Multiscale Numerical Methods and Their Interplay with Artificial Inteligence

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The tremendous development of massively parallel computing architectures has led to a rethinking of computational simulators, which increasingly need to incorporate asynchronous and communication-avoiding algorithms. In this context, where accuracy and robustness remain fundamental requirements while algorithms must efficiently exploit modern computing architectures, multiscale numerical methods based on a "divide-and-conquer" philosophy have emerged as an attractive alternative.

In the first part of this lecture, we present a brief historical overview of a family of multiscale methods known as the Multiscale Hybrid-Mixed (MHM) method [1,2]. We then discuss recent developments of the MHM framework, including the reconstruction of dual variables [4] and robust a posteriori error estimators. We also introduce a novel variant, the Multiscale Hybrid-Hybrid Method (MH^{2}M) [3], whose main features are illustrated through an elliptic model problem with rough coefficients.

In the final part, we explore the interplay between multiscale numerical methods and artificial intelligence. In particular, we discuss how Physics-Informed Neural Networks (PINNs) and neural operator learning approaches, such as DeepONet, can be integrated into the multiscale framework, for instance to approximate or parameterize local multiscale problems and basis functions. This perspective opens the possibility of preserving the mathematical and physical structure of multiscale methods while substantially reducing their computational cost.

References

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