standard clocks that are at rest in i , and synchronized à l' Einstein .

Let be,

$$\Gamma_{1*} = \frac{\partial}{\partial x^0} \circ \Gamma_1 , \quad 0 \le x^0 \le T_1$$

$$\vec{\Gamma}_{2*} = (1-\nu^2)^{-1/2} \frac{\partial}{\partial x^0} \circ \vec{\Gamma}_2 + \nu (1-\nu^2)^{-1/2} \frac{\partial}{\partial x^1} \circ \vec{\Gamma}_2 , \quad 0 \le x^0 \le T_1/2$$

$$\overleftarrow{\Gamma}_{2*} = (1-\nu^2)^{-1/2} \frac{\partial}{\partial x^0} \circ \overleftarrow{\Gamma}_2 - \nu (1-\nu^2)^{-1/2} \frac{\partial}{\partial x^1} \circ \overleftarrow{\Gamma}_2 , \quad T_1/2 \le x^0 \le T_1 \qquad (9)$$

In eq.(9) $0 \le \nu < 1$ is a positive real constant . If clocks 1 and 2 are put at the same fase at $x^0 = 0$, we get the canonical result

$$T_2 = (1-\nu^2)^{1/2} \, T_1 \qquad (10)$$

We now must invetigate if $T_1 > T_2$ when clock 1 is left at rest in the inertial frame i and clock 2 is at rest in an accelerated frame. We will distinguish two cases,

(i) Clock 2 is at rest in the accelerated frame Q and the tangent vector to its world line Γ_2 is given by

$$[1-\nu (x^0)^2] ^{-1/2} \frac{\partial}{\partial x^0} \circ \Gamma_2 + \nu (x^0)([1-\nu (x^0)^2] ^{-1/2} \frac{\partial}{\partial x^1} \circ \Gamma_2 , \quad 0 \le x^0 \le \tau$$

$$\Gamma_{2*} = [1-\nu^2]^{-1/2} \frac{\partial}{\partial x^0} \circ \Gamma_2 + \nu [1-\nu^2]^{-1/2} \frac{\partial}{\partial x^1} \circ \Gamma_2 ; \quad , \tau \le x^0 \le 2\tau \qquad (11)$$

$$[1-\nu '(x^0)^2] ^{-1/2} \frac{\partial}{\partial x^0} \circ \Gamma_2 + \nu '(x^0) [1-\nu '(x^0)^2] ^{-1/2} \frac{\partial}{\partial x^1} \circ \Gamma_2 , \quad 2\tau \le x^0 \le 3\tau$$

where $\nu (x^0)$ and $\nu '(x^0)$ are the standard velocities functions [7] of clock 2 with constant accelerations (in the $\partial/\partial x^1$ - direction) a and $-a$ respectively . If clocks 1 and 2 are put at the same fase at $x^0 = 0$, we get from eq.(11) using Definition 2 and eq.(4) again

$$T_2 = \tau[1-\nu^2]^{1/2} + \frac{2}{a} \ln\left(a\tau + \sqrt{a^2\tau^2+1}\right) < T_1 = 3\tau \qquad (12)$$

(ii) Clock 2 is at rest in the frame Q at rest relative to the inertial frame $i = \partial/\partial x^0 = \partial/\partial t$, but is rotating with constant angular velocity ω. For this problem we use polar coordinates and write the flat metric of $\mathcal{M}$ as

$$g = dt \otimes dt - dr \otimes dr - r^2 d\phi \otimes d\phi - dz \otimes dz \qquad (13)$$

and

$$Q = [1-\omega^2 r^2]^{-1/2}\partial/\partial t + \omega\,[1-\omega^2 r^2]^{-1/2}\partial/\partial\phi \tag{14}$$

defined in $M \supseteq U = \{ -\infty < t < \infty;\ 0 < r < 1/\omega\ ;\ 0 \le \phi \le 2\pi\ ;\ -\infty < z < \infty \}$. Then,

$$\alpha = g(\,Q,\,) = [1-\omega^2 r^2]^{-1/2}dt - \omega\,[1-\omega^2 r^2]^{-1/2}d\phi \tag{15}$$

A (nacs|Q) is $\langle t, r, \phi, z \rangle$, with $\phi = \phi + \omega t$. In the canonical non coordinate basis ($\partial/\partial t, \partial/\partial r, r\partial/\partial\phi, \partial/\partial z$) associated with this coordinate system we get for the rotation vector [3, 8] Ω associated to α,

$$\Omega = \frac{1}{2}\hat{g}[\, * (d\alpha \wedge \alpha\,),\,] = \omega\partial/\partial z \tag{16}$$

which shows that Q is indeed rotating with constant angular velocity ω relative to the z - axis of i . Note that in the in the non coordinate basis ($\partial/\partial t, \partial/\partial r, r\partial/\partial\phi, \partial/\partial z$) associated with the polar coordinate functions of the inertial reference frame i ,

$$\Omega = \omega\,[1-\omega^2 r^2]^{-1/2}\partial/\partial z \tag{16'}$$

Now, the tangent vector field to the world line Γ_2 of clock 2 is

$$\Gamma_{2*} = [1-\omega^2 R^2]^{-1/2}\frac{\partial}{\partial t}\circ\Gamma_2 + \omega\,[1-\omega^2 R^2]^{-1/2}\frac{\partial}{\partial\phi}\circ\Gamma_2 \tag{17}$$

If clocks 1 and 2 are put at the same fase at $x^0 = 0$, we get from eq.(17) using Definition 2 and eq.(4) that

$$T_2 = [1-\omega^2 R^2]^{1/2}\, T_1 \tag{18}$$

We now come to comments concerning Cornille's paper:

A - Cornille quote correctly that several experiments [9, 10, 11, 12, 13] done (using the Mossbauer effect) with atomic systems that follow world lines as in eq.(17) are compatible with eq. (18). From this he concludes that eq.(12) is false since it is eq.(18) that is observed experimentally. Well, since both equations are derived for the opperationally distinct motions from the same assumptions (Definition 2 plus eq.(4)) it is epistemological non sequitur to claim that only one of the equations is valid within Relativity Theory . Obviously both equations are theoretically true statements. If these statements are realized in the physical world it is a question that pure mathematics cannot say nothing - only experiments can solve the issue.

B - Cornille say that the experiment [14] shows that eq.(12) is false and is in accord with his own eq. (13) . Well, first of all his eq.(13) is non sequitur as a theoretical statement within Relativity Theory. This point is clear from the theoretical analysis we did above. Also in the experiment [14] the rest mean life - time of muons is determined in a statistical way from muons that are " quickly " stoped after they are produced and the mean life- time of moving muons are compared with the life time of the muons put to rest in the laboratory. In particular it must be said that each muon produced in an elementary particle collision born with a fixed velocity v . It is not accelerated from zero velocity to the velocity v contrary to Cornille supposition. Of course, the muon suffers accelerations due to its electric charge after they have been produced . The effect of a constant angular acceleration (equivalent to 10^{20} times the gravity acceleration) on muons has been measured in experiment [15] . The agreement between experiment and eq.(18) is not so good. Indeed, Apsel [16] found that there is a better agreement if the " proper time " of the moving muons are associated with a Finslerian metric in R^4 involving the electromagnetic potentials. This may implies that muons are not standard clocks or that Relativity is after all wrong. More experimentation is need, of couse, to have any answer concerning this point.

C - Cornille said: " Moreover, if there was a time difference after a round trip in the case of a rectilinear motion and iff this effect was attributed to a pure velocty effect as most authors think, then we will have an experiment which allows to discriminate a state of rest from a state of rectilinear uniform motion which is contradictory to the Milchelson Morley experiment which fails to measure the rectilinear uniform motion of the earth through space ".Well, besides the fact that Cornille did note said which is the experiment his talking about the fact is that just the opposite is true. More precisely we showed in a rigorous mathematical way [17] . that in $\mathcal{M}$ Relativity Theory forbids the existence of Lorentz invariant clocks, i.e., a clock that when set in motion relative to an inertial reference frame i does not lag behind relative to a series of clocks synchronized à l' Einstein in i . Indeed in [17] we showed that the existence of one such clock implies the breakdown of Lorentz invariance.

D - Eqs. (16) and (16') are presenteded only to show that Cornille's comments concerning Davis and Jennison paper [18]are indeed trivialities, having nothing to do with the problem at issue.

Concerning the Hafele - Keating [19,20] experiment it is clear that Cornille's analysis and formulas cannot be applied since they are wrong within Relativity Theory. Here, we must say that the original Hafele - Keating theoretical analysis is also a litle bit mislaeading. Indeed, to predict correctly the proper times registred by the three set of clocks in their experiment it is necessary to use the Kerr - metric instead the Scharzschild metric , write the parametric equations of the world lines of the clocks and finally use Definition 2 and eq.(4). However the final equation presented in [20] is a good approximation if we are to believe the precision of the measurements presented by Hafele and Keating. In this respect we unfortunately have to quote that Essen [21], the " builder " of the atomic clocks used in [20] says that the clocks do not have the precision in order to provide a test of Relativity Theory! For the same reason, of course, the Hafele - Keating data cannot be used to test Cornille's odd formulas.

Before ending we quote Einstein .In his autobiografical notes [22] he said:

<< A clock at rest relative to the system of inertia defines a local time. The local time of all space points taken together are the " time " which belongs to the selected system, if a means is given to " set " these clocks relative to each other . >>

<<.... The presuposition of the existence (in principle) of (ideal, viz, perfect) measuring rods and clocks is not indenpendent of each other, since a light signal, which is reflected back and forth between the ends of a rigid rod, constitutes an ideal clock, provided that the postulatede of the constancy of the light velocity in vacuum does not lead to contradictions.

<< According to the rules of connection , used in classical physics, of the spatial co-ordinates and of the time of events in the transition from one inertial frame to another the two assumptions of

(1) the constancy of the light velocity

(2) the independence of the laws (thus specially also the law of the constancy of the light velocity) of the choice of the inertial system (principle of relativity) are mutually incompatible (despite the fact that both taken separately are based on experience).

The insigth which is fundamental for the special theory of relativity is thi The assumptions (1) and (2) are compatible if relations of a new type (" Lorentz transformations ") are postulated for the conversion of co - ordinates and the time of events. With the given physical interpretation of co - ordinates and time, this is by no means a conventional step, *but implies certain hypothesis concerning the actual behaviour of measuring - rods and clocks, which can be experimentally validated or disproved* .>>(our italics)

The hypothesis concerning the behaviour of clocks is the one introduced by Definition 2 and eq.(4) as proved rigorously in [3]. The question of which real clocks are the standard clocks of Relativity Theory are not experimentally solved yet in view of the above discussion. (Dirac[23] ,for example, is of the oppinion that atomic clocks do not realize the Lorentzian metric of relativity Theory, i.e., they do not satisfy Definition 2 and eq.(4)). What is out of question is the theoretical result (presented above) for the behaviour of standard clocks in Relativity Theory. We hope that the present paper put an end in the "phoenix like " carrer of the twin paradox at least within the pages of this journal.

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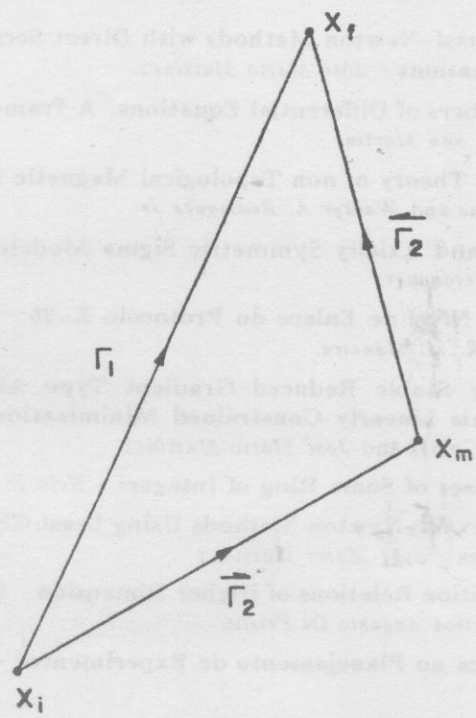

Fig. 1 — Paths of two clocks (1) and (2) synchronized of x_i and that meet again in x_f in M

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